

Stock Dependent EOQ Model of Imperfect Quality under Cloudy Fuzzy Environment

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Abstract:

In this article, an EOQ deteriorating model is developed where demand is stock dependent. Shortages are not allowed for this model. This model is developed for both crisp and cloudy fuzzy environments where discount increases if number of defects increase in the lots for imperfect quality items. Yager's Ranking index is used for defuzzification. The results are numerically verified. Also, sensitivity analysis of the model is considered to validate the crisp model for optimality.

Keywords: deterioration, EOQ, stock dependent demand, cloudy fuzzy number, Yager's Ranking Index.

1. Introduction

Today, proper administration and control of inventory (stock of items produced or available in a business organization) is a big challenge for every business organization. Proper management of inventory in business satisfies the market behaviour well and business people in general. Customer demand is the one of the most relevant market behaviours, which is directly controlled by efficient and effective regulation of inventory. Until now, many inventory control models have been developed assuming different types of demand along with various other aspects related to inventory control, and therefore a suitable solution procedure is followed to maximize profits or minimize costs. We have developed an inventory model considering various business-like assumptions, such as stock dependent demand, constant deterioration and proportionate discount plays an important role in competitive business. Generally, this type of model suitable for different types of industrial sectors of the economy, for instance food (processed foods and raw foods), garment (clothes and dress materials), automobile (parts and finished products), electronic equipment's, etc. In the present study deterioration, in the form of decay or expiration of

the product, is another important issue and it has wide impact in the inventory analysis, so we can't ignore this issue. In the proposed model, depending on the quality of all units of defective items, a proportionate rate of discount has been introduced. In any inventory process, initially the uncertainties viewed are high and as the time progresses everything gradually begins to get clear for a decision maker. After a long period of time the ambiguities underlying in the inventory system starts reducing; when the inventory cycle time is low the ambiguity is high and reversely if the cycle time is high then uncertainty is low; it is the most common phenomenon of day to day life. Let us consider about the ambiguity over the demand rate, a most vital parameter of an inventory process. In beginning the ambiguity over demand rate is high because, people usually take much time to accept and adopt the process. The basic insight in the public opinion is that 'the system is less reliable' as the decision makers take more time to run the process. This feeling directly affects the customers' satisfactions as well as the demand rate. However, as the cycle time increases the customers also start getting a sense of satisfaction as saturation on adoptability and reliability reaches. So the ambiguities have been removed from the process and a grand paradigm

shift on progress (financial development, cost minimization, achievement of large customer) of that system has been viewed. Since the cloud of uncertainties removes over time from the process so such uncertainties are named as ‘cloudy fuzzy’. We have utilized a cloudy fuzzy number and defuzzification technique via the extension of Yager (1981)’s ranking index method.

2. Literature survey

Assuming constant rate of demand Harris (1913) first developed the EOQ model, which was then extended considering linearly increasing demand by Resh et al. (1976). Kim et al. (1995) further considered price-dependent demand pattern model and calculated maximize profit, optimal order size and unit wise retail price. Considering stock dependent demand Giri et al. (1996) developed a model for deteriorating items. Thereafter, Mandal and Maiti (1997) using profit maximization principle, determined optimum order quantity and assuming stock dependent demand developed a shortage model for damageable items. Under stock dependent selling rate Chang (2006) established a time dependent partially backlogging EOQ model for perishable items. Panda(2010) discussed an EOQ

model with stock dependent demand and also assumed that each lot received contains percentage defective items with known probability distribution. The effects of imperfect quality items in lot sizing policy were noted and discussed by Khan (2011) and Jaggi (2011). Accordingly, De and Mahata (2017) under cloudy fuzzy demand rate proposed a fuzzy backorder model. Under trade-credit policies with price dependent demand Khanna(2017) developed shortage and fully backlogged model. Fuzzy EOQ model for deteriorating items have been developed by Indrajitsingha(2018) where in the first interval demand is dependent on stock and in second interval it is constant demand. Under cloudy fuzzy demand rate Karmakar(2018) studied simple EOQ model, and for growing items with imperfect quality Sebatjane(2019) developed Economic order quantity model. On the basis of two level supply chain policy Pervin(2019) formed Multi-item deteriorating two-echelon inventory model with price-and stock-dependent demand. A partially backlogged model had been developed by Shaikh(2019) considering stock dependent demand pattern and price discount facility for deteriorating items. The comparison table gives the main dimension of our study.

