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Comparison of Precise Sagnac Effect corrections for Navigation with Indian Constellation (NAVIC) Satellites and Global Positioning System (GPS) satellites

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Abstract

Precise clock information helps Global Positioning System receivers to achieve high positional accuracy. One of the predominant error source on GPS time is Sagnac effect. An effect arising due to the daily movement of observer on earth because of its rotation. In this paper, the pattern of Sagnac corrections for each second is developed for a regional navigation system named Navigation with Indian Constellation (NAVIC). The clear variations for Geo-Stationary Earth orbit satellites (GEO's) and Geo-Synchronous orbit satellites (GSO's) in Sagnac corrections is calculated for the ephemeris received from a NAVIC receiver that is placed at low-latitude Indian region (lat:16.32° N; lon:80.44° E). The Sagnac correction for GPS satellite data recorded from a GNSS Ionospheric Scintillation and TEC Monitor (GISTM) receiver (Novatel) at KLEF, Guntur, India are also presented. The maximum Sagnac corrections for a day obtained for NAVIC and GPS satellites is also foreseen.

Keywords : Navigation with Indian Constellation, Global Positioning System, Relativistic effects, Sagnac effect.

I. INTRODUCTION

The Global Navigation satellite system has been serving the purpose of giving the positional information for both military and civilian users from the past decade. Though it was developed for military, the civilian applications are predominant. A good positioning system should even out all the errors. Though the satellites are equipped with high precise atomic clocks, for if they are thought as a flywheel; which will show an error of about a degree for 30 million revolutions [1].

According to special relativity a clock in a moving reference frame ticks slower than the one in a non-moving reference frame. This effect is considerable as the satellites (GPS) move at a speed of 3.874 km/sec. Based on General relativity the clock which is close to heavier objects ticks slower than the one far away, this is due to the gravitational field of the object. As the satellite is far away from the earth the satellite clock ticks faster than the one on the Earth. For Global Positioning System satellites with average orbital radius of 29,634 km, these two effects do not totally counterbalance each other. The net effect is a −38.58 ns/d gain for the satellite clock [2]. The atomic clocks are set at 10.22999999543 MHz to receive 10.23 MHz. The difference being the relativistic correction which includes time dilation and Einstein gravitational blue shift. The clocks on the satellites need to be synchronized with the clocks on the Earth so that they tick at same rate. Clocks in an inertial frame can be synchronized by a light pulse. The clocks need to be continuously corrected for precise positioning.

Along with those two effects the system should also account for Sagnac effect; an effect arising due to rotation of the Earth. This effect is significant and can go up to hundreds of nanoseconds. A

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nanosecond can produce an error of 30 cm [2]. Altitude has a major impact on Sagnac effect. The effect is proportional to the area swept out by the tip of a vector from the rotation axis to the signal pulse as it propagates from transmitter to receiver on the equatorial plane.

The importance of Sagnac effect on GPS time is presented by several researchers [1-2, 3-4]. This effect left uncorrected can produce an error of 30 m in the positioning [4]. Ashby et al., gave mathematical formulations that can deduce the relativistic effects on GPS navigation [1]. Caliguri et al., gave a new synchronization procedure and the role of Sagnac effect is studied [5]. Hecimovic et al., gave detail analysis on relativistic effects on multi-constellation satellite navigation systems [6]. The experiment on relativistic effect on clock transport also suggests that the time gain due to Sagnac effect is more compared to other relativistic effects [7]. For a stationary terrestrial receiver on the earth surface the Sagnac correction is seen as maximum of 133 ns that must be corrected at the receiver [8]. Wang et el., re-examined the Sagnac effect using the GPS range measurements and presented a new way to look at it by deducing it from range measurement equation [9]. All the past research supports that the influence of Sagnac effect on positioning, navigation and timing services of a satellite- based navigation system. In this paper, we present the Sagnac correction for a newly developed Indian regional navigation satellite system named as NAVIC (NAVigation with Indian Constellation).

