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Numerical Simulation of two Dimensional Flutter in Simple 2D Airfoil

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Abstract:

Flutter is one of the dynamic aeroelastic problems, it mainly occurs at lifting surfaces when the airplane cruises at high speeds. At relatively low speeds, the torsional stiffness of the wing is enough to counteract the twisting. However, the variation in flutter frequency causes the instabilities in aircraft motion. The numerical simulation of airfoil vibrations is done. To understand Differential Governing Equation for Dynamic Aeroelastic System. This paper focus on theuse of Modal Analysis Method to find the pitching, plunging motion of airfoil& to calculate the frequency, damping ratio when an Airfoil is subjected into Dynamic Aeroelastic instability. Modal analysis method is performed through MATLAB. In ANSYS we analyze the turbulence flow patterns due to pitching and plunging of Airfoil.

Keywords:*Building Information Model (BIM); Cost Estimation (5D); visualization model (3D); Quantity Take-Offs (QTO), two dimensions (2D).*

1.INTRODUCTION

Aeroelasticity phenomena involve the study of the interaction between aerodynamic forces and elastic forces are known as static aeroelasticity, also aerodynamic forces, inertia forces and elastic forces are known as dynamic aeroelasticity, finally aerodynamic forces, inertia forces, elastic forces and control laws are known as aero-servoelasticity.

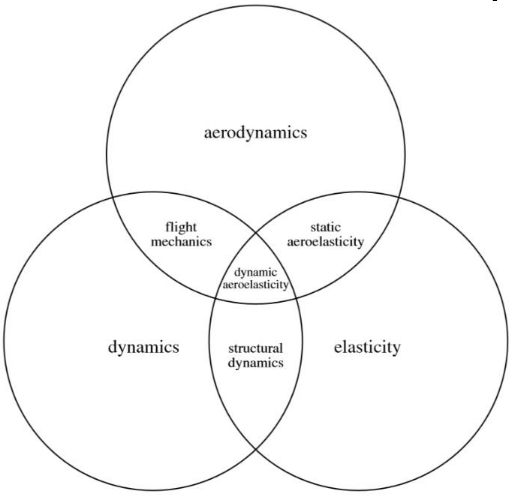

Fig: 1 Schematic of the field of Aeroelasticity

The flexibility of the modern aircraft structures makes aeroelastic study an important aspect of aircraft design and stability verification procedures.The wing torsional divergence and the flutter are the two major aeroelastic phenomena considered in aircraft design. The divergence is a static instability which occurs when the static aerodynamic effects counteract the torsional stiffness of the structure. The flutter is a dynamic aeroelastic instability characterized by sustained oscillation of structure arising from interaction between elastic, inertial and aerodynamic forces acting on the body.

1.1 DYNAMIC AEROELASTICITY

A dynamic instability of a flight vehicle associated with the interaction of aerodynamic, elastic and inertial forces. Flutter problems is considered from the field of dynamic stability. Then, the modal representation is used to set up a lifting surface flutter analysis as a linear set of ordinary differential equations. These are transformed into an eigenvalue problem, and the stability characteristics are then discussed in terms of the eigenvalues.

To complete the set of analytical tools needed for

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flutter analysis, two very different unsteady aerodynamic theories are outlined, one suitable for use with classical flutter analysis and its derivatives, and the other suitable for eigenvalue-based flutter analysis.

A. AEROELASTIC ANALYSIS OF A TYPICAL SECTION

We demonstrate the flutter analysis of a linear aeroelastic system. To do this a simple model is needed. This configuration could represent the case of a rigid, two-dimensional wind-tunnel model that is elastically mounted in a wind-tunnel test section, or it could correspond to a typical airfoil section along a finite wing. In the latter case the discrete springs would reflect the wing structural bending and torsional stiffnesses, and the reference point would represent the elastic axis.

