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Distribution of Temperature in a Viscous Incompressible Fluid Flow using Differential Homotopy Perturbation Method

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Abstract

This paper is about the analysis of distribution of temperature in viscous incompressible fluid flow which is caused by the sheet stretched with uniform heat flux. This is a new kind of technique called differential homotopy perturbation method is developed for various solutions for the distribution of temperature and velocity. The obtained series solution for nonlinear equation is occurred by temperature field over a scattering medium and the final result is compared by exact solution so that the accuracy is by the study of distributed perturbation technique.

Keywords: *Kinematic viscosity, Similarity transforms series solution, variation iterative method, and prandtl number.*

I. INTRODUCTION

The study of heat to a fluid streaming in divert has applications in innovative fields, heat exchanger, reactor cooling and so on. Every one of these examinations are confined to hydrodynamic stream and warmth move issues, as of late these issues have turned out to be progressively essential to industry. Because of its wide scope of uses, the extending sheet issues have been considered by various specialists. Most arrangements accessible depend on numerical strategies, for example, kellor box technique, Runge-Kutta strategy and limited component technique. [1] Examined fragmented gamma capacity to ponder the conduct of temperature dissemination over an extending sheet. [2] Tackled the higher dimensional introductory limit esteem issues by variation Homotopy bother strategy. In [3] utilized variation homotopy annoyance technique for Fishers conditions. There are not many agents who have attempted to consider the progression of fluid over an extending sheet and their conduct under various conditions. He [4-9] presented the homotopy bother technique, which is created by joining the standard homotopy and irritation strategy. In these strategies the arrangement is given in a vast arrangement as a rule uniting to an exact arrangement. Because of various mechanical procedures the limit layer idea for stream of an incompressible fluid over an extending sheet is very famous among the scientists as of late. An expansive scope of investigative and numerical techniques has been utilized in the examination of these logical models. A viable technique is required to break down the scientific model which gives arrangements complying with physical reality. The greater part of the intrigue meant to the warmth move in Engineering applications is the investigation of the warm reaction of the course divider and fluid temperature and uniform warmth transition (Neumann issue). Wazwaz [10] examined the disintegration strategy for explaining higher dimensional introductory limit esteem issues of variable coefficients. He [11-17] read homotopy annoyance procedures for some sort of nonlinear issues. NureSyuhada Ismail et al [18] researched the impact of surface pressure angle and warmth move in a parallel stream with steady surface warmth transition by utilizing steadiness investigation. Shivaraman et al [19] investigated Marangoni consequences for constrained convection of intensity law fluids in dainty film over a precarious flat extending surface with warmth source. Bachok et al [20] talked about the limit layer stream and warmth move of a nano fluid over an exponentially contracting sheet.

The point of this paper is to research the speed and temperature dispersion in the progression of a gooey incompressible fluid brought about by extending sheet and contrasting and the definite arrangements.

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II. Differential Homotopy perturbation method (DHPM)

Now we express those fundamental thought of the changed differential iteration method, we look upon the given differential equation.

$$Lu + Nu = g(x) \qquad (1)$$

The L may be a linear operator; N is a nonlinear operator, and g(x) the forcing term. As stated by differential technique can make a improvement as takes after:

$$u_{n+1}(x) = u_n(x) + \int_{\Lambda}^{x} \lambda(\xi)(Lu_n(\xi) + Nu_n(\xi))d(\xi) \qquad (2)$$

The λ may be a Lagrange multiplier, which can be identifier ideally by differential iteration strategy. The subscripts n indicate that nth rough calculation; f˜ will be viewed as constrained differentiation. That is, δf˜n = 0. Now, will apply the homotopy perturbation strategy.

$$\sum_{n}^{\infty} = 0 p^n f_n = u_0(x) + p\int_0^x \lambda(\xi)(\sum_{n}^{\infty} = 0 p^{(n)} L(u_n) + \sum_{n}^{\infty} = 0 p^{(n)} N(u_n))d\xi - \int_0^x \lambda(\xi) g(\xi) d\xi,$$

$\sum_{n}^{\infty} = 0\, p^n\, f_n = u_0(x) + p \int_0^x \lambda(\xi)(\sum_{n}^{\infty} = 0p^{(n)}\, L(u_n) + \sum_{n}^{\infty} = 0p^n N(u_n)d\xi - \int_0^x \lambda(\xi) g(\xi) d\xi$,

Here the differential homotopy perturbation method (DHPM) gives the solution by paring differential iteration method and domains polynomials has a comparative study of powers (P)which give a solution for various orders.