NAVIC satellites are placed at altitudes of 35,786 km. The system will be fully equipped with seven satellites in its constellation. Currently, it has six satellites deployed of which three are Geo-Stationary Earth Orbit: IRNSS-C (I03), IRNSS-F (I06), and IRNSS-G (I07) and three are in Geo-Synchronous Orbit: IRNSS-1B (I02), IRNSS-1D (I04), and IRNSS-1E (I05) (Table 1) [8].

Table 1. Orbital Characteristics for NAVIC Satellites

NAVIC satellite orbit parameters	**I02**	**I03**	**I04**	**I05**	**I06**	**I07**
Orbit	GSO	GEO	GSO	GSO	GEO	GEO
Semi-major axis (km)	42,164	42,164	42,164	42,164	42,164	42,164
Longitude (degree)	550	830	111.750	111.750	32.50	129.50
Eccentricity	0.01	0.01	0.01	0.01	0.01	0.01
Inclination (degree)	290	00	290	290	00	00
Argument of perigee	0	0	0	0	0	0
Apogee altitude (km)	35,874.8	35,874.8	35,874.8	35,874.8	35,874.8	35,874.8
Perigee altitude (km)	35,711.8	35,711.8	35,711.8	35,711.8	35,711.8	35,711.8
Ascending node altitude (km)	35,711.8	35,711.8	35,711.8	35,711.8	35,711.8	35,711.8
Period of revolution (min)	1436.1	1436.1	1436.1	1436.1	1436.1	1436.1
Mean velocity (km/s)	3.075	3.075	3.075	3.075	3.075	3.075
Mean motion (rev/d)	1.0	1.0	1.0	1.0	1.0	1.0

II. Mathematical Formulation for Sagnac effect

Earth rotation causes the radio signal to travel an extra distance. Hence the signal takes extra time to reach the receiver on the Earth surface. $\omega_E = 7.2921151247 \times 10^{-5}\ \mathrm{rad/s}$ [10] being the

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Earth's angular velocity. Earth's linear velocity vector is given by:

$$\mathbf{V} = \omega_{\mathbf{E}} \times \mathbf{r}_{\mathrm{R}} (1)$$

When the receiver is at rest, the Earth rotation velocity becomes receiver velocity and the Sagnac correction term is given by [3]:

$$\Delta t_{\mathrm{Sagnac}} = \frac{\omega_{\mathrm{E}} \times \mathbf{r}_{\mathrm{R}} \cdot \mathbf{R}}{c^2} (2)$$

$$= \frac{2\omega_{\mathrm{E}}}{c^2} \cdot \frac{1}{2} (\mathbf{r}_{\mathrm{R}} \times \mathbf{R}) \quad (3)$$

Where, $\frac{2\omega_{\mathrm{E}}}{c^2} = 1.6227 \times 10^{-21}\ \mathrm{s/m^2}$,$\mathbf{r}_{\mathrm{R}}$ is vector from the center of the Earth to the receiver position and **R** is the vector joining the satellite position and the receiver on the Earth and θ is the angle between them(Fig.1). These two vectors and the local horizontal lie on a same plane. ∅ is the angle made by the vector R with the local horizontal plane (Fig.1).

$$\mathbf{A} = \frac{1}{2} (\mathbf{r}_{\mathrm{R}} \times \mathbf{R}) (4)$$

$$\Delta t_{\mathrm{Sagnac}} = \frac{2\omega_{\mathrm{E}}}{c^2} \cdot \mathbf{A} (5)$$

The vector product in (4) is area of parallelogram and it's half is the area of triangle that is swept out by the tip of the vector from the axis of rotation along the propagation of the radio signal. It is represented by A. The dot product in (5) projects the area onto the plane parallel to equatorial plane. Which is the Sagnac correction (5) [3],[10].

