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Anti – Fuzzy Ideals (AFI) in Boolean Near Rings (BNR)

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Abstract:

We establish the view of anti-fuzzy ideals(AFI) in Boolean Near Rings(BNR) N and also obtain their properties in this paper. We prove every fuzzy set is an AFI of N (BNR) iff compliment of fuzzy set is FI of N and also prove that every homomorphic pre-image of an AFI is an AFI.

Keywords: Near ring, Boolean Near ring, Fuzzy set, fuzzy ideal, Anti fuzzy ideal, Anti fuzzy left ideal

I. INTRODUCTION

This concept BNR was earlier studied by S.Ligh & etl [7] and many other researchers. Zadeh [8] proposed the fuzzy subset of a nonempty set. Ziu proposed Fuzzy ideals (FI's) of rings, and was considered by many authors. The theory of FI and its properties are useful to many areas like Semi rings, Semi groups (Etc.), the idea of anti fuzzy subgroups was recognized by R. Bidwas and etl,[10] considered the concept of AFI's in near rings. We establish the concept of AFI in BNR's and study related properties of AFI's in this paper.

We now define the preliminaries of AFI in BNR.

.PRELIMINARIES

Definition 2.1: *A right near-ring (RNR) $N \neq \varphi$ equipped with two dyadic operations '+' and '.'such that*

(i) (N, +) is a group
(ii) (l.m).n = l.(m.n) for all l, m, n$\in$ N
(iii) (m+l).n= m.n + l.n for all l, m, n $\in$ N (right distributive law)

Using the left distributive law (iii)l.(m+n) = l.m+l.n in the place of (iii) would yield left near-rings.

Remark: N is not necessarily abelian .

Definition 2.2**:** A NR 'N' is said to be a BNR if $n^2 = n$ for each n $\in$ N.

Definition 2.3: Let I be a Normal subgroup of a NR (N, +, .) I is said to be a right ideal if i.n $\in$ I for all i $\in$ I, n $\in$ N. I is said to be a left ideal if n.(m+a) – n.m $\in$ I for each a$\in$ I, n, m $\in$ N. I is said to be an ideal if I is a left and a right ideal.

Note that the clause for left ideal I can also be written as n.(a+m) – n.m $\in$ I for all n, m $\in$ N, i $\in$ I.

Example 2.1 **:** (i) Every BR is a BNR.

(ii)Each constant NR N: $xy = x$ for all $x,y \in N$ is a BNR.

(iii)Let *N={0,1,2,3,4,5}* be the additive group of integers modulo 6. N is a BNR with the multiplication given in the following table

TABLE 1

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	1	1	3	1	1
2	0	2	2	0	2	2
3	3	3	3	3	3	3
4	0	4	4	0	4	4
5	3	5	5	3	5	5

Now clearly, N is a Non-zero-symmetric BNR and *{0,2,4}* , *{0,3}* are ideals.

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Definition 2.2: If Ψ is a fuzzy set on N, then Ψ is called a FI of N if Ψ satisfies the following conditions :

i. $\Psi(p-q) \geq \min\{\Psi(p), \Psi(q)\}, \forall\ p, q \in N$

ii. $\Psi(ua) \geq \Psi(a), \forall\ u, a \in N$.

iii. $\Psi((u+a)+uv) \geq \Psi(a), \forall\ u, a, v \in N$.

II. MAIN RESULTS

Definition 3.1: A fuzzy set Ψ in a BNR N is called an anti fuzzy left ideal (AFLI) of M, if

i. $\Psi(p-q) \leq \max\{\Psi(p), \Psi(q)\}$, for every $p, q \in N$

ii. $\Psi(u.a) \leq \Psi(a)$, for every $u, a \in N$.

iii. $\Psi((u+a)+u.v) \leq \Psi(a)$, for each $u, a, v \in N$.

N is a zero-symmetric BNR.

Example 3.1:

TABLE 2
Multiplication table

.	0	1	2	3
0	0	0	0	0
1	0	0	1	1
2	0	0	2	2
3	0	1	2	3

TABLE 3
Addition table

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	1	2	1	0

Clearly it is a BNR. Let Ψ be an AFI defined on N by $\Psi(x)$ - 0.65 for all $x \in M$.

Then Ψ is an AFI of M.

Theorem 3.1**:** Consider N is a BNR & Ψ is an AFI (resp. right) of N. At that moment the set $N_\Psi = \{x \in N / \Psi(p) = \Psi(0)\}$ is a left (resp. right) ideal of N.

