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Improved Method for Optimal Solution of Unbalanced Assignment Problems under Trapezoidal Fuzzy Numbers Using Ranking Functions

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Abstract:
Proposed in this paper an improved approaches to solving an Unbalanced Assignment Problem (UBAP). This process involves providing an IBFS optimality. I indicated both Row Penalty Allocation Method (RPAM) and also the Column Penalty Allocation Method (CPAM) in order to maintain an optimum UBAP solution. To less reductions and steps we get the optimum. The proposed algorithms are clear to understand and easy to access when it comes to discussing the fuzzy unbalanced assignment problems social and economic problems in real life situations.

Keywords: *Unbalanced Assignment Problem (UBAP): Row Penalty Allocation Method (RPAM) :Column Penalty Allocation Method (CPAM): Triangle Fuzzy Number (TFN):*

Introduction:

An assignment problem is a special case of transportation problem where the aim is to assign a wealth of resources to an equal variety of activities in possible to minimize planned costs or significantly increase total profits from assignments.The challenge of assignment occurs to originate as available resources, such as employees, highways, factories, areas, etc., have varying degrees of applicability in pursuing development projects. The real problem, instead, is how to build the assignments to maximize the given objective. One of the specific combinatorial optimization problems in the management branch or computational modeling in mathematics is the problem of availability. Dantzig.G.B, Thapa.M.N[1], the assignment method is a special linear programming technique to solve these problems, along with other choosing the right man for the right job for more than Every choice is possible and every single individual can still do jobs.The main objective is to allocate a number of events at minimal cost (or full profit) or some other unique purpose to an incredible number of facilities. Hungarian mathematician Konig (1931) developed the Hungarian assignment method which provides us with an efficient way to obtain the optimal solution without achieving a comparable evaluation on each solution. It depends on the premise that the given cost variable is transformed to a virtual world of opportunity costs.The assignment method presented above involves the number of columns and rows in the assignment matrix to always be equal. However, R.K.Gupta, [2],Kanti, Swarup, P.K, Gupta[3], the assignment problem is called an unbalanced problem if the given cost matrix is not a square matrix.From such cases the matrix needs to add a dummy row(s) or column(s) to make it a square matrix (with zeros as cost elements). Mukherjee, S., and Basu, K.[5], describe an application-fuzzy ranking system for solving Fuzzy Cost assignment problems. Nagraj Balakrishnan[6], present a modified Vogel's Approximation method for the unbalanced transportation problem, Ignore the penalty that involved a dummy row or column. D.G.Shimshak.D.G,J.A.Kaslik.J.A, and Barclay.T.D[7], A Modification of Vogel's approximation method through the use of heuristics, Ignore the penalty involved in a dummy row or column.our proposed method is determine row / column penalty for each row / column by differentiating between both the smallest and also the lowest effectiveness respectively.

2. Therortical Development:

Acceding to 'n' resources (or facilities) and 'n' activities (or jobs) and efficiency (in terms of potential costs, benefits, time, etc.) of each resource (facility) for each activity (job), the problem lies in allocating the resource to one and only one activity (job) in order to make sure optimum job satisfaction level. The objective function is to,

$$Minimize Z = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \text{for all i(Resource Availability)}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \text{for all j(Activity Requirements)}$$

Where

$$x_{ij} = \begin{cases} 1 & , if\ i^{th}\ machine\ is\ assigned\ to\ the\ j^{th}\ job \\ 0 & , if\ i^{th}\ machine\ is\ not\ assigned\ to\ the\ j^{th}\ job \end{cases}$$

3.3 Definition: Trapezoidal Fuzzy Number:

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$, is said to be generalized trapezoidal fuzzy numbers if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : \Re \to [0, \omega]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b,c]$, where $0 < \omega \leq 1$.

