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Inventory Models with Proportionate Discount under Learning Effect to Maximize Profit on Imperfect Quality Items

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***Abstract*:**
Two economic order quantity (EOQ) models for items with imperfect qualities were developed by takingallowable proportionate discount for scrap type defective items and a fixed selling price for both good and rework type items after the end of screening process under without and with learning effectfor different parameters: Inthe Type-I model(without learning):the incoming lot has fractions of both scrap and rework items and these fractions are considered random variables with known probability density functions. In theType-II model (with learning): learning is taken for both the holding cost and the ordering cost. Demands for both models are fulfilled from perfect and reworked items. These concepts are best fitted in automotive industries where due to learning, holding cost and ordering cost reduces from one shipment to another. The automotive industries can earn more profits by considering the effect of learning on holding cost and ordering cost in each lot. The objective is to obtain the maximum total profits for both the models. Numerical results are provided to illustrate the developed models and sensitivity analysis is conducted to understand the effect of scrap and reworked items on the total profit and lot size with and without learning effect on different parameters.

Keywords:Imperfect quality;Proportionate discount; Rework rate; Screening rate; Learning effect.

1. Introduction

In every manufacturing organization inventory plays the role of lifeblood. Although, thetraditional economic order quantity (EOQ) model has been successfully applied in the area of inventory management since long; it contains a few unrealistic assumptions. One of the unrealistic assumptions in the EOQ model is that all the items are of perfect quality whereas, in reality, the production process is not always free from defects. A fraction of the produced itemsis always producedwith defects. During the last decades a lot of work has been done in the area of inventory problems on imperfect quality items.The two common assumptionsfor any traditional EOQ models arethe presence of all perfect quality items andno shortages of products. Both of these assumptions fail to copewith the realistic situations in the business scenario. Imperfect quality items in an inventory system are due tothe presence of imperfect production process, damages, or for other uncertain reasons. Rosenblatt & Lee (1986), and Porteus (1986)

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discussedon deterministic inventory models by considering a low cost for there-work ofdefective items. Zhang & Gerchak(1990) developed an inventory model with a joint lot sizing and inspection policy by assuming that a random proportionof lot sizes are defective.Salameh &Jaber (2000)assumed that the defective items could be sold at a discounted price at the end of a cent per cent screening of all theitems. Goyal &Cárdenas-Barrón (2000) extended the work of Salameh &Jaber (2000) and optimized the lot size. Papachristos &Konstantaras (2006) revised the EOQ inventory model of Salameh &Jaber (2000) by introducing randomness for the imperfect items.Wee et al. (2007) developed aninventory modelfor imperfect quality items having shortages with a 100% screening. Maddah &Jaber (2008) developed an EOQ model characterized by a random fraction of imperfect quality items under screening process. Maddah et al. (2009) established a production inventory model where the produced/purchased items are of two different qualities. Salamah &Alsawafy (2012) developed an economic order quantity model ofboth scrap and re-workable type of defective items along with perfect quality items.

Research shows that the effect of learning plays a very important role in the case of economic production/order models.The ‘S’-shaped learning curve was introduced by Jordan (1958).Jaber et. al. (2008) introduced an inventory model for imperfect quality items subjected to learning effect. Jaber (2011) introduced learning in different business sectors. Yadav et al. (2012) established an inventory model where defective rate, ordering cost and holding cost are subjected to learning effect and the optimum result for the profit is obtained algebraically. Mukhopadhyay &Goswami (2013) considered an EPQ inventory model for items of imperfect quality with learning in the set-up costs. Singh et al. (2013) developed an inventory model of items with imperfect quality and used the learning effect under two limited storage capacity. Alamri et al. (2016) presented an EOQ model for items that are subjected to inspection for imperfect quality where the percentage of defective items in each lot reduces with the effect of the learning curve.Patro et al. (2017a)considered the learning effect for defective rate. Thereafter,Patro et al. (2017b) consideredEOQ model where learning is taken for ordering and holding costs. Later,Patro et al. (2017c) have extended the work of Patro et al. (2017b) by considering two warehouses. Bazan et al. (2017) developed an environmentally responsible closed-loop supply chain models with anominal proportion of items returned for remanufacturing.The contribution and differences of previous work and our work presented by Table 1.