Table-1: Observation about Published work and present work

Author	Types of demand	Deterioration	Imperfect Items	Fuzzy	Profit
Harris(1915)	Constant	no	no	no	
Resh(1976)	Linearly increasing	no	no	no	
Kim(1995)	Price	no	no	no	
Giri(1996)	Stock	yes	no	no	
Chang, Goyal & Teng (2006)	Stock	no	no	no	
Panda(2010)	Stock	no	yes	no	
Khan, Jaber & Bonney, (2011)	Constant	no	yes	no	
Jaggi & Mittal(2011)	Constant	yes	yes	no	
Sarkar & Sarkar (2013)	Stock	yes	no	no	
De & Mahata (2017).	Constant	no	no	yes	
Khanna, Gautam & Jaggi (2017)	Price	yes	yes	no	
Indrajitsingha, Samanta & Misra (2018)	Stock	yes	no	yes	
Karmakar, De & Goswami(2018)	constant	no	no	yes	

Sebatjane & Adetunji (2019)	constant	no	yes	no	
Pervin, Roy & Weber (2019)	Stock	yes	no	no	
Shaikh, Khan, Panda, & Konstantaras, I. (2019)	Stock	yes	no	no	
This paper	Stock	yes	yes	yes	

3. Research gap and our contribution

Based on the literature survey we have developed our present model, in which deterioration effect on the inventory model is being considered. The model is assumed to have a constant deteriorating effect on the inventory system, and a stock-dependent demand pattern also. Practically, there are always options, which gives rise to uncertainties in making decisions in the model, that can be avoided rarely. Fuzziness or vagueness is the kind of uncertainty which is being mostly considered in the economic order quantity models. As uncertainties can be removed over time, like clouds, we consider cloudy fuzzy number in demands, as demand is highly volatile depending upon time.

$$\mu_{\tilde{A}}(x, t) = \begin{cases} 0 & \text{if } x < b\left(1 - \frac{\gamma}{1+t}\right) \text{ and } x > b\left(1 + \frac{\delta}{1+t}\right) \\ \frac{x - b\left(1 - \frac{\gamma}{1+t}\right)}{\frac{b\gamma}{1+t}} & \text{if } b\left(1 - \frac{\gamma}{1+t}\right) \leq x \leq b \\ \frac{b\left(1 + \frac{\delta}{1+t}\right) - x}{\frac{b\delta}{1+t}} & \text{if } b \leq x \leq b\left(1 + \frac{\delta}{1+t}\right) \end{cases}$$

Rule:1 Let A_L and A_R are the left and right alpha cuts of the fuzzy number $\tilde{A}$ then the defuzzification rule under Yager's Ranking Index is given by

$$I(\tilde{A}) = \frac{1}{2} \int_0^1 (A_L + A_R) d\alpha$$

Rule:2 The left and right alpha-cut of the membership function $\mu_{\tilde{A}}(x, t)$ is denoted as $L^{-1}(\alpha, t)$ and $R^{-1}(\alpha, t)$ then the defuzzification rule under time extension of Yager's ranking index is

$$I(\tilde{A}) = \frac{1}{T} \int_{\alpha=0}^{\alpha=1} \int_{t=0}^{t=T} (L^{-1}(\alpha, t) + R^{-1}(\alpha, t)) da dt$$

4. Definitions and Rules.

Following definitions and rules are used to developed cloudy type fuzzy model.

Definitions: A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a cloudy normalized triangular fuzzy number if after an infinite times the set its self converges to a crisp singleton. That means as time t tends infinity both $a, c \rightarrow b$. Let as consider the fuzzy number

$$\tilde{A} = \left[b\left(1 - \frac{\gamma}{1+t}\right), b, b\left(1 + \frac{\delta}{1+t}\right) \right], \text{ for } 0 < \gamma, \delta < 1.$$

Note that $\lim_{t \rightarrow \infty} b\left(1 - \frac{\gamma}{1+t}\right) = b$ and $\lim_{t \rightarrow \infty} b\left(1 + \frac{\delta}{1+t}\right) = b$, so $\tilde{A} \rightarrow \{b\}$.

Then the membership function for $0 \leq t$ is as:

Where α and t independent variables and $\tilde{A}$ be cloudy normalised triangular fuzzy number.

5. Assumptions and Notations

5.1 Assumptions

The following assumptions are made in developing the models.