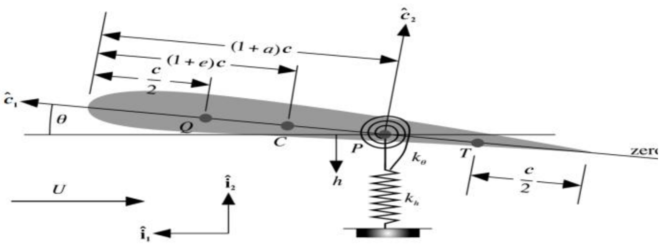

Fig:2 Schematic showing geometry of the wing section with pitch and plunge spring restraints.

In the above model,
P - reference point (i.e., where the plunge displacement h is measured)
C - the center of mass
Q - aerodynamic center (presumed to be the quarter-chord in thin-airfoil theory)
T - the three- quarter-chord (an important chordwise location in thin-airfoil theory).
The rigid plunging and pitching of the model is restrained by light, linear springs with spring constants kh and kθ. It is convenient to formulate the equations of motion from Lagrange's equations. To do this, one needs kinetic and potential energies as well as the generalized forces resulting from aerodynamic loading. The potential energy can be written as

$$P = \frac{1}{2} K_h \mathrm{h}^2 + \frac{1}{2} K_\theta \theta^2$$

To deduce the kinetic energy, one needs the velocity of the mass center C, which can be found as

$$V_C = V_P + \dot{\theta}\,\hat{c}_3 \times c\,[(1 + a) - (1 + e)]\,\hat{c}_1$$

Where the inertial velocity of the reference point P is

$$V_P = -\dot{h}\,\hat{\imath}^2$$

$$V_C = -\dot{h}\,\hat{\imath}_2 + c\,\dot{\theta}(a-e)\,\hat{c}_2$$

$$K = \frac{1}{2} m V_C{}^2 + \frac{1}{2} I_c \dot{\theta}^2$$

Where IC is the moment of inertia about C. By virtue of the relationship between $\hat{c}_2$ and the inertially fixed unit vectors $\hat{\imath}_1$ and $\hat{\imath}_2$, assuming θ to be small,

$$K = \frac{1}{2} m\,(\dot{h} + c^2 \dot{\theta}^2 x_\theta{}^2 + 2c\,x_\theta \dot{h}\dot{\theta}) + \frac{1}{2} I_c \dot{\theta}^2$$

$$K = \frac{1}{2} m\,(\dot{h} + 2c\,x_\theta \dot{\theta}\,\dot{h}) + \frac{1}{2} I_p \dot{\theta}^2$$

$$I_p = I_c + mc^2 x_\theta{}^2$$

The generalized forces associated with the degrees of freedom h and θ are easily derived from the work done by the aerodynamic lift through a virtual displacement of the point Q and by the aerodynamic pitching moment about Q through a virtual rotation of the model.
The velocity of Q is

$$V_Q = -\dot{h}\,\hat{\imath}_2 + c\dot{\theta}\left(\frac{1}{2} + a\right)\hat{c}_2$$

$$\delta P_Q = -\dot{h}\,\hat{\imath}_2 + c\delta\theta\left(\frac{1}{2} + a\right)\hat{c}_2$$

The virtual displacement of the point Q can be obtained simply by replacing the dot over each unknown in V_Q equation with a δ in front of it

$$\delta P_Q = -\delta h\,\hat{\imath}_2 + c\delta\theta\left(\frac{1}{2} + a\right)\hat{c}_2$$

δP_Q is the virtual displacement at Q. The angular velocity of the wing is $\dot{\theta}\hat{c}_3$, so that the virtual rotation of the wing is simply $\delta\theta\hat{c}_3$. Therefore, the virtual work of the aerodynamic forces is

$$\overleftrightarrow{\delta W} = L\left(-\delta h + c\,\delta\,\theta\left(\frac{1}{2} + a\right)\right) + M_{(1/4)}$$

and the generalized forces become

$$Q_h = -L$$

$$Q_\theta = M_{(1/4)} + c\left(\frac{1}{2} + a\right)L$$

It is clear that the generalized force associated with h is the negative of the lift, whereas the one associated with θ is the pitching moment about the reference point P.
Lagrange's equations are here specialized for the case in which the kinetic energy K depends only on $\dot{q}_1, \dot{q}_2, \ldots$, and so