Table 1. Contribution of this paper

Reference	Objective	Contributions	Limitations	Remarks
Jaber 2011	Finding EOQ	EOQ is given by Numerical Technique	Constant demand	Discount is constant
Patro et al. 2017(a)	Finding EOQ	Same as above	Demand rate is fuzzy	Discount is proportionate and effect of learning on defective rate, fixed ordering cost and holding cost
Patro et al. 2017(b)	Finding EOQ	Same as above	Demand rate is fuzzy	Discount is proportionate and effect of learning on holding cost, ordering cost and the

				number of lot wise defectives
Patro et al. 2017(c)	Finding EOQ	Same as above	Constant demand	Discount is proportionate
Bazan et al. (2017) 2017	Finding Supply Chain Model	Same as above	Constant demand and having no shortages	A nominal proportion of items returned for remanufacturing where the model has both manufactured and re-manufactured products
This paper	Finding EOQ	Same as above	Constant demand	Discount is proportionate

A discount pricing strategy can produce great results if used under the right circumstances and in the right form. Offeringdiscounts to customers on purchases is a way to quickly draw people into the stores. Discounts not only help the shoppers; they also help the business. From increased sales to improved reputation, discounts may be that one ingredient that can bring business success.Till date, researchers used learning for different parameters with the constant/proportionate rate of discount for lot wise defective items to find both total profits and optimal order lot size. Therefore, the objective of this paper is to formulate the models both for without and with learning for both scrap and re-workable defective items and then to know the total profit and lot size for each cycle by introducing learning for parameters present in the models and also to introduce proportionate discounts for the scrap items.

Two different EOQ models are developed fortwo kinds of imperfect quality items by introducing allowable proportionate discount without and with learning effect on different parameters. The incoming lot has fractions of both scrap and rework items. A 100% screening process is conducted for each lot.After the screening processfor both types of models (with and without learning) the scrap items are sold as a single batch at a proportionate discount and both good and rework items are sold in a fixed price. In the case of without learning the incoming lot has both fractions of scrap and rework able items and theseare considered to be random variables with known probability density functions (PDF). In case of with learning,the learning effect is taken for both the holding cost and the ordering cost. Finally, numerical examples are provided and a sensitivity analysis is conducted to know the effect of scrap and reworked items on the total profit and lot size with and without learning effect on different parameters.

The remainder section of the paper is organized as follows: 2-assumptions and notations. 3- brief introduction to learning curve. 4- model formulation. 5-numerical examples and result analysis. 6- conclusion and future scope.

2. Notations and Assumptions

2.1.Notations

Decision variables

- z : Size of the ordering quantity.
- $ETPU(z)$: Expected total profit per unit time in z.

Input and other parameters

- C_p: Variable unit cost.
- C_k: Fixed ordering cost.
- D: Demand rate.
- C_h: Holding cost per unit.
- P_S: % of scrap items.
- P_R: % of rework items.
- p: % of scrap and rework items in z.
- $f(P_S)$:PDF of P_S.
- $f(P_R)$:PDF of P_R.

- t_1: Period of screening.
- t_2: Period of rework.
- t_3: Leftover time period to utilize the total inventory after receiving re-worked items.
- I_1: Available level of inventory after screening.
- I_2: Available level of inventory after selling of the scrap and returned reworked items.
- I_3: Available level of inventory before receiving the reworked items.
- S_g: Per unit selling price of good quality item.
- w: Rate of screening.
- L: Rate of rework.
- R: Unit rework cost.
- C_s: Unit screening cost.
- T: Length of one cycle.
- TR: Total revenue.
- TP: Total profit.
- TPU: Total profit in unit time.
- $C_k(n)$:Ordering cost per order includes a fixed constant C_{k_0} and a decreasingcycle $\frac{C_{k_1}}{n^{\beta_1}}$.
- $C_h(n)$: Fixed holding cost per order C_{h_0} and a decreasing cycle $\frac{C_{h_1}}{n^{\beta_2}}$.

2.2.Assumptions

- Constant demand rate.
- Absence of shortage /no shortage.
- Instantaneous delivery of lot.
- Known probability density function for percentage of defective items.
- Fixed selling price of good quality items.
- Single batch selling of defective items at a proportionate discount.
- 100% inspection of shipment items.
- Zero lead time.
- Error free screening and rework processes.
- Sufficient availability of good quality itemsto satisfy the demand during screening period.

3. Learning curves

Learning (or experience) curve theory has a wide range of applications in inventory problems. Learning in inventory problems relates to a repetitive job and productivity (cost) i.e., the productivity increases (and the cost decreases) as there is more experience, which is/because of the (number of times the same job is repeated). In this context, it has been observed that the time required to produce a unit item decreases as the operator produces more units of same type. In the present model learning is used to get both $C_k(n)$ and $C_h(n)$ depending on the number of shipments. Because of learning, $C_k(n)$ and $C_h(n)$ are two decreasing functions of the number of shipments. The graphs of $C_k(n) = C_{k_0} + \frac{C_{k_1}}{n^{\beta_1}}$ and $C_h(n) = C_{h_0} + \frac{C_{h_1}}{n^{\beta_2}}$ under learning effect are as follows.