1. Stock dependent demand is denoted by $D_R = a + bI(t)$, where $a(> 0)$ be the initial demand, $0 \leq b \leq 1$ is stock sensitivity parameter and $I(t)$ is the instantaneous level of inventory at time t .
2. The constant rate of deterioration is θ , where $0 < \theta \ll 1$.
3. Instantaneous delivery and no shortage.

4. Fixed selling price for non-defective items.
5. Rate of Screening is greater than rate of demand.
6. The time horizon of the inventory system is infinite.

5.2 Notations

The following notations are used

- Q_S : Order size for each cycle.
 C_V : Variable cost.
 K_C : Fixed ordering cost.
 H_C : Cost associated holding of items
 P_D : The defective % in Q_S .
 S_P : Unit wise selling price for good quality items.
 S_R : Rate of Screening
 S_C : One unit item screening cost.
 T : Cycle time.
 C_R : Total revenue
 TC : Total cost.
 C_{TP} : Total profit cycle wise.
 TP_U : Total profit unit wise

6. Mathematical models

Crisp model

Based on the above assumptions, this model has a stock dependent demand with initial lot size Q_S units at time $t=0$ with defective and deteriorating items. After 100% screening of all the items, good quality items are separated from the defective ones, and these defective items are collected as a single batch which is then sold at proportionately discounted price but good quality items are sold at a fixed selling price. As we have assumed, the cycle length T , screening time t and the number of defective and good quality items drawn from the inventory as $P_D Q_S$ and $(1 - P_D) Q_S$, then the cycle starts with the initial lot size Q_S at time $t=0$, due to the combined effect of deterioration and customer demand the inventory level decreases during the time $[0, t_1]$, at time $t = t_1$ the inventory level $I(t_1)$ becomes $(1 - P_D) Q_S - D_R t_1$ and time $t=T$ inventory level becomes zero. To avoid shortage

within screening time t_1 , the defective percentage is restricted $P_D \leq 1 - \frac{D_R}{S_R}$, where $t_1 = \frac{Q_S}{S_R}$.

Instantaneous inventory level over the period $[0, T]$ as described by following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -D_R, 0 \leq t \leq t_1 \quad (6.1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D_R, t_1 \leq t \leq T \quad (6.2)$$

where $0 < \theta \ll 1$ and $D_R = a + bI(t)$.

Using boundary condition $I(0) = Q_S$, $t=t_1$, $I(t_1) = (1 - P_D) Q_S - D_R t_1$ and $t=T$, $I(t) = 0$ in equ.(6.1) and equ.(6.2) get as follows:

$$I(t) = Q_S e^{-(\theta+b)t} + \frac{a}{\theta+b} (e^{-(\theta+b)t} - 1), \quad 0 \leq t \leq t_1 \quad (6.3)$$

$$I(t) = ((1 - P_D) Q_S - D_R t_1) e^{(\theta+b)(t_1-t)} + \frac{a}{\theta+b} (e^{-(\theta+b)t} - 1), \quad t_1 \leq t \leq T \quad (6.4)$$

$$\text{And } T = t_1 - \frac{1}{\theta+b} \ln \left(\frac{\frac{a}{\theta+b}}{(1-P_D) Q_S - D_R t_1 + \frac{a}{\theta+b}} \right) \quad (6.5)$$

The cycle wise total cost is:

$TC(Q_S) = \text{Ordering cost} + \text{variable cost} + \text{Screening cost} + \text{holding cost}$

$$= K_C + C_V Q_S + S_C Q_S + H_C \left[\frac{Q_S^2 (1-P_D)}{D_R} \right] \quad (6.6)$$

Total revenue during time period $(0, T)$:

$C_R(Q_S) = \text{sum of total sale with good quality and imperfect quality items}$

$$\frac{2S_P (1-P_D) Q_S + (Q_S P_D + 1) \left[K_C + C_V Q_S + S_C Q_S + H_C \left(\frac{Q_S^2 (1-P_D)}{D_R} \right) \right]}{2Q_S + Q_S P_D + 1} \quad (6.7)$$

The cycle wise total profit:

$$C_{TP}(Q_S) = C_R(Q_S) - TC(Q_S)$$

$$= \frac{2S_P Q_S^2 - 2Q_S \left[K_C + C_V Q_S + S_C Q_S + H_C \left(\frac{Q_S^2 (1-P_D)}{D_R} \right) \right]}{2Q_S + Q_S P_D + 1} \quad (6.8)$$