Proof: Suppose A be an AFLI.

i) Suppose $p, q \in N_\Psi$ implies $\Psi(p) = \Psi(0)$ & $\Psi(q) = \Psi(0)$ follows that $\Psi(p-q) \leq \max\{\Psi(p), \Psi(q)\}$ which n_Ψ lies that $\Psi(p-q) \leq \max\{\Psi(0), \Psi(0)\} = \Psi(0)$. Hence $p-q \in N_\Psi$

ii) Now $\forall\ u, v \in N$, $a \in N_\Psi$

$\Psi((u+a)v+uv) \leq \Psi(a) = \Psi(0) \Rightarrow \Psi((u+a)v+uv) = \Psi(0)$ [resp. $\Psi(ua) \leq \Psi(a) = \Psi(0) \Rightarrow \Psi(ua) = \Psi(0)$]

ie. $(u+a)v+uv \in N_\Psi$ [resp. $ua \in N_\Psi$]

Theorem 3.2**:** Suppose $\{\psi_i / i \in \wedge\}$ is a family of AFI's of a BNR 'N' then so is $\vee_{i\in I} \Psi i$

Proof: Consider $\{A_i / i \in \wedge\}$ a relative of AFI's of N & let $p, q \in N$**.**

Then, $(\vee_{i\in I} \Psi_i)(p-q) = \text{Sup}\{A_i(p-q) / i \in \wedge\}$

$\leq \text{Sup}\{\max\{(\Psi_i(p), \Psi_i(q) / i \in \wedge\}$

$= \max\{\text{Sup}\{\Psi_i(p) / i \in \wedge\}, \text{Sup}\{\Psi_i(q) / i \in \wedge\}$

$= \max\{(\vee_{i\in I} \Psi_i)(p), \{(\vee_{i\in I} \Psi_i)(q)\}$

Consider $u, a \in N$**.**

Then, $(\vee_{i\in I} \Psi_i)(ua) = \text{Sup}\{\Psi_i(ua) / i \in \wedge\}$

$\leq \text{Sup}\{\Psi_i(a) / i \in \wedge\} = (\vee_{i\in I} \Psi_i)(a)$.

Now, let $u, a, v \in N$**.**

Then, $(\vee_{i\in I} \Psi_i)((u+a)v+ua) = \text{Sup}\{\Psi_i((u+a)v+ua) / i \in \wedge\} \leq \sup\{\Psi_i(a) / i \in \wedge\}$

$= (\vee_{i\in I} \Psi_i)(a)$.

Theorem 3.3**:** Suppose $\bigcap_{i\in I} \Psi_i$ of AFLI (resp. right)'s of a BNR **'N'** is an AFLI (resp. right) of N.

Proof**:** Suppose N is a BNR and suppose $\{\Psi_i / i \in I\}$ be the family of AFLI (resp. right) of N and agree to $a_1, a_2 \in N$**.** followed by, we have

i) $(\cap_{i\in I} \Psi_i)(a_1 - a_2) = \inf_{i\in I}\{\Psi_i(a_1 - a_2)\}$

$\leq \inf_{i\in I}\{\max\{\Psi_i(a_1), \Psi_i(a_2)\}\}$

$= \max\{\inf_{i\in I} \Psi_i(a_1), \inf_{i\in I} \Psi_i(a_2)\}$

$= \max(\bigcap_{i\in I} \Psi_i)(a_1), (\bigcap_{i\in I} \Psi_i)(a_2)]$

ii) Let $u, a \in N$**.** followed by, $(\bigcap_{i\in I} \Psi_i)(ua)$

$= \{\inf_{i\in I} \Psi_i(ua)\} \leq \inf_{i\in I}\{\Psi_i(a)\}$

$= (\bigcap_{i\in I} \Psi_i)(a)$

Let $u, a, v \in N$**.** Then, $(\bigcap_{i\in I} \Psi_i)((u+a)v+uv) = \{\inf_{i\in I} \Psi_i((u+a)v+uv)\} \leq \inf_{i\in I}\{\Psi_i(a)\} = (\bigcap_{i\in I} \Psi_i)(a)$

Theorem 3.4: Suppose N is a BNR. Follows that a fuzzy set Ψ is an AFI of N iff Ψ^c is a FI of N.