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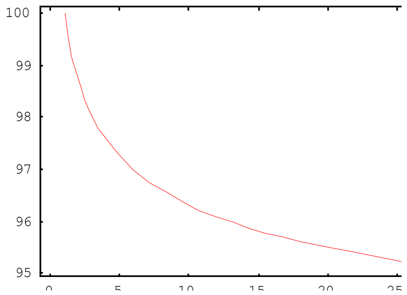
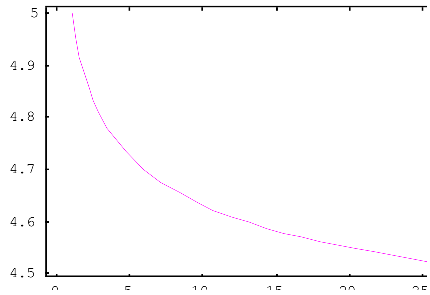


[Fig.1. Behavior of $C_k(n)$ versus n]

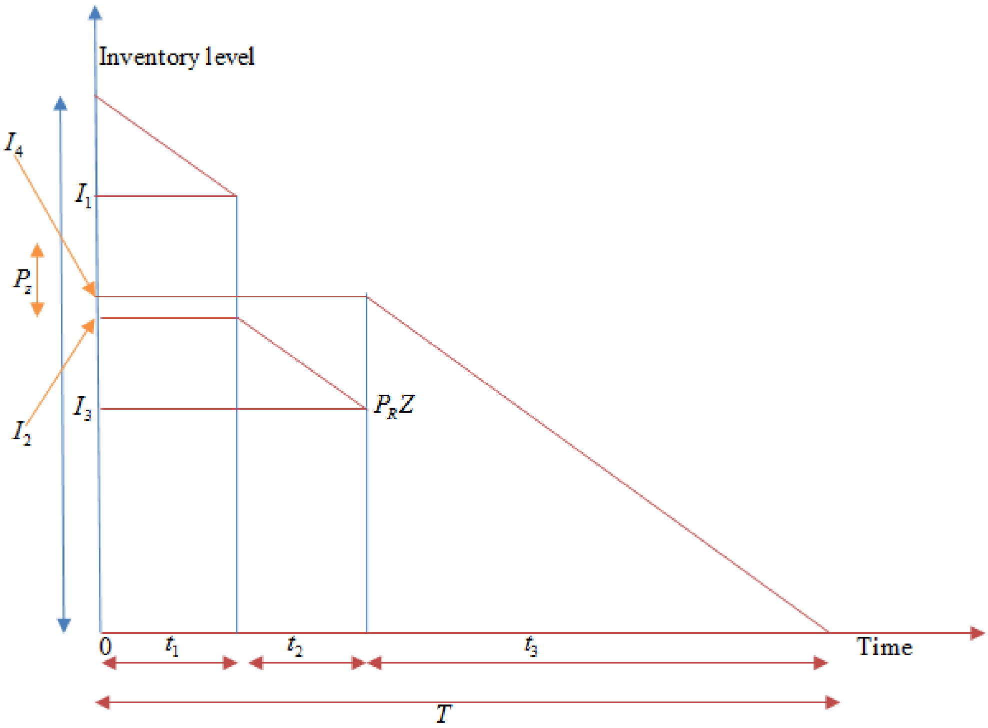


[Fig.2. Behavior of $C_h(n)$ versus n]

Here, C_{k_0} (or C_{h_0}), C_{k_1} (or C_{h_1}) and β_1 (or β_2) present in $C_k(n)$ (or $C_h(n)$) are three fixed positive constants and the ordering cost $C_k(n)$ (or $C_h(n)$) decreases as the number of shipment increases under learning effect.

4. Description of mathematical model

In the present model, the lot size z is deliveredinstantaneously with cycle length T ,fixed C_k and unit purchasing price C_p . Each received lot contains a fraction of scrap items P_S and rework items P_R with known probability density functions $f(P_S)$ and $f(P_R)$ respectively. In each lot,screening is allowed with C_s as the unit screening cost at a rate of w units per unit time. The screening process is done for all the lot at a rate of w units per unit time.The good quality and reworked items are sold with a fixed price S_g per unit where the scrap items are sold as a single batch with a proportionate rate of discount(in arithmetic progression (AP)). The behavior of the inventory level is shown in Fig.3.