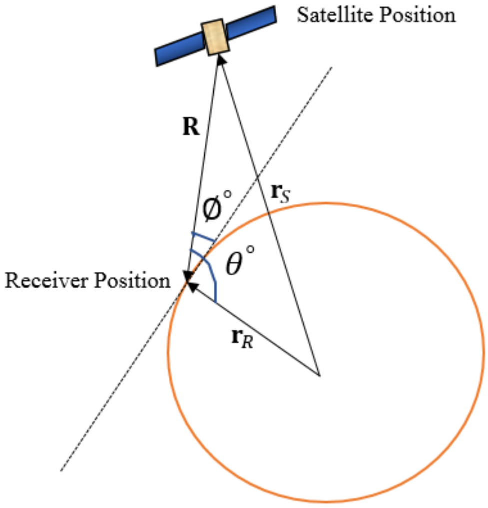

Fig1:Sagnac Effect for ECEF observer

The NAVIC receiver provides satellite ephemeris data for each second. The algorithm steps to compute Sagnac correction for NAVIC satellites is given below:

2.1Algorithm Steps for NAVIC Satellites

1. Fix the receiver position in ECEF-frame: $\{(r_R) = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}\}$
2. for each satellite sat=1:6
3. for time t = 1:86400 seconds
4. Choose the satellite position for each second in ECEF-frame: $\{(r_S(t)) = S_x(t)\hat{\imath} + S_y(t)\hat{\jmath} + S_z(t)\hat{k}\}$
5. Compute R: $(r_R - r_S(t))$ for each second and $\{(R) = R_x(t)\hat{\imath} + R_y(t)\hat{\jmath} + R_z(t)\hat{k}\}$
6. Calculate the receiver magnitude $|r_R| = \sqrt{r_x{}^2 + r_y{}^2 + r_z{}^2}$
7. Calculate magnitude for R: $|R| = \sqrt{(r_x - S_x(t))^2 + (r_y - S_y(t))^2 + (r_z - S_z(t))^2}$
8. The angle between $r_R \& R$ is given by $\theta(t) = \cos^{-1}\left\{\frac{(r_x R_x(t) + r_y R_y(t) + r_z R_z(t))}{|r_R||R|}\right\}$
9. The area swept out because of rotation is given by $A(t) = \frac{1}{2}|r_R||R|\sin(\theta(t))$
10. Compute the Sagnac correction for each second as $\Delta t_{\mathrm{Sagnac}}(t) = (1.6227 \times 10^{-21}).A(t)$

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11. End
12. end

The RINEX navigation file for GPS satellites are obtained from NOVATEL GISTM receiver. The GPS satellites ephemeris can be deduced from navigation file in MATLAB. The process for Sagnac computation for each GPS satellite is given below:

2.2Algorithm Steps for GPS Satellites

1. Fix the receiver position in ECEF-frame: $\{(r_R) = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}\}$
2. for each satellite sat=1:32
3. for time t=1:1440
4. Repeat the above algorithm from step:4 to step:10
5. End
6. end

III. Results and Discussions

3.1Sagnac corrections for GPS satellites

GPS satellites have higher mean velocity than earth's rotation. They revolve twice around the earth for a day. These are placed at Medium Earth Orbit (MEO) at an altitude of 20,200 km. Each satellite has its own visibility period based on the elevation cut-off at the receiver. Based on their visibility, the Sagnac correction has been carried out. Fig.2 gives the Sagnac corrections for each GPS satellite observed from KLEF Guntur with a multi-constellation dual frequency GISTM receiver(NOVATEL). The RINEX navigation data recorded on 16th February 2018 was chosen and processed to obtain Sagnac corrections.

Almost, all the GPS pseudo random numbers (PRN's) have the maximum Sagnac correction to be approximately 137 ns/d. The maximum Sagnac values are confined to the values that in the literature [6]. The minimum Sagnac corrections are obtained for GPS PRN 29 with 2.25 ns. Table.2. gives the Sagnac corrections for all the GPS satellites. The maximum Sagnac is obtained when the angle obtained from above algorithm step 8 reaches to unity. So, each satellite reached to 137 ns at their respective time instants based on their orbital characteristics.