3. Mathematical formulation of the Problem

compare the instance of a level sheet issuing from a thin opening at x = 0, y = 0, and in this manner being extended, as in a polymer preparing application. The stream brought about by the extending of this sheet is thought to be laminar. Expecting limit layer approximations, the conditions of progression, force and warmth move in the standard documentation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \;\; \text{..........} \; (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \;\; \text{..............} \; (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{v}{\sigma}\frac{\partial^2 T}{\partial y^2} \;\; \text{............} \; (3)$$

Here u and v are the speed segments in the x and y bearings separately, σ is the Prandtl number and ν is the kinematic thickness subject to the limited conditions

$$u = \alpha(x), v = 0, -\lambda\frac{\partial T}{\partial y} = A'aty = 0, u \to 0, T \to T_\infty asy \to \infty$$

……….. (4)

Define a stream function

$$\psi = -(\alpha v)^{1/2} x f(\eta), \eta = (\alpha/v)^{1/2} y \;\; \text{..........} \; (5)$$

$$u = axf'(\eta), v = -(\alpha v)^{1/2} f(n) \;\; \text{...........} \; (6)$$

Which are consistent with equations (1) and (2)

4. Problem solution is given by

Substituting equations 4, 5, 6 in 1 and 2 so to get equation 7 and 8

$$f'^2(\eta) - f''(\eta) = f'''(\eta) \;\; \text{................} \; (7)$$

$$g''(\eta) - \sigma f(\eta) g'(\eta) = 0 \;\; \text{.........} \; (8)$$

Boundary conditions for the subject is given by

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0 \;\; \text{..........} \; (9)$$

$$g'(0) = -1, g(\infty) = 0 \;\; \text{..........} \; (10)$$

From this part we are solving equations (7), (8) with boundary conditions given in equation (9) and (10) by He's variational iterative method. The initial guess for *f* &*g* is given below

$$f_0(\eta) = \eta + \frac{\alpha_1 \eta^2}{2} \;\; \text{.........} \; (11)$$

$$g(0) = 1 + \alpha_2 \eta \;\; \text{.........} \; (12)$$

Where $f''(0) = \alpha 1 < 0$ and $g'(0) = \alpha 2 < 0$. To solve (7), (8), (9) (10), with the help of Variational iterative method, we create a correctional functional which is given by

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^n \lambda_1(\xi)\left(\frac{\partial^3 f_n(\xi)}{\partial \xi^3} - (f_n^1(\xi))^2 + f_n^{'}(\xi) f_n^{''}(\xi)\right) d\xi$$

…….. (13)

$$g_{n+1}(\eta) = g_n(\eta) + \int_0^n \lambda_2(\xi)\left(\frac{\partial^2 g_n(\xi)}{\partial \xi^2} + \sigma \tilde{f}_n(\xi)\frac{\partial^2 \tilde{g}_n(\xi)}{\partial \xi}\right) d\xi$$

……… (14)

"Making the correction functional stationary, the Lagrange multipliers can easily be identified"

$$\lambda_1 = -\frac{1}{2}(\xi - \eta)^2, \lambda_2 = (\xi - \eta) \;\; \text{.........} \; (15)$$

Consequently

$$f_{n+1}(\eta) = f_n(\eta) - \frac{1}{2}\int_0^n (\xi - n)^2 \left(\frac{\partial^2 g_n(\xi)}{\partial \xi^2} - (\tilde{f}_n(\xi))^2 + \tilde{f}_n(\xi)\tilde{f}^{"}_n(\xi)\right) d\xi$$

………. (16)

$$g_{n+1}(\eta) = g_n(\eta) + \int_0^n (\xi - \eta)\left(\frac{\partial^2 \tilde{g}_n(\xi)}{\partial \xi} + \sigma \tilde{f}_n(\xi)\frac{\partial \tilde{g}_n(\xi)}{\partial \xi}\right) d\xi$$

……………. (17)

"Applying the variational homotopy perturbation method (VHPM)", we get

$$f_0 + pf_1 \text{......} = f_0(\eta) - \frac{p}{2}\int_0^n (\xi - \eta)^2 \left(\frac{\partial^3 f_0}{\partial \xi^3} + p\frac{\partial^3 f_1}{\partial \xi^3}\right) - \left(\frac{\partial^3 f_0}{\partial \xi} + p\frac{\partial^3 f_1}{\partial \xi} + \text{...........}\right)^2 + (f_0 + pf_1 + \text{......})\left(\frac{\partial^2 f_0}{\partial \xi^2} + p\frac{\partial^2 f_1}{\partial \xi^2} + \text{...........}\right)^2$$