3.4 Arithmetic Operations:

The arithmetic operations of two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers $\Re$,are presented.If $\tilde{A}_1 = (k_1, k_2, k_3, k_4; \omega_1)$ and $\tilde{A}_2 = (l_1, l_2, l_3, l_4; \omega_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 + \tilde{A}_2 = (k_1 + l_1, k_2 + l_2, k_3 + l_3, k_4 + l_4; \text{minimum}(\omega_1, \omega_2))$

(ii) $\tilde{A}_1 - \tilde{A}_2 = (k_1 - l_4, k_2 - l_3, k_3 - l_2, k_4 - l_1; \text{minimum}(\omega_1, \omega_2))$

4.Proposed algorithms for Unbalanced Assignment problem Row/ Column Penalty Allocationt Method (RPAM/CPAM):

Step1:Check the problem is balanced or unbalanced, if the problem is unbalanced goto step 2,

otherwise ignore.

Step2:Determine row / column penalty for each row / column by differentiating between both the smallest and also the lowest effectiveness respectively.

Step3:Observe the maximum penalty row / column, identify and encircle the smallest performance corresponding to that row / column, remove the corresponding row and column. Whenever there is a tie in the maximum row/column penalty always choose the maximum efficiency which always leads to the minimum

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effectiveness.If there is tie again then select the best one of them.

***Step 4*:**Repeat step2 and step3until only one row/column is remained uncrossed. In the last row, assign the minimum value, encircle it and cancel out both the corresponding row and column. If there is a tie in the minimum effectiveness then assign the smallest effectiveness which correspond to the minimum next smallest effectiveness in the column until optimum assignment is reached.

5. Numerical Examples 1:

A batch of three jobs can be assigned to four different machines. The set up time in hours for each job on various machines is given.Find an optimal assignment of jobs to minimize the total set up time.

Case (i) :Proposed Method of unbalanced assignment problem:

Table 1:Matrix Entries Represent the Processing Time (in Hours)

$$\begin{pmatrix} & M_1 & M_2 & M_3 & M_4 \\ J_1 & 18 & 24 & 28 & 32 \\ J_2 & 8 & 13 & 17 & 19 \\ J_3 & 10 & 15 & 19 & 22 \end{pmatrix}$$

Solution:

The above problem is unbalanced, ignore the balanced assignment problem.Our proposed method for solving only unbalanced assignment problemas follows:

Table 2:Matrix Entries Represent Row Penalty allocation Processing Time (in Hours)

$$\begin{pmatrix} & M_1 & M_2 & M_3 & M_4 & Row\,Penalty & Row\,Penalty & Row\,Penalty \\ J_1 & 18 & 24 & 28 & 32 & 6 & - & - \\ J_2 & 8 & 13 & 17 & 19 & 5 & 4 & - \\ J_3 & 10 & 15 & 19 & 22 & 5 & 4 & 3 \end{pmatrix}$$

The maximum penalty is used to classify the first row penalties, then join the row to determine the total table value. Supposing value is tie means objectively crossed. After identifying the table's minimum value simultaneously eliminate the concern row and column. Repeat the apply the final value to both the process.Next to identify the maximum penalties are identified the second row then enter the row, to assign the minimum cost of table. Suppose cost is tie means broken arbitrarly. After assign the minimum cost of the table remove the concern row and column simultaneously. Repeat the process assign the final value.

The optimum job schedule is allocate the machines are

$$J_1 \to M_1\ , J_2 \to M_2\ , J_3 \to M_3, No\ Job \to M_4$$

The optimal solution of unbalanced assignment problems is =18+13+19+0=50 hours.

Case (ii): Proposed Method for Trapezoidel Fuzzy Unbalanced Assignment Problem:

Our proposed method is solving only for unbalanced assignment problem is applicable otherwise use regular method for Hungarian method is suitable.It is not required for add dummy row/column .