[Fig.3. Behavior of inventory level over time]

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4.1. Without learning

The number of good quality items is at least equal to the demand during the screening time t_1 i.e., $(1-P_S-P_R)z \geq Dt_1$ (4.1.1)

where the shortages are avoided. In eq. (4.1.1), D is the yearly constant demand and substituting the value of $t_1=\frac{z}{w}$, we get $P_S+P_R \leq 1-\frac{D}{w}$; (4.1.2)

where $w \geq D$.

Now the following holds for P_S and P_R.

$E[P_S]+E[P_R] \leq 1-\frac{D}{w}$, $w \geq D$.(4.1.3)

During the time period t_1 the screening process is conducted, where $t_1=\frac{z}{w}$. (4.1.4)

Before the end of the screening process the inventory level I_1 reaches at the following level.

$$I_1=\left(1-\frac{D}{w}\right)z. \quad (4.1.5)$$

During the time period t_2 the items needing rework are sent and again received back:

$$t_2=\frac{E(P_R)z}{L}. \quad (4.1.6)$$

After the scrap items are removed from the lot and the reworked item are received, the inventory level I_2 is given as follows:

$$I_2=\left(1-P-\frac{D}{w}\right)z. \quad (4.1.7)$$

Just before the reworked items are received, the inventory level I_3 reaches the following level:

$$I_3=\left(1-P-\frac{D}{w}-\frac{D\ E[P_R]}{L}\right)z. \quad (4.1.8)$$

When the reworked items are included along with the good quality items, the inventory level I_4 reaches the following level:

$$I_4=\left(1-P_S-\frac{D}{w}-\frac{D\ E[P_R]}{L}\right)z. \quad (4.1.9)$$

Hence, $t_3=\frac{I_4}{D}$

(4.1.10)Different types of costs:

Procurement cost: $C_k+C_p z$; Screening cost: $C_s z$; Rework cost: $R\ E[P_R]\ z$.

Expected inventory holding cost: $C_h\left[\frac{E(1-P_S)zE(T)}{2}+\frac{E(P_S)z^2}{x}-\frac{E(P_R)^2 z^2}{L}\right]$.

Expected total cost :

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$$E(TC(z)) = C_k + C_p z + C_s z + R\ E[P_R]\ z + C_h\left[\frac{E(1-P_S)zE(T)}{2} + \frac{E(P_S)z^2}{x} - \frac{E(P_R)^2 z^2}{L}\right]. \quad (4.1.11)$$

The estimated proportionate rate of discount for the scrap items is as follows.

$\left(1-\dfrac{zE(P_S)-i}{zE(P_S)}\right)\left(\dfrac{E(TR(z))-E(TC(z))}{z}\right)$; where $i = 1,2,3.....,\left[zE(P_S)\right]$; where, $\left[zE(P_S)\right]$ is the greatest integer $\le zE(P_S)$.

Hence,

$$E(TR(z)) = S_g z(1-E(P_S)) + \sum_{i=1}^{[zE(P_S)]}\left(S_g - \left(1-\frac{zE(P_S)-i}{zE(P_S)}\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)\right) \quad (4.1.12)$$

Simplifying (4.1.12), we get:

$$E(TR(z)) = \frac{2S_g z^2 + \left\{\left(C_k + C_p z + C_s z + R\ E[P_R]\ z + C_h\left[\frac{E(1-P_S)zE(T)}{2} + \frac{E(P_S)z^2}{x} - \frac{E(P_R)^2 z^2}{L}\right]\right)(zE(P_S)+1)\right\}}{2z + zE(P_S)+1}. \quad (4.1.13)$$

Hence, expected total cycle profit $E\left(TP\left(z\right)\right) = E\left(TR\left(z\right)\right) - E\left(TC\left(z\right)\right)$

$$= \frac{2S_g z^2 + \left\{\left(\begin{array}{l} C_k + C_p z + C_s z + R\ E[P_R]\ z \\ +C_h\left[\frac{E(1-P_S)zE(T)}{2} + \frac{E(P_S)z^2}{x} - \frac{E(P_R)^2 z^2}{L}\right]\end{array}\right)(zE(P_S)+1)\right\}}{2z + zE(P_S)+1}$$
$$-\left(C_k + C_p z + C_s z + R\ E[P_R]\ z + C_h\left[\frac{E(1-P_S)zE(T)}{2} + \frac{E(P_S)z^2}{x} - \frac{E(P_R)^2 z^2}{L}\right]\right). \quad (4.1.14)$$

Simplifying (4.1.14), we get:

$$E\left(TP\left(z\right)\right) = \frac{2S_g z^2 - 2z\left(C_k + C_p z + C_s z + R\ E[P_R]\ z + C_h\left[\frac{E(1-P_S)z\ E(T)}{2} + \frac{E(P_S)z^2}{x} - \frac{E(P_R)^2 z^2}{L}\right]\right)}{2z + zE(P_S)+1}. \quad (4.1.15)$$