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$$\frac{d}{dt}\left(\frac{\partial K}{\partial q_i}\right)+\frac{\partial P}{\partial q_i} = Q_i \qquad (i = 1,2,...,n)$$

Here n = 2, q1 = h, and q2 = θ and the equations of motion become

$$m\,(\ddot{h} + c\,x_\theta\,\ddot{\theta}) + K_h\,h = -L$$

$$I_P\ddot{\theta} + m\,c\,x_\theta\,\ddot{h} + K_\theta\theta = M_{(1/4)} + c\left(\frac{1}{2} + a\right)L$$

Consider, once again, the two-dimensional aerofoil with the flexural axis positioned a distance ec aft of the aerodynamic centre and ab aft of the mid chord, where

$$ec = \frac{c}{4} + ab$$

$$ec = \frac{c}{4} + \frac{ac}{2}$$

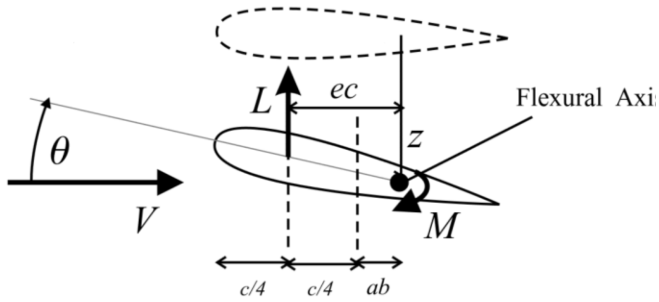

Fig: 3 Two-dimensional airfoil.

The lift and moment per unit span for an airfoil may be expressed, for a particular reduced frequency, as

$$L = \rho V^2 \left(L_Z Z + L_{\dot{Z}}\frac{s\dot{Z}}{V} + L_\theta s\theta + L_{\dot{\theta}}\frac{s^2\dot{\theta}}{V}\right);$$

$$M = \rho V^2 \left(M_Z sZ + M_{\dot{Z}}\frac{s^2\dot{Z}}{V} + M_\theta s^2\theta + M_{\dot{\theta}}\frac{s^3\dot{\theta}}{V}\right)$$

V will be taken as the true air speed and ρ is the density at a prescribed altitude. Taking the quasi-steady assumption (k → 0, F → 1, G→ 0)for all of the aerodynamic derivatives, then the lift and pitching moment per unit span about the flexural axis become

$$L = \frac{1}{2}\rho V^2 c\, a_1\left(\theta + \frac{\dot{Z}}{V}\right)$$

$$M = \frac{1}{2}\rho V^2 e\, c^2 a_1\left(\theta + \frac{\dot{Z}}{V}\right)$$

The $M_{\dot{\theta}}$ unsteady aerodynamic derivative term will retain as it has been shown that this has an important effect on the unsteady aerodynamic behaviour. It adds a pitch damping term to the pitching moment Equation and the model then becomes

$$M = \frac{1}{2}\rho V^2 c^2 \left(e\, a_1\left(\theta + \frac{\dot{Z}}{V}\right) + M_{\dot{\theta}}\frac{\dot{\theta}c}{4V}\right)$$

Where M is negative and will initially be assumed to be constant. This „simplified unsteady aerodynamic" model will now be used to develop a binary aeroelastic model. The simple unswept/untapered (i.e. rectangular) wing model shown in Figure (4). The rectangular wing of span s and chord c is rigid but has two rotational springs at the root to provide flap (k) and pitch (θ) degrees of freedom. Note that there is no stiffness coupling between the two motions. The springs are attached at a distance ec behind the aerodynamic centre (on the quarter chord), defining the position of the flexural axis. The wing is assumed to have a uniform mass distribution and thus the mass axis lies on the mid- chord.

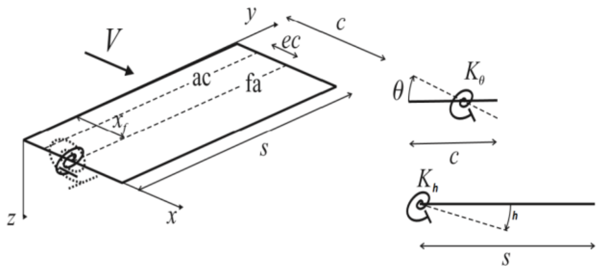

Fig: 4 Binary aeroelastic model.