The Unit wise total profit :

$$TP_U(Q_S) = \frac{C_{TP}(Q_S)}{T} = \frac{2D_R(S_P Q_S - K_C - C_V Q_S - S_C Q_S)}{(1-P_D)(2Q_S + Q_S P_D + 1)} - \frac{2H_C Q_S^2}{(2Q_S + Q_S P_D + 1)} \quad (6.9)$$

Differentiate $TP_U(Q_S)$ with respect to Q_S two times are given as follows:

$$\frac{dTP_U(Q_S)}{dQ_S} = \frac{1}{(2Q_S + Q_S P_D + 1)^2} \left[\frac{2D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{(1-P_D)} - 2H_C Q_S \right] \quad (6.10)$$

$$\text{and } \frac{d^2 TP_U(Q_S)}{dQ_S^2} < 0$$

The negative value of $\frac{d^2 TP_U(Q_S)}{dQ_S^2}$ shows $TP_U(Q_S)$ is concave function and setting $\frac{dTP_U(Q_S)}{dQ_S} = 0$ and solving we get optimal order size that represent the maximum annual profit. After some basic manipulation we get

$$(Q_S)_{\max} = \sqrt{\frac{D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{H_C(2 + P_D)(1 - P_D)}} \quad (6.11)$$

When $P_D = 0$, $S_P - C_V - S_C = 2K_C$ then $(Q_S)_{\max}$ reduce to the traditional EOQ formula.

$$(Q_S)_{\max} = \sqrt{\frac{2K_C D_R}{H_C}} \quad (6.12)$$

Fuzzy Model

In proposed model rate of demand is flexible in nature. So we assume the customer demand D_R (demand rate) behaves like a cloud type fuzzy number. Since, functionally $Q_S \left(= \frac{(1-P_D)T}{D_R} \right)$ (that is good quality items) is related to the rate of demand.

So from equation (6.9) the fuzzy problem becomes

$$\text{Max } \tilde{Z} = \frac{A \tilde{D}_R \tilde{Q}_S}{B \tilde{Q}_S + 1} - \frac{2 \tilde{D}_R K_C A'}{B \tilde{Q}_S + 1} - \frac{A'' \tilde{Q}_S^2}{B \tilde{Q}_S + 1} \quad (6.13)$$

$$\text{where, } A = \frac{2S_P - 2C_V - 2S_C}{1 - P_D}, B = 2 + P_D, A' = \frac{1}{1 - P_D}, A'' = 2H_C$$

$$\text{Subject to } \tilde{Q}_S = \frac{\tilde{D}_R T}{1 - P_D}$$

Therefore utilizing (6.13) the Index value of the fuzzy objective function is given by

$$I(\tilde{Z}) = I \left[\frac{A \tilde{D}_R \tilde{Q}_S}{B \tilde{Q}_S + 1} - \frac{2 \tilde{D}_R K_C A'}{B \tilde{Q}_S + 1} - \frac{A'' \tilde{Q}_S^2}{B \tilde{Q}_S + 1} \right] \quad (6.14)$$

Solving equation (6.14) get as :

$$I(\tilde{Z}) = \left[\frac{1}{\frac{B D_R T}{(1 - P_D)} \left[\frac{\tau (\gamma - \delta)}{4} \left\{ 1 - \frac{\log(1 + \tau)}{\tau} \right\} \right] + 1} \right] \left[D_R T \left\{ 1 + \gamma - \delta \log 1 + \tau 4 \tau A D_R T (1 - P_D) \tau 2 - \gamma - \delta 4 1 - \log 1 + \tau \tau 2 - 2 K_C A' - A'' D_R T 2 1 - P_D 2 \tau 2 - \gamma - \delta 4 1 - \log 1 + \tau \tau 2 \right\} \right] \quad (6.15)$$