Expected total profitper unit time $E\left(TPU(z)\right)$:

$$E\left(TPU(z)\right) = \frac{E\left(TP\left(z\right)\right)}{E(T)} = \frac{2D\left(S_g z - C_k - C_p z - C_s z - R\ E[P_R]z\right)}{E(1-P_S)\left(2z + zE[P_S]+1\right)}$$

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$$-\frac{C_h z^2\left[\left(E(1-P_S)\right)^2+\frac{2DE[P_S]}{w}-\frac{2DE(P_R)^2}{L}\right]}{E(1-P_S)\left(2z+zE[P_S]+1\right)}; \qquad (4.1.16)$$

where, $E(T)=\frac{E(1-P_S)z}{D}$. The optimality condition meant for the concavity of $E(TPU(z))$, is demonstrated by finding the first and second derivatives of it i.e.,

$$\frac{d(E(TPU(z)))}{dz}=\left(\frac{1}{E(1-P_S)}\right)\left(\frac{1}{\left(2z+zE[P_S]+1\right)^2}\right)$$

$$\times\left[\begin{array}{l}\left(2DS_g-2DC_p-2DC_s-2DRE[P_R]+4DC_k+2DC_kE[P_S]\right)\\ -\left(\begin{array}{l}2C_hz^2\left(E(1-P_S)\right)^2+\frac{4C_hz^2DE[P_S]}{w}-\frac{4DC_hz^2\left(E[P_R]\right)^2}{L}\\ +C_hz^2E[P_S]\left(E(1-P_S)\right)^2+\frac{2C_hz^2D\left(E[P_S]\right)^2}{w}-\frac{2DC_hz^2\left(E[P_R]\right)^2(E[P_S])}{L}\\ +2C_hz\left(E(1-P_S)\right)^2+\frac{4C_hzDE[P_S]}{w}-\frac{4DC_hz\left(E[P_R]\right)^2}{L}\end{array}\right)\end{array}\right]. \qquad (4.1.17)$$

$$\frac{d^2(E(TPU(z)))}{dz^2}=-\left(\frac{1}{E(1-P_S)}\right)\left(\frac{2}{\left(2z+zE[P_S]+1\right)^3}\right)$$

$$\times\left[\begin{array}{l}8DS_g-8DC_p-8DC_s-12DRE[P_R]+16DC_k+16DC_kE[P_S]\\ +4DS_gE[P_S]-4DC_pE[P_S]-4DC_sE[P_S]+4DC_k\left(E[P_S]\right)^2\\ +2C_h\left(E(1-P_S)\right)^2+\frac{4C_hDE[P_S]}{w}-\frac{4DC_h\left(E[P_R]\right)^2}{L}\end{array}\right]. \qquad (4.1.18)$$

Following is the observed from (4.1.18). $\frac{d^2(E(TPU(z)))}{dz^2}<0$.

i.e., $E(TPU(z))$ is concave in z. Hence, we obtain the following result for the lot size:

$$\Rightarrow\left(2DS_g-2DC_p-2DC_s-2DRE[P_R]+4DC_k+2DC_kE[P_S]\right)$$

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$$= z^2 \begin{pmatrix} 2C_h\left(E(1-P_S)\right)^2 + \dfrac{4C_h DE[P_S]}{w} - \dfrac{4DC_h\left(E[P_R]\right)^2}{L} \\ +C_h E[P_S]\left(E(1-P_S)\right)^2 + \dfrac{2C_h D\left(E[P_S]\right)^2}{w} - \dfrac{2DC_h\left(E[P_R]\right)^2 (E[P_S])}{L} \\ +\dfrac{2C_h\left(E(1-P_S)\right)^2}{z} + \dfrac{4C_h DE[P_S]}{wz} - \dfrac{4DC_h\left(E[P_R]\right)^2}{Lz} \end{pmatrix}. \quad (4.1.19)$$

For large value of z, making $\frac{1}{z} = 0$, and the maximum lot size z^* of the EOQ is given as follows:

$$z^* = \sqrt{\frac{\left(2DS_g - 2DC_p - 2DC_s - 2DRE[P_R] + 4DC_k + 2DC_k E[P_S]\right)}{\begin{matrix} 2C_h\left(E(1-P_S)\right)^2 + \dfrac{4C_h DE[P_S]}{w} - \dfrac{4DC_h\left(E[P_R]\right)^2}{L} \\ +C_h E[P_S]\left(E(1-P_S)\right)^2 + \dfrac{2C_h D\left(E[P_S]\right)^2}{w} - \dfrac{2DC_h\left(E[P_R]\right)^2 (E[P_S])}{L} \end{matrix}}}. \quad (4.1.20)$$