The displacement z (downwards+ve) of a general point on the wing is

$$Z(x, y,t) = y\,h(t)+(x - x_f)\,\theta(t)$$
$$= \varphi_h h + \varphi_\theta\,\theta$$

Whereh and θ are generalized coordinates and φ_h and φ_θ are simple assumed shapes. They are actually normal mode shapes (i.e. pure flap and pitch) if there is no inertia coupling about the flexural axis. The equations of motion can be found using Lagrange’s equations. The kinetic energy now exists due to the dynamic motion and is

$$T = \int_{wing} \frac{1}{2}\, dm\,\dot{Z}^2$$

$$T = \frac{m}{2}\int_0^s\int_0^c (y\dot{h} + (x - x_f)\dot{\theta})^2\, dx\, dy$$

Where m is the mass per unit area of the wing. The potential (or strain) energy is due solely to the springs at the root, such that

$$U = \frac{1}{2} K_h h^2 + \frac{1}{2} K_\theta \theta^2,$$

Whereas for a general bending and torsional vibration of a flexible wing it would take the form

$$U = \frac{1}{2}\int EI\left(\frac{d^2z}{dy^2}\right)^2 dy + \frac{1}{2}\int GJ\left(\frac{d\theta}{dy}\right)^2 dy$$

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$$\frac{dT}{dt}\left(\frac{dT}{d\dot{h}}\right) = m\int_0^s\int_0^c\left(y^2\ddot{h} + y(x - x_f)\ddot{\theta}\right)dx\,dy$$

$$= m\left(\frac{s^3c}{3}\ddot{h} + \frac{s^2}{2}\left(\frac{c^2}{2} - x_f c\right)\ddot{\theta}\right)$$

$$\frac{dT}{dt}\left(\frac{dT}{d\dot{\theta}}\right) = m\int_0^s\int_0^c\left[y(x - x_f)\ddot{h} + (x - x_f)^2\ddot{\theta}\right]dx\,dy$$

$$= m\left(\frac{s^2}{2}\left(\frac{c^2}{2} - x_f c\right)\ddot{h} + s\left(\frac{c^3}{3} - c^2 x_f + x_f^2 c\right)\ddot{\theta}\right)$$

and $\frac{\partial U}{\partial h} = K_h h$,

$$\frac{\partial U}{\partial \theta} = K_\theta\,\theta,$$

Applying Lagrange's equations for both generalized coordinates gives leading to the equations of motion for the wing, without any aerodynamic forces acting, as

$$\begin{bmatrix} \frac{ms^3c}{3} & \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) \\ \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) & ms\left(\frac{c^3}{3} - c^2 x_f + x_f^2 c\right) \end{bmatrix}\begin{Bmatrix}\ddot{h}\\ \ddot{\theta}\end{Bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix}\begin{Bmatrix}h\\ \theta\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Applying strip theory, together with the simplified unsteady aerodynamics representation, leads to expressions for lift and pitching moment (about the flexural axis) for each elemental strip dy of

$$dL = \frac{1}{2}\rho V^2 c\,dy\,a_W\left(\frac{y\dot{h}}{V} + \theta\right)$$

$$dM = \frac{1}{2}\rho V^2 c^2 dy\left(e a_W\left(\frac{y\dot{h}}{V} + \theta\right) + M_{\dot{\theta}}\frac{\dot{\theta}c}{4V}\right)$$

Where $y\dot{h}$ is the effective heave velocity (+ve downwards) and M_{θ} <0). Thus, the full aeroelastic equations of motion become and it may be seen that the mass and stiffness matrices are symmetric while the aerodynamic matrices are Non symmetric. Thus the two DOF are coupled and it is this coupling that can give rise to flutter.