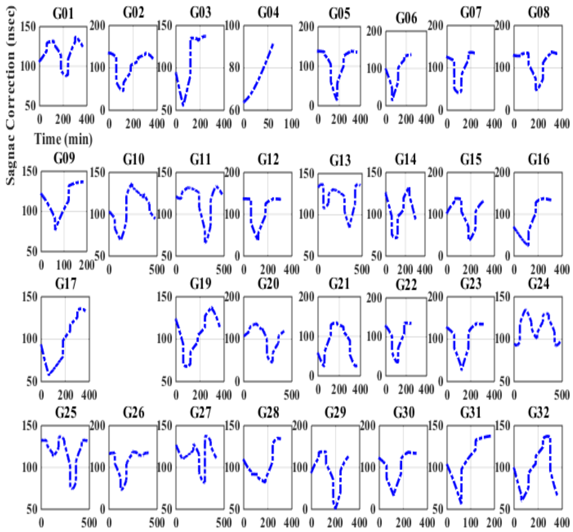

Fig.2. GPS satellites Sagnac corrections for all PRN's on 16th February 2018.

3.2Sagnac corrections for NAVIC satellites

The NAVIC satellites I02, I04, I05 are in Geo-Synchronous orbit and I03, I06, I07 are in Geo-Stationary Earth orbit. The GSO satellites I04 and I05 are placed at same longitude of 111.750 E having equatorial crossings at 550 E. So, these two satellites have same patterns of "8" (Fig.3). The GEO satellites are stationary through-out the day claiming the less change movement in their ECEF directions (Fig.3). The each second Sagnac corrections for the NAVIC satellites are given in Fig.4. GSO and GEO satellites show a different pattern for the Sagnac corrections. The GSO satellites have higher Sagnac changes compared to GEO satellites (Table.2). All the satellites have maximum Sagnac value above 170 ns/day except for I03 GEO satellite. Above all, I04 and I05 have maximum Sagnac correction of 178 ns. The GEO satellites have maximum Sagnac correction of approximately 172 ns. As, the GEO satellites are constant in their movement there is lesser change in maximum to minimum in Sagnac correction, that is approximately 5 ns to 19 ns per day. The Sagnac

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change is approximately in the range of 81 ns to 93 ns for NAVIC GSO satellites, that is higher compared with GEO satellites (Table.3).

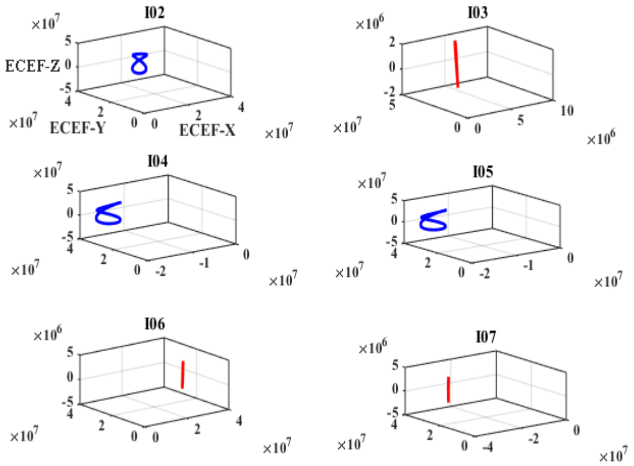

Fig.3. NAVIC satellites position in ECEF coordinate system (red - GSO & blue - GEO).

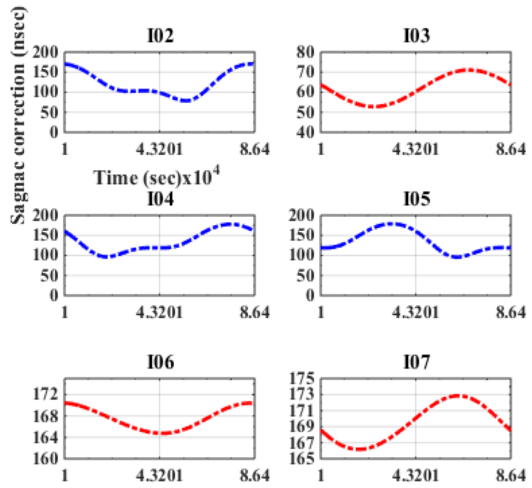

Fig.4. Sagnac corrections obtained for the NAVIC satellites for one complete day.