and when, $S_g = C_p + C_s$, $P_S = 0$ and $P_R = 0$, Eq. (4.1.20) reduces to the traditional EOQ formulae,

$$z = \sqrt{\frac{2C_k D}{C_h}}. \quad (4.1.21)$$

Following the work of Bazan et al.(2017), an inventory model with proportionate discountwhich is in geometric progression (GP) has been proposed. The proportionate discount is estimated by introducing an approximate per unit selling price of defective items, which are known after the a screening of the lot conducted at a rate of w units in every unit time and the poor quality items are kept in stock. These defective items are sold prior to the arrival of the next lot at the estimated rate of discount. The unit selling price A of imperfect items in GP is as follows:

$$A = \left(S_g - \left(a - ar^i\right)\left(\frac{E(TR(z)) - E(TC(z))}{z}\right)\right). \quad (4.1.22)$$

where i takes the discrete values $1, 2, ..., [zE(P_S)]$ and simplifying eq.(4.1.22) we get the following total selling price of the defective items:

$S_g zE(P_S) - \left\{\left(azE(P_S) - ra\left(\dfrac{1 - r^{zE(P_S)-1}}{1-r}\right)\right)\left(\dfrac{E(TR(z)) - E(TC(z))}{z}\right)\right\}$. In the present model, the first defective item is sold with minimum discount and subsequently the other defectives are sold with a proportionately higher rate of discount which leads toa maximum discount for the last defective item and the selling price of the last item is

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less than or equal to the actual cost price of the item.As discussed in the model, a proportionate rate of discount is introduced depending on the number of scrap items. The estimated proportionate rate of discount for the scrap items is given as follows.

$\left(a-ar^{i}\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)$; where $i=1,2,3.....,\left[zE(P_S)\right]$; where $E(TC(z))$ the expected total cost isgiven in eq. (4.1.11).Now,the expected total cycle revenue $E(TR(z))$ is given as follows:

$E(TR(z))=$ total sales with regard to good quality items $+$ total sales with regard to scrap Items

$$=S_g z(1-E(P_S))+\sum_{i=1}^{[zE(P_S)]}\left(S_g-\left(a-ar^{i}\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)\right). \qquad (4.1.23)$$

Simplifying the above eq. (4.1.23), the following result is obtained.

$$=S_g z-\left\{azE(P_S)-r\left(a+ar+ar^2+ar^3+.........+ar^{zE(P_S)-1}\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)\right\}$$

$$=S_g z-\left\{\left(azE(P_S)-ra\left(\frac{1-r^{zE(P_S)-1}}{1-r}\right)\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)\right\}.$$

Neglecting the higher degree terms we find the following, where $0<r<1$; we get:

$$S_g z-\left\{\left(azE(P_S)-ra\left(\frac{1-(1+E(P_S)\ln r)\frac{1}{r}}{1-r}\right)\right)\left(\frac{E(TR(z))-E(TC(z))}{z}\right)\right\}$$

$$\Rightarrow E\left(TR\left(z\right)\right)=S_g z-a^2 zE(P_S)+a^2\left(\frac{r-1-E(P_S)\ln r}{1-r}\right). \qquad (4.1.24)$$

The expected total cycle profit $E\left(TP\left(z\right)\right)$ is given as follows:

$$E\left(TP\left(z\right)\right)=E\left(TR\left(z\right)\right)-E\left(TC\left(z\right)\right)=\left\{S_g z-a^2 zE(P_S)+a^2\left(\frac{r-1-E(P_S)\ln r}{1-r}\right)\right\}$$

$$-\left\{C_k+C_p z+C_s z+R\ E[P_R]\ z+C_h\left[\frac{E(1-P_S)zE(T)}{2}+\frac{E(P_S)z^2}{x}-\frac{E(P_R)^2 z^2}{L}\right]\right\}. \qquad (4.1.25)$$

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The expected total profit per unit time $E\left(TPU(z)\right)$ is given as follows.

$$E\left(TPU(z)\right)=\frac{E\left(TP\left(z\right)\right)}{E(T)}=\frac{S_g z-a^2 zE(P_S)-a^2-a^2\dfrac{E(P_S)\ln r}{1-r}-C_k-C_p z-C_s z-R\ E[P_R]\ z-C_h\dfrac{E(1-P_S)z^2}{2}-C_h\dfrac{E(P_S)z^2}{x}+C_h\dfrac{E(P_R)^2 z^2}{L}}{E(T)}; \quad (4.1.26)$$

where $E(T)=\dfrac{E(1-P_S)z}{D}$.