$$\begin{bmatrix} \frac{ms^3c}{3} & \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) \\ \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) & ms\left(\frac{c^3}{3} - c^2 x_f + x_f^2 c\right) \end{bmatrix}\begin{Bmatrix}\ddot{h}\\ \ddot{\theta}\end{Bmatrix} + \rho V\begin{bmatrix} \frac{cs^3 a_W}{6} & 0 \\ \frac{-ec^2 s^2 a_W}{4} & \frac{-sc^3}{8}M_{\dot{\theta}} \end{bmatrix}\begin{Bmatrix}\dot{h}\\ \dot{\theta}\end{Bmatrix} + \rho V^2\begin{bmatrix} 0 & \frac{cs^2 a_W}{4} \\ 0 & \frac{-e a_W s c^2}{2} \end{bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix}\begin{Bmatrix}h\\ \theta\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

I. MODAL ANALYSIS METHOD

The matrix form of equation of vibration for a force response of multiple degree of freedom system can be calculated by use of modal analysis, the equation of motion takes the form

$M\ddot{x} + C\dot{x} + Kx = F(t)$

M = Mass Matrix, C = Damping Matrix, K = Stiffness Matrix.

F (t) = Force Matrix;

Steps to Solve Modal Analysis:

1. Calculate $(M)^{-\frac{1}{2}}$.
2. Calculate $\tilde{K} = (M)^{-\frac{1}{2}}\,K\,(M)^{-\frac{1}{2}}$, the mass normalised stiffness matrix.
3. Calculate $\tilde{C} = (M)^{-\frac{1}{2}}\,C\,(M)^{-\frac{1}{2}}$, the mass normalised damping matrix.
4. Calculate the symmetric Eigen value Problem for $\tilde{K}$ to get w_i^2 and v_i.
5. Normalize v_i and form the magtrix P = $[v_1\ v_2]$.
6. Calculate S = $(M)^{-\frac{1}{2}}$ P and $S^{-1} = P^T(M)^{-\frac{1}{2}}$.
7. Calculate $P^T\tilde{C}P$ and $P^T\tilde{K}P$.
8. Find out the values of ξ_1 and ξ_2 (for $0 < \xi < 1$).
9. Now find the Decoupled model equation. (r (t)).
10. x = $(M)^{-\frac{1}{2}}$ P r (t). ; here x = $\begin{Bmatrix}h(t)\\ \theta(t)\end{Bmatrix}$
11. Now write the equation of h (t) and θ (t).
12. Now plot the graph for h(t) and θ(t) Vs Time where,

    Time (sec) along x- axis and h (t) in meters and θ(t) in radians along y axis.

$$\ddot{x} = \begin{Bmatrix} \ddot{h} \\ \ddot{\theta} \end{Bmatrix} \;;\quad \dot{x} = \begin{Bmatrix} \dot{h} \\ \dot{\theta} \end{Bmatrix} \;;\quad x = \begin{Bmatrix} h \\ \theta \end{Bmatrix}$$

BOUNDARY CONDITIONS:	
Semi-span(s)	8m
Chord (c)	2m
Flexural axis (x_f)	0.48c
Mass axis (x_c)	0.5c
Mass per unit area	100 kg/m^2
Flap stiffness (K_h)	$I_h(5\times 2\pi)^2$Nm/rad
Pitch stiffness (K_θ)	$I_\theta(10\times 2\pi)^2$Nm/rad
Lift curve slope (a_w)	2π
Nondimensional pitch damping derivative (M_θ)	-1.2
Air density (ρ)	1.225 kg/m^3

The modal analysis method is done through MATLAB using the boundary conditions.

A. Calculated Damped frequency:

$$w_h = \sqrt{\frac{K_h}{I_h}} \quad w_\theta = \sqrt{\frac{K_\theta}{I_\theta}}$$

w_h = 5 Hz. $\qquad$ w_θ = 10Hz.