Table. 2. Sagnac Corrections observed for GPS Satellites

Satellite PRN GPS	Maximum Sagnac Correction (ns/day)	Minimum Sagnac Correction (ns/day)	Sagnac Change (ns/day)
1	136.824980774808	88.3596563572687	48.4653244175397
2	135.007056524926	46.3901621571703	88.6168943677554
3	137.294767922855	55.7751263502240	81.5196415726311
4	90.7414212305882	64.0869595771709	26.6544616534173
5	137.878681333106	15.0630030747648	122.815678258342
6	136.576685794027	14.0157172268617	122.560968567165
7	136.520901878613	41.7197181522124	94.8011837264009
8	137.763928398991	48.6326858602805	89.1312425387104
9	136.843306043458	76.9399377173122	59.9033683261457
10	136.456471154946	68.9470041533749	67.5094670015715
11	133.560298760910	65.9830620421776	67.5772367187321
12	137.021322011484	37.1587965621922	99.8625254492919
13	136.939703613197	84.6896584178508	52.2500451953467
14	132.821080160162	71.8712843703847	60.9497957897775

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15	137.753485628755	38.7173503917342	99.0361352370204
16	137.700748384490	25.3945512270330	112.306197157457
17	135.852562859445	57.6383089902909	78.2142538691538
18	0	0	0
19	137.456859702261	67.8570081269762	69.5998515752851
20	136.642619185658	45.7578330204525	90.8847861652052
21	134.079272729271	24.9918593611229	109.087413368148
22	135.652619122074	35.5637300325491	100.088889089525
23	136.442675834310	26.5265358516268	109.916139982683
24	134.853868018769	92.6403411217714	42.2135268969979
25	136.818739155397	74.4709832461919	62.3477559092046
26	137.667548034321	46.3284190280859	91.3391290062352
27	137.468243170541	82.0547933780667	55.4134497924747
28	134.840301326659	82.5033065934988	52.3369947331603
29	137.440640431202	2.25355636381555	135.187084067387
30	136.510769978190	33.0716518059476	103.439118172243
31	137.402442685593	56.4513040584786	80.9511386271146
32	137.206379242864	60.2373867825149	76.9689924603487

Table3. Sagnac Corrections observed for NAVIC Satellites

Satellite PRN NAVIC	Maximum Sagnac Correction (ns/day)	Minimum Sagnac Correction (ns/day)	Sagnac Change (ns/day)
I02	170.634151858923	78.6064011208768	92.0277507380466
I03	71.1325233660480	52.8007103323240	18.3318130337240
I04	177.927386341880	96.3907161870364	81.5366701548440
I05	178.954467147813	95.6039116408877	83.3505555069251
I06	170.431604763745	164.745520740684	5.68608402306097
I07	172.848250992328	166.165955664660	6.68229532766827

IV. CONCLUSION

In this paper, a comparison inSagnac corrections for the newly developed region based navigation system NAVIC is analyzed for a day along with GPS satellites. The maximum Sagnac corrections are higher for NAVIC satellites for a day (178 ns) compared with GPS satellites (137 ns). As GPS

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satellites are at MEO where as NAVIC satellites are at GEO, that creates an excessive Sagnac of 41 ns (if θ in Algorithm A.8 is taken 90° for both). This analysis is useful in precise time estimation for navigation systems by correcting the Sagnac value in the receiver. The Sagnac effect is predominant effect that influences the positioning along with other effects. Based on the above analysis, it is an added advantage in the receiver clock bias estimation.

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