Proof: Consider $l, m \in N$ and Ψ is an AFI of N then we have,

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i) $\Psi^c(l - m) = 1 - \Psi(l – m) \geq 1 - \max\{\Psi(l),\Psi(m)\} = \min\{1 - \Psi(l), 1 - \Psi(m)\} = \min\{\Psi^c(l),\Psi^c(m)\}$

ii) Suppose $u, a \in N$. Then, $\Psi^c(ua) = 1 - \Psi(ua) \geq 1 - \Psi(a) = \Psi^c(a)$

iii) Let $u, a, v \in N$. Then, $\Psi^c((u+a)v + uv) = 1 - \Psi((u+a)v + uv) \geq 1 - \Psi(a) = \Psi^c(a)$

Hence Ψ^c is a FI of N. Similarly the converse follows.

Theorem 3.5: A BNR homomorphic pre-image of an AFI is an AFI.

Proof: Suppose N & S are BNR's. Consider, $f: N \to S$ is a BNR homomorphism θ is an AFI of S and Ψ be the pre image of θ under f. consider p, q a,u,v$\in$ N. follows that,

i) $\Psi(p – q) = \theta(f(p – q)) = \theta(f(p) – f(q)) \leq \max\{\theta(f(p)), \theta(f(q))\} = \max\{\Psi(p), \Psi(q)\}$

ii) $\Psi(ua) = \theta(f(ua)) = \theta(f(ua)) = \theta(f(u) f(a)$

$\leq \theta(f(ua)) = \Psi(a)$.

iii) $\Psi((u+a)v + uv)) = \theta(f(u+a)v + uv)$

$= \theta((f(u+a)v + uv)$

Hence Ψ is an AFI of **N.**

Theorem 3.6**:** Suppose Ψ is an AFLI (resp. right) of a BNR . 'N' & Ψ^+ is a fuzzy set in N given by $\Psi^+(p) = \Psi(p) + 1 - \Psi(1)$ for all $p \in N$**.** follows that Ψ^+ is an AFLI (resp. right) of **N**.

Proof**:** Suppose Ψ is an AFLI of a BNR 'N'**,** for all p, q, u, a, v $\in$ N**.** Then,

i)$\Psi^+(p– q) = \Psi(p – q) + 1 - \Psi(1)$

$\leq \max\{\Psi(p), \Psi(q)\} + 1 - \Psi(1)\}$

$= \max\{\Psi(p) + 1 - \Psi(1), \Psi(q) + 1 - \Psi(1)\}$

$= \max\{\Psi^+(p), \Psi^+(q)\}$

ii) $\Psi^+(ua) = \Psi(ua) + 1 - \Psi(1)$

$\leq \Psi(a) + 1 - \Psi(1) = \Psi^+(a)$

iii)$\Psi^+((u+a)+uv) = \Psi((u + a)v + uv) + 1 - \Psi(1)$

$\leq \Psi(a) + 1 - \Psi(1) = \Psi^+(a)$

Hence Ψ^+ is an AFLI of a BNR **N.**

Theorem 3.7**:** Suppose **N** is a BNR. a fuzzy set Ψ is normal- AFLI (resp. right) of BNR **N** $\Leftrightarrow$ $\Psi^+ = \Psi$

Proof**:** Adequate is directly follows.

T.P: The Necessary Part Suppose Ψ is normal-AFLI (res. right) of N**.**

Then $\Psi^+(x) = \Psi(x) + 1 - \Psi(1) = \Psi(x) + 1 – 1 = \Psi(x)$ for all $x \in N$. Hence, $\Psi^+ = \Psi$.

Theorem 3.8**:** suppose Ψ is an AFLI (res. right) of a BNR N follows that $(\Psi^+)^+ = \Psi^+$.

Proof**:** For any $x \in R$, we have $(\Psi^+)^+(x) = \Psi^+(x) + 1 - \Psi(1) = \Psi(x) + 1 - \Psi(1) = \Psi^+(x)$

Hence $(\Psi^+)^+ = \Psi^+$.

Theorem 3.9 **:** Suppose A be an AFLI (resp. right) of a BNR **N** & $\phi: [0, \Psi(0)] \to [0, 1]$ be an rising function. Consider Ψ_ϕ a fuzzy set in N defined by $\Psi_\phi(t) = \phi(\Psi(t))$, for all $t \in N$**.** follows that Ψ_ϕ is an AFLI (resp. right) of N.

III. Conclusion

The main theme of this paper is the study of Anti fuzzy ideals in BNR.

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