$$\Rightarrow E\left(TPU(z)\right)=D\left(S_g-a^2E(P_S)-C_p-C_s-R\ E[P_R]\right)\left(\frac{1}{E(1-P_S)}\right)$$

$$-\frac{D}{z}\left(a^2+a^2\frac{E(P_S)\ln r}{1-r}+C_k\right)\left(\frac{1}{E(1-P_S)}\right)-\frac{C_hE(1-P_S)z}{2}-\frac{C_hE(P_S)z}{xE(1-P_S)}+\frac{C_hE(P_R)^2 zD}{LE(1-P_S)} \quad (4.1.27)$$

The optimality condition meant for the concavity of the expected total profit per unit time $E(TPU(z))$, given inEq. (4.1.27), is demonstrated by finding the first and second derivatives is $\dfrac{d(E(TPU(z)))}{dz}=\dfrac{D}{z^2}\left(a^2+a^2\dfrac{E(P_S)\ln r}{1-r}+C_k\right)\left(\dfrac{1}{E(1-P_S)}\right)$

$$-\frac{C_hE(1-P_S)}{2}-\frac{C_hE(P_S)}{xE(1-P_S)}+\frac{C_hE(P_R)^2 D}{LE(1-P_S)}. \quad (4.1.28)$$

and $\dfrac{d^2(E(TPU(z)))}{dz^2}=-\dfrac{2D}{z^3}\left(a^2+a^2\dfrac{E(P_S)\ln r}{1-r}+C_k\right)\left(\dfrac{1}{E(1-P_S)}\right)$. (4.1.29)

Following is observed from (4.1.29) is $\dfrac{d^2(E(TPU(z)))}{dz^2}<0$. $E(TPU(z))$ is concave in z, and hence the maximum value of $E(TPU(z))$ will occur at the point that satisfies $\dfrac{d(E(TPU(z)))}{dz}=0$.

$$\Rightarrow \frac{D}{z^2}\left(a^2+a^2\frac{E(P_S)\ln r}{1-r}+C_k\right)\left(\frac{1}{E(1-P_S)}\right)-\frac{C_hE(1-P_S)}{2}-\frac{C_hE(P_S)}{xE(1-P_S)}+\frac{C_hE(P_R)^2 D}{LE(1-P_S)}=0$$

$$\Rightarrow D\left(a^2+a^2\frac{E(P_S)\ln r}{1-r}+C_k\right)\left(\frac{1}{E(1-P_S)}\right)=z^2\left[\frac{C_hE(1-P_S)}{2}+\frac{C_hE(P_S)}{xE(1-P_S)}-\frac{C_hE(P_R)^2 D}{LE(1-P_S)}\right]$$

$$\Rightarrow z=\sqrt{\frac{D\left(a^2+a^2\frac{E(P_S)\ln r}{1-r}+C_k\right)\left(\frac{1}{E(1-P_S)}\right)}{\left[\frac{C_hE(1-P_S)}{2}+\frac{C_hE(P_S)}{xE(1-P_S)}-\frac{C_hE(P_R)^2D}{LE(1-P_S)}\right]}}. \qquad (4.1.30)$$

and when, $S_g=C_p+C_s$, $P_S=0$, $P_R=0$ and $a=0$ Eq. (4.1.30) reduces to $z=\sqrt{\frac{2C_kD}{C_h}}$ (traditional EOQ formulae).

4.2. With learning effect

In this section, we derive an economic order quantity model by considering learning effects. The holding cost and the ordering cost follow the learning effect. In this case, the total profit per unit time isthe same as that of the expression in Eq. (4.1.16). However, C_h and C_k are replaced by $C_h(n)$ and $C_k(n)$ which are respectively the shipmentholding cost and the ordering cost under learning effects. The holding cost $C_h(n)$ and the ordering cost $C_k(n)$ are $C_k(n)=C_{k_0}+\frac{C_{k_1}}{n^{\beta_1}}, \beta_1>0$ and $C_h(n)=C_{h_0}+\frac{C_{h_1}}{n^{\beta_2}},\ \beta_2>0$.