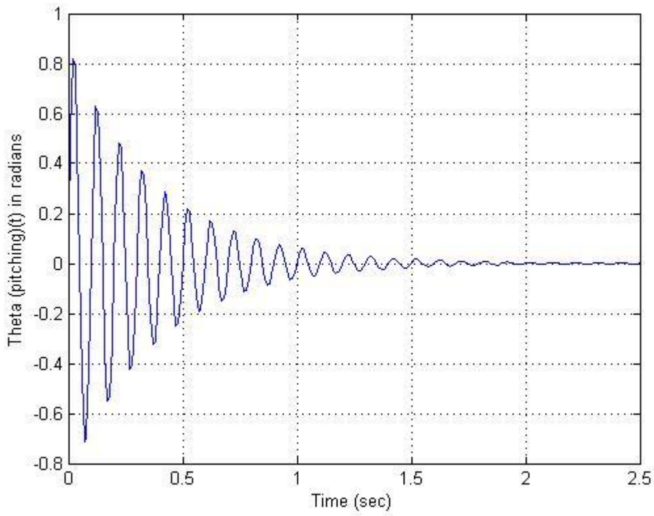

Fig: 5 Time (sec) Vs Pitching Graph (radian)

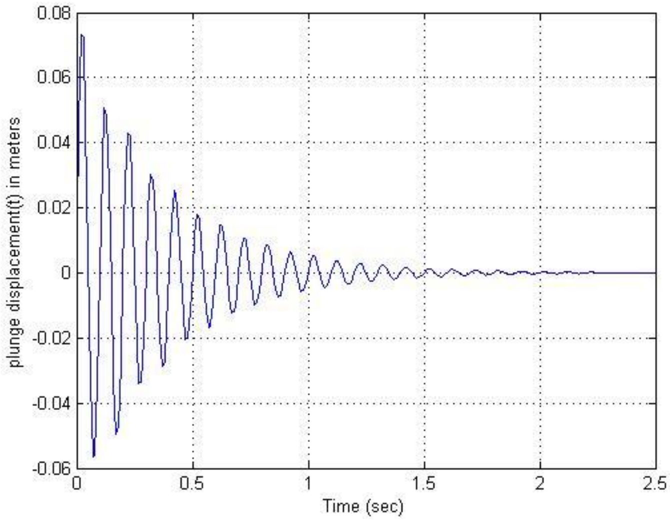

Fig: 6Time (sec) Vs Plunging Displacement (meter)

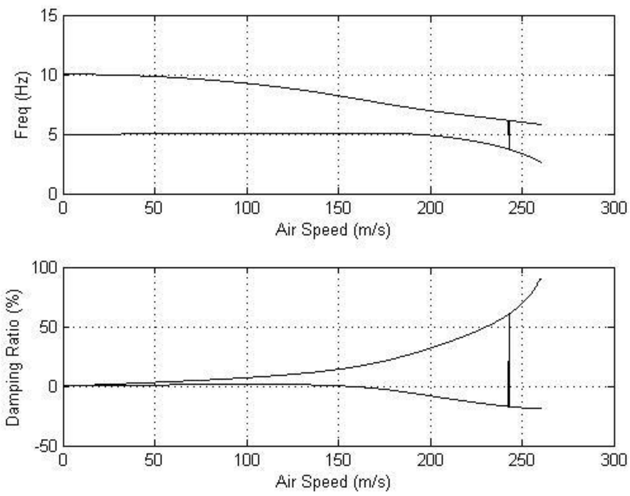

Fig: 7 Air Speed (m/s) Vs Frequency (Hz) and Damping Ratio(%).

II. Conclusion

Thus, the full aeroelastic equations of motion is solved with the help of modal analysis method through MATLAB obtaining the graph with pitching angle 0.8 radians and plunging displacement 0.08 meter with a frequency of 10Hz and 5Hz Frequency and damping trends for the modified system with the unsteady aerodynamics term included.

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Nomenclature

L - Lift (N)

M - Pitching moment (N-m)

ρ - Density (Kg/m3)

V - Airspeed (m/s)

s- Semi-Span (m)

h- Linear displacement (m)

θ - Angular displacement (m)

c - Chord (m)

a_w - Lift curve slope

M_θ- Unsteady Aerodamping derivative

m - mass/Unit Area (Kg/m2)

M_Z, L_Z - Non-dimensional numbers

K_h- Flap Stiffness (Nm/rad)

K_θ- Pitch Stiffness (Nm/rad)

ξ- Damping Ratio

w_h - Flap frequency (Hz)

w_θ - Pitch frequency (Hz)

x_f - Flexural axis (m)

x_c - Mass axis (m)