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Table 3:Matrix Entries Represent the Processing Time(in Hours) Trapezoidal Fuzzy Numbers

$$\begin{array}{cc} & \begin{array}{cccc} \quad A \quad & \quad B \quad & \quad C \quad & \quad D \quad \end{array} \\ \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} & \left(\begin{array}{cccc} (26,31,41,46:0.5) & (38,43,53,58:0.5) & (46,51,61,66:0.5) & (54,59,69,74:0.5) \\ (6,11,21,26:0.5) & (16,21,31,36:0.5) & (26,29,39,44:0.5) & (28,33,43,48:0.5) \\ (10,15,25,30:0.5) & (20,25,35,40:0.5) & (28,33,43,48:0.5) & (34,39,49,54:0.5) \end{array}\right) \end{array}$$

Table 4:Matrix Entries Represent Row Penalty Allocation(in Hours)

$$\begin{array}{ccc} & \begin{array}{cccc} \quad A \quad & \quad B \quad & \quad C \quad & \quad D \quad \end{array} & Row\,Penalty \\ \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} & \left(\begin{array}{cccc} (26,31,41,46:0.5) & (38,43,53,58:0.5) & (46,51,61,66:0.5) & (54,59,69,74:0.5) \\ (6,11,21,26:0.5) & (16,21,31,36:0.5) & (26,29,39,44:0.5) & (28,33,43,48:0.5) \\ (10,15,25,30:0.5) & (20,25,35,40:0.5) & (28,33,43,48:0.5) & (34,39,49,54:0.5) \end{array}\right) & \begin{array}{c} (-8,2,22,32:0.5) \\ (-10,0,20,30:0.5) \\ (-10,0,20,30:0.5) \end{array} \end{array}$$

The maximum penalty is used to classify the first row penalties, then join the row to determine the total table value. Supposing cost is tie means objectively crossed. After identifying the table's minimum cost simultaneously eliminate the concern row and column. Repeat the apply the final value to both the process.

Table 5:Modified Matrix Entries Row Penalty Allocation(in Hours)

$$\begin{array}{cccc} & \begin{array}{cccc} A & \quad B \quad & \quad C \quad & \quad D \quad \end{array} & Row\,Penalty & Row\,Penalty \\ \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} & \left(\begin{array}{cccc} - & - & - & - \\ - & (16,21,31,36:0.5) & (26,29,39,44:0.5) & (28,33,43,48:0.5) \\ - & (20,25,35,40:0.5) & (28,33,43,48:0.5) & (34,39,49,54:0.5) \end{array}\right) & \begin{array}{c} - \\ (-12,-2,18,28:0.5) \\ (-12,-2,18,28:0.5) \end{array} & \begin{array}{c} - \\ - \\ (-14,-4,16,26:0.5) \end{array} \end{array}$$

The maximum penalty is identified the second row penalties then enter the row, to assign the minimum cost of table. Suppose cost is tie means broken arbitrarly. After assign the minimum cost of the table remove the concern row and column simultaneously. Repeat the process assign the final value.

The optimal assignment problem is

Table 6:Optimum Matrix Entries Represent the Processing Time (in Hours)

$$\begin{array}{cc} & \begin{array}{cccc} \quad A \quad & \quad B \quad & \quad C \quad & \quad D \quad \end{array} \\ \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} & \left(\begin{array}{cccc} (26,31,41,46:0.5) & (38,43,53,58:0.5) & (46,51,61,66:0.5) & (54,59,69,74:0.5) \\ (6,11,21,26:0.5) & (16,21,31,36:0.5) & (26,29,39,44:0.5) & (28,33,43,48:0.5) \\ (10,15,25,30:0.5) & (20,25,35,40:0.5) & (28,33,43,48:0.5) & (34,39,49,54:0.5) \end{array}\right) \end{array}$$

The optimum job schedule is allocate the machines are

$$J_1 \to M_1\ , J_2 \to M_2\ , J_3 \to M_3, No\,Job \to M_4$$

The optimal solution of unbalanced trapeizoidel fuzzy assignment problems using ranking function is