Particular cases

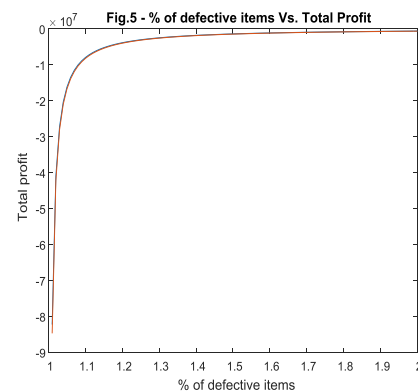
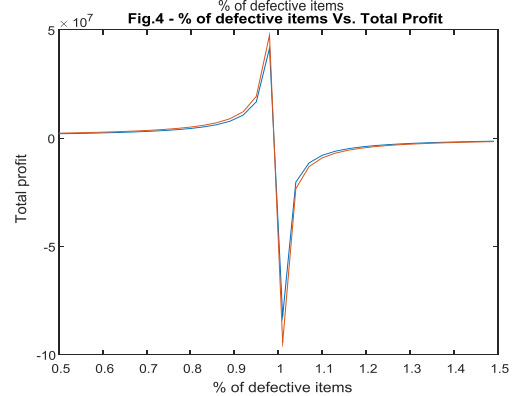
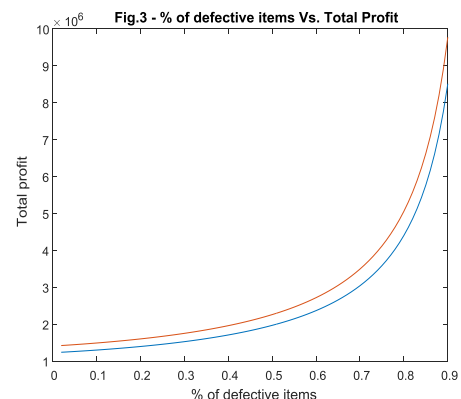
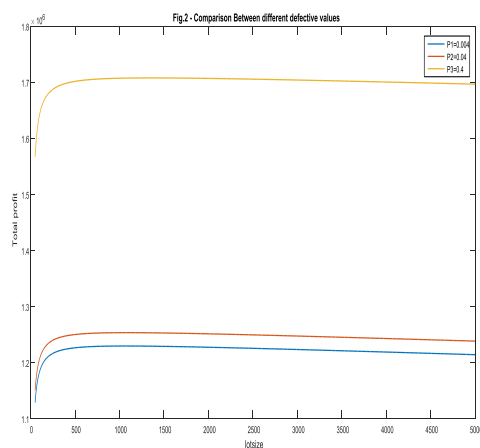
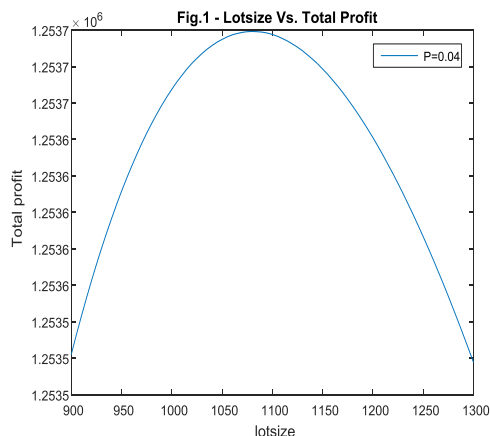
$$\text{If } (\gamma - \delta) \rightarrow 0 \text{ then } Z = \frac{A D_R Q_S}{B Q_S + 1} - \frac{2 D_R K_C A'}{B Q_S + 1} - \frac{A'' Q_S^2}{B Q_S + 1} \text{ this is similar to crisp objective function.}$$

7. Numerical results

To illustrate the behavior of the optimal lot sizes, we have considered an example with following parameters for both models: variable cost \$25/unit, fixed ordering cost \$100/cycle, holding cost \$5/unit/year, selling price of good quality items \$50/unit, screening cost \$0.25/unit, rate of deterioration 0.02, rate of defective 0.04, and initial demand $a=50000$, scale parameters $b=0.02$ and stock level $=1000$, $\gamma = 0.9$, $\delta = 0.3$ and initial demand $a = \langle 50000(1 - \gamma), 50000, 50000(1 + \delta) \rangle$.

The maximize profit ($TP_U(Q_S)$) and optimal order size $((Q_S)_{\max})$ of EOQ model is 1,253,699.263/year and 1080.97 units, screening time

=0.0061 and expected cycle length 0.0268 year but for the fuzzy model 1,360,916/year and 1154.83 units are the total profit and optimal order size. The graphical analysis of the model is being carried out in the following figures considering the above example. Figure 1 is a graph of total lot size against Total Profit from the model, the maximum profit in the model occurs when % of defective items is considered to be 0.04 and the value of lot size is in the range of (1050,1100). The maximum profit of this model varies if the percentage of defective items changes, which can be seen in figure -2. In this figure, we can note that as the value increases for % of defective items from 0.004 to 0.4, the profit of the model increases, whereas the lot size almost remains same.



Figures 3, 4 and 5 depicts the graphs of % of defective items versus Total profit in the items. In the first figure (fig-3) as the % of defective items increases from 0.1 towards 1 there is increase in the total profit whereas in the last figure (fig – 5) there is a decrease in the total profit when the % of defective items becomes more than 1. In fig - 4, we can see that there is a gap when the value of % of defective item is exactly 1. So, the total profit in the model increases only if the value of % of defective items is below 1.