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$$g_0 + pg_1 + \text{......} = g_0(\eta) + p\int_0^n (\xi - \eta)\left(\left(\frac{\partial^2 g_0(\xi)}{\partial \xi^2} + p\frac{\partial^2 g_1(\xi)}{\partial \xi^2} + \text{...........}\right) + \sigma(f_0(\xi) + pf_1(\xi) + \text{.....})\left(\frac{\partial g_0(\xi)}{\partial \xi} + p\frac{\partial g_1(\xi)}{\partial \xi} + \text{...........}\right)\right) d\xi$$

………. (19)

Comparing the coefficient of like powers of *p*, we get

$$p^{(0)} = f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} \quad \ldots\ldots\ldots\ldots\ (20)$$

$$p^{(1)} = f_1(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120} \quad \ldots\ldots\ (21)$$

$$p^{(2)} = f_2(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha_1\eta^4}{24} + \frac{\alpha_1^2\eta^5}{120} + \frac{\alpha_1\eta^6}{720} + \frac{\alpha_1^2\eta^7}{5040} + \frac{\alpha^3{}_1\eta^8}{40320} + \frac{\alpha_1^2\eta^9}{362880} + \frac{\alpha^3{}_1\eta^{10}}{3628800} + \frac{\alpha_1^4\eta^{11}}{39916800}$$

$\ldots\ldots\ldots\ldots$ (22)

The series solution is given by

$$f_2(\eta) = \lim_{n\to\infty} f_n \quad \ldots\ldots\ldots\ldots\ (23)$$

$$f(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha_1\eta^4}{24} + \frac{\alpha_1^2\eta^5}{120} + \frac{\alpha_1\eta^6}{720} + \frac{\alpha_1^2\eta^7}{5040} + \frac{\alpha^3{}_1\eta^8}{40320} + \frac{\alpha_1^2\eta^9}{362880} + \frac{\alpha^3{}_1\eta^{10}}{3628800} + \frac{\alpha_1^4\eta^{11}}{39916800}$$

$\ldots\ldots\ldots\ldots$ (24)

$$p^{(0)} = g_0 = 1 + \alpha_2\eta \quad \ldots\ldots\ldots\ldots\ (25)$$

$$p^{(1)} = g_1 = 1 + \alpha_2\eta - \frac{\sigma\alpha_2\eta^3}{6} - \frac{\sigma\alpha_1\alpha_2\eta^4}{24} \quad \ldots\ldots\ldots\ldots\ (26)$$

$$g(\eta) = \lim_{n\to\infty} g_n \quad \ldots\ldots\ldots\ldots\ (27)$$

$$g(\eta) = 1 + \alpha_2\eta - \frac{\sigma\alpha_2\eta^3}{6} - \frac{\sigma\alpha_1\alpha_2\eta^4}{24} - \frac{\sigma\alpha_2\eta^5}{120} + \frac{\sigma^2\alpha_2\eta^6}{240} - \frac{\sigma\alpha_1\alpha_2\eta^6}{720} - \frac{\sigma^2\alpha_1\alpha_2\eta^6}{72} + \frac{\sigma\alpha^2{}_1\alpha_2\eta^7}{504} - \frac{\sigma^2\alpha^2{}_1\alpha_2\eta^7}{5040}$$

$\ldots\ldots\ldots$ (28)

III. RESULT AND DISCUSSION

Table 1: the examination comes about to speed of the DHPM with the accurate result as shown.

Exact solution	VHPM		
η	f(η)	η	f(η)
0.1	0.0952	0.1	0.0997
0.2	0.1813	0.2	0.1993
0.3	0.2592	0.3	0.3000
0.4	0.3297	0.4	0.4026
0.5	0.3935	0.5	0.5081
0.6	0.4512	0.6	0.6175

From the table 1 it can be seen that present solution method DHPM results are better than the exact solution results. Subsequently by watching the outcomes about gotten by DHPM Also correct result technique we discovered that those arrangement result gotten by DHPM converges speedier over the correct result in the mulled over the event.

Table 2: the correlation comes about to high temperature flux of the DHPM for the correct result.

Exact solution	VHPM		
η	g(η)	η	g(η)
0.1	-0.0952	0.1	-0.9952
0.2	-0.9815	0.2	-0.9814
0.3	-0.9600	0.3	-0.9601
0.4	-0.9321	0.4	-0.9330
0.5	-0.8990	0.5	-0.9023
0.6	-0.8617	0.6	-0.8711

IV. CONCLUSION

From this above discussion, we arrive at the conclusions that the differential homotopy perturbation technique which is effectively useful for series solution of border equation for 2 dimensional flows over scattering sheet with uniform heat flux, so the result obtained is perfect solution for comparing velocity and temperature distribution. It can be concluded that the present solution is approximately very near to exact solution and is perfect by differential homotopy perturbation method (DHPM)

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