=(26,31,41,46:0.5)+ (16,21,31,36:0.5)+ (28,33,43,48:0.5)

= (50,75,125,150:0.5)

R(A)=50 hours

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5.1.Numerical Examples 2:Case (i) :Proposed Method of Unbalanced Assignment Problem:

A department has four employees with six jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table 7:Matrix Entries Represent the Processing Time (in Hours)

	E_1	E_2	E_3	E_4
J_1	4	7	3	7
J_2	8	2	5	5
J_3	4	9	6	9
J_4	7	5	4	8
J_5	6	3	5	4
J_6	6	8	7	3

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

Solution: The above problem is unbalanced, ignore the balanced assignment problem.Our proposed method for solving only unbalanced assignment problemas follows:

Table 8:Matrix Entries Column Penalty Allocation Time (in Hours)

	E_1	E_2	E_3	E_4
J_1	4	7	3	7
J_2	8	2	5	5
J_3	4	9	6	9
J_4	7	5	4	8
J_5	6	3	5	4
J_6	6	8	7	3
Column Penalty	0	1	1	1
Column Penalty	0	–	1	1
Column Penalty	2	–	–	1
Column Penalty	–	–	–	1

Maximum column penalty is identified enter into second column,select the minimum values are identified . ignore the row and column.now assigned the jobs are second row and second column of job assigned.Maximum column penalty is identified enter into third column,select the minimum values are identified . ignore the row and column.now assigned the jobs are first row and third columnof job assigned.

Repeat process of the above algorithm for column penalty allocation from step 2, step 3 and step 4 as follows then the jobs are allocated one per employee, so as to the optimal assignment as follows

The jobs are allocated one per employee, so as to minimize the total man in trapeizodel fuzzy number in hours

$$J_1 \to E_{3,} J_2 \to E_2, J_3 \to E_1, J_4 \to No\,Employ, J_5 \to No\,Employ, J_6 \to E_4$$

The optimal solution of unbalanced assignment problems is =3+2+4+0+0+3=12 hours.

Case(ii): Tripeizoidel Fuzzy Unbalnced Assignment Problem using Proposed Method:

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Table 9: Matrix Entries Represent Processing Time (in Hours) Trapeizodal Fuzzy Number

	E_1	E_2	E_3	E_4
J_1	(4,5,11,12 : 0.5)	(10,11,17,18 : 0.5)	(2,3,9,10 : 0.5)	(10,11,17,18 : 0.5)
J_2	(12,13,19,20 : 0.5)	(0,1,7,8 : 0.5)	(6,7,13,14 : 0.5)	(6,7,13,14 : 0.5)
J_3	(4,5,11,12 : 0.5)	(14,15,21,22 : 0.5)	(8,9,15,16 : 0.5)	(14,15,21,22 : 0.5)
J_4	(10,11,17,18 : 0.5)	(6,7,13,14 : 0.5)	(4,5,11,12 : 0.5)	(12,13,19,20 : 0.5)
J_5	(8,9,15,16 : 0.5)	(2,3,9,10 : 0.5)	(6,7,13,14 : 0.5)	(4,5,11,12 : 0.5)
J_6	(8,9,15,16 : 0.5)	(12,13,19,20 : 0.5)	(10,11,17,18 : 0.5)	(2,3,9,10 : 0.5)

To find the optimal solution usingproposed method of column penalty allocation methodas follows

Table 10:Modified Matrix Entries Column Penalty Allocation Time (in Hours)