8. Sensitivity Analysis

From the above example the sensitivity analysis of the model had been done by changing the values of one parameter at a time in a range from $\pm 10\%$. The following table below shows the influence of

Table-3

Parameters	Percentage change	Screening time	T	$(Q_s)_{\max}$	$TP_U(Q_s)$
A	+10	0.0064	0.0262	1133.71	1379560.874
	+5	0.0063	0.0265	1107.66	1316626.991
	-5	0.0060	0.0272	1053.61	1190778.159
	-10	0.0058	0.0276	1025.52	1127864.208
B	+10	0.0061	0.0268	1080.99	1253749.603
	+5	0.0061	0.0268	1080.98	1253724.433
	-5	0.0061	0.0268	1080.96	1253674.093
	-10	0.0061	0.0268	1080.95	1253648.923
K_C	+10	0.0064	0.0280	1128.14	1253236.795
	+5	0.0063	0.0274	1104.81	1253465.561
	-5	0.0060	0.0262	1056.60	1253938.237
	-10	0.0058	0.0256	1031.65	1254182.855
S_P	+10	0.0062	0.0271	1092.72	1508996.648
	+5	0.0062	0.0270	1086.86	1381347.800
	-5	0.0061	0.0267	1075.05	1126051.043
	-10	0.0061	0.0265	1069.09	998403.144
C_V	+10	0.0061	0.0267	1075.05	1126051.043
	+5	0.0061	0.0268	1078.02	1189875.113
	-5	0.0061	0.0269	1083.92	1317523.492
	-10	0.0062	0.0270	1086.86	1381347.800
P_D	+10	0.0061	0.0268	1083.14	1256554.873
	+5	0.0061	0.0268	1082.06	1255148.112
	-5	0.0061	0.0268	1079.93	1252359.300
	-10	0.0061	0.0269	1078.86	1250977.159
S_C	+10	0.0061	0.0268	1080.91	1252422.779
	+5	0.0061	0.0268	1080.94	1253061.021
	-5	0.0061	0.0268	1081.00	1254337.505
	-10	0.0061	0.0268	1081.03	1254975.747
H_C	+10	0.0058	0.0256	1030.67	1253182.476
	+5	0.0060	0.0262	1054.92	1253437.789
	-5	0.0063	0.0275	1109.05	1253967.366
	-10	0.0065	0.0283	1139.45	1254242.628

From the above table, it is quite evident that the screening time is dependent on parameters a, K_C and holding cost but there is no change in screening time with respect to the change in values of b, S_P , P_D , C_V and S_C . In a positive way, the cycle length depends upon the values of K_C , S_P but in negative way it depends on H_C , C_V and initial demand. On other hand, with the change of b, S_C and P_D there is no effect in cycle time. The order quantity is increased, when there is a change in value of the

different parameters on screening time, cycle length, optimal order size and total profit of the system.

parameters a, K_C , P_D , S_P and decreases when there is change in parameters H_C , C_V but remains constant with changes in remaining parameters. From the above table we see that initial demand, holding cost, selling price of good quality items are highly sensitivity in regard to profit.

7. Conclusion

Classical EOQ models help us decide on the terms of how much to order, in order to manage an inventory.

Any company dealing with physical products needs to manage an inventory to improve and avoid shortages occurring due to stock. Many a times, the lots come with defective items due to which there is loss in the effectiveness of the EOQ models. In present study, we have considered a constant rate of deterioration of items where the demand depends on the stock and a proportionate discount is being considered for defective items. We have utilized a cloudy fuzzy number and defuzzification technique via the extension of Yager (1981)'s ranking index method. The problem is optimised to get the best solution, utilizing Yager's Ranking. Moreover, we have compared the numerical results obtained from crisp and fuzzy environments. Lastly graphical and sensitivity illustration justify the models. We observe some managerial implications such as:

- Positive and negative impact on the seller total profit depends on initial demand, selling price and carrying cost on the seller. Hence, to maintain customer demand the manager is suggested to follow a proper marketing policy.
- Variable cost has the second negative impact on the marketer's total profit. For higher order size, the manager assures the supplier or manufacturer to reduce the unit variable cost.
- Average maximum profit gives the fuzzy model.
- Overall process depends on choice of perfect order size and specifies cycle time.

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