	E_1	E_2	E_3	E_4
J_1	(4,5,11,12 : 0.5)	(10,11,17,18 : 0.5)	(2,3,9,10 : 0.5)	(10,11,17,18 : 0.5)
J_2	(12,13,19,20 : 0.5)	(0,1,7,8 : 0.5)	(6,7,13,14 : 0.5)	(6,7,13,14 : 0.5)
J_3	(4,5,11,12 : 0.5)	(14,15,21,22 : 0.5)	(8,9,15,16 : 0.5)	(14,15,21,22 : 0.5)
J_4	(10,11,17,18 : 0.5)	(6,7,13,14 : 0.5)	(4,5,11,12 : 0.5)	(12,13,19,20 : 0.5)
J_5	(8,9,15,16 : 0.5)	(2,3,9,10 : 0.5)	(6,7,13,14 : 0.5)	(4,5,11,12 : 0.5)
J_6	(8,9,15,16 : 0.5)	(12,13,19,20 : 0.5)	(10,11,17,18 : 0.5)	(2,3,9,10 : 0.5)
Column Penalty	(–8,–6,6,8 : 0.5)	(–6,–5,8,10 : 0.5)	(–6,–4,8,10 : 0.5)	(–6,–4,8,10 : 0.5)

Maximum column penalty is identified enter into second column,select the minimum values are identified ignore the row and column.now assigned the jobs are second row and second column.

Table 12:Modified Matrix Entries Column Penalty Processing Time (in Hours)

	E_1	E_2	E_3	E_4
J_1	(4,5,11,12 : 0.5)	–	(2,3,9,10 : 0.5)	(10,11,17,18 : 0.5)
J_2	–	–	–	–
J_3	(4,5,11,12 : 0.5)	–	(8,9,15,16 : 0.5)	(14,15,21,22 : 0.5)
J_4	(10,11,17,18 : 0.5)	–	(4,5,11,12 : 0.5)	(12,13,19,20 : 0.5)
J_5	(8,9,15,16 : 0.5)	–	(6,7,13,14 : 0.5)	(4,5,11,12 : 0.5)
J_6	(8,9,15,16 : 0.5)	–	(10,11,17,18 : 0.5)	(2,3,9,10 : 0.5)
Column Penalty	(–8,–6,6,8 : 0.5)	–	(–6,–4,8,10 : 0.5)	(–6,–4,8,10 : 0.5)

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Repeat process of the above algorithm for column penalty allocation from step 2, step 3 and step 4 as follows then the jobs are allocated one per employee, so as to minimize the total man in trapeizodel fuzzy number in hours

$$J_1 \rightarrow E_{3,} J_2 \rightarrow E_2, J_3 \rightarrow E_1, J_4 \rightarrow No\,Employ, J_5 \rightarrow No\,Employ, J_6 \rightarrow E_4$$

The optimal solution of unbalanced assignment problems is

=(2,3,9,10:0.5)+(0,1,7,8:0.5)+(4,5,11,12:0.5)+ (0,0,0,0:1)+ (0,0,0,0:1)+(2,3,9,10:0.5)

=(8,12,36,40:0.5)

R(A)=12 Hours

6. Results and Discussion:

Numerical Examples	Crisp Values for Hungarian method[1,2,3]	Trapeizoidel Fuzzy Number Using Ranking function for Hungarian method[[1,2,3]	Proposed Method for crisp values	Proposed Method Trapeizoidel Fuzzy Number Using Ranking function
1	50	R(A)= (50,75,125,150:0.5) =50	50	R(A)=(50,75,125,150:0.5) =50
2	12	R(A)=(8,12,36,40:0.5) R(A)=12	12	R(A)=(8,12,36,40:0.5) R(A)=12

By our proposed method then the minimum number reduction to easily and acknowledged the optimal solution. Our proposed algorithms are simple to understand and easy to apply.

CONCLUSION

In this paper, the problem for unbalanced assignment is considered to be an imprecise number explained by numbers with trapeizoidel fuzzy which are more feasible and general throughout nature. We have formulated row penalty allocation methods and column penalty allocation methods without balancing assignment problems and minimizing everything respectively.By our proposed method then the minimum number reduction to easily and acknowledged the optimal solution for any problems without changing the order of assigning less reductions and computations. The proposed algorithms are easy to understand and easy to apply whenever it comes to identifying the fuzzy unbalanced assignment problems that occur in real life situations.

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