By considering the learning effects, the total profit per unit time is given as follows:

$$\left(TPU\left(z_n\right)\right)=\frac{2D\left(S_gz_n-C_k(n)-C_pz_n-C_sz_n-RP_Rz_n\right)}{(1-P_S)\left(2z_n+z_nP_S+1\right)}-\frac{C_h(n)z_n{}^2\left[\left(1-P_S\right)^2+\frac{2DP_S}{w}-\frac{2D(P_R)^2}{L}\right]}{(1-P_S)\left(2z_n+z_nP_S+1\right)}$$

(4.2.1)

$$\frac{d(TPU(z_n))}{dz_n}=\left(\frac{1}{(1-P_S)}\right)\left(\frac{1}{\left(2z_n+z_nP_S+1\right)^2}\right).$$
$$\left[\begin{array}{l}\left(2DS_g-2DC_p-2DC_s-2DRP_R+4DC_k(n)+2DC_k(n)P_S\right)\\ -\left(\begin{array}{l}2C_h(n)z_n{}^2\left(1-P_S\right)^2+\frac{4C_h(n)z_n{}^2DP_S}{w}-\frac{4DC_h(n)z_n{}^2\left(P_R\right)^2}{L}\\ +C_h(n)z_n{}^2P_S\left(1-P_S\right)^2+\frac{2C_h(n)z_n{}^2D\left(P_S\right)^2}{w}-\frac{2DC_h(n)z_n{}^2\left(P_R\right)^2P_S}{L}\\ +2C_h(n)z_n\left(1-P_S\right)^2+\frac{4C_h(n)z_nDP_S}{w}-\frac{4DC_h(n)z_n\left(P_R\right)^2}{L}\end{array}\right)\end{array}\right]. \qquad (4.2.2)$$

$$\frac{d^2(TPU(z_n))}{dz_n{}^2}=-\left(\frac{1}{(1-P_S)}\right)\left(\frac{2}{\left(2z_n+z_n[P_S]+1\right)^3}\right)$$

$$\times\begin{bmatrix} 8DS_g - 8DC_p - 8DC_s - 12DRP_R + 16DC_k(n) + 16DC_k(n)E[P_S] \\ +4DS_gP_S - 4DC_pP_S - 4DC_sP_S + 4DC_k(n)\left(P_S\right)^2 \\ +2C_h(n)\left(1-P_S\right)^2 + \frac{4C_h(n)DP_S}{w} - \frac{4DC_h(n)\left(P_R\right)^2}{L} \end{bmatrix}. \quad (4.2.3)$$

By setting the first derivative equal to zero, we get the following equation.

$$\Rightarrow\left(2DS_g - 2DC_p - 2DC_s - 2DRP_R + 4DC_k(n) + 2DC_k(n)P_S\right)$$

$$= {z_n}^2\left(\begin{array}{l} 2C_h(n)\left(1-P_S\right)^2 + \frac{4C_h(n)DP_S}{w} - \frac{4DC_h(n)\left(P_R\right)^2}{L} \\ +C_h(n)P_S\left(1-P_S\right)^2 + \frac{2C_h(n)D\left(P_S\right)^2}{w} - \frac{2DC_h(n)\left(P_R\right)^2 P_S}{L} \\ +\frac{2C_h(n)\left(1-P_S\right)^2}{z_n} + \frac{4C_h(n)DP_S}{wz_n} - \frac{4DC_h(n)\left(P_R\right)^2}{Lz_n} \end{array}\right).$$

For the large value of z_n, $\frac{1}{z_n} \to 0$, we get the following result:

$$z_n = \sqrt{\frac{\left(2DS_g - 2DC_p - 2DC_s - 2DRP_R + 4DC_k(n) + 2DC_k(n)P_S\right)}{\begin{array}{l} 2C_h(n)\left(1-P_S\right)^2 + \frac{4C_h(n)DP_S}{w} - \frac{4DC_h(n)\left(P_R\right)^2}{L} \\ +C_h(n)P_S\left(1-P_S\right)^2 + \frac{2C_h(n)D\left(P_S\right)^2}{w} - \frac{2DC_h(n)\left(P_R\right)^2 P_S}{L} \end{array}}}. \quad (4.2.4)$$

4.3. *Special cases*

When $C_p + C_s = S_g$,due to the impact of learning and for large n, ordering cost and holding cost become constant or reduces to (C_{k_0} and C_{h_0})constants.Then, the above equation reduces to the classical EOQ formulae:

$$z_n = \sqrt{\frac{2C_{k_0}D}{C_{h_0}}} \quad (4.2.5)$$

and $$z_n = \sqrt{\frac{2C_kD}{C_h}}. \quad (4.2.6)$$

5. Numerical Results

We take the example taken in case of Salameh & Jaber (2000) to have the numerical clarification of models. D =50 000 units/year, C_k =\$100/cycle, C_h =\$5unit/year, C_s =\$0.5/unit, R =\$2.5/unit, C_p = \$25/unit, S_g = \$50/unit, and w =1 unit/min, L =0.5unit/min [under the assumption that it operates 8 hours a day, for 365 days a year. Thereby, the annual inspection rate is w =175 200 units/year, L =43800unit/year]. In addition, we assume scrap and rework fractions, P_S and P_R, are uniformly distributed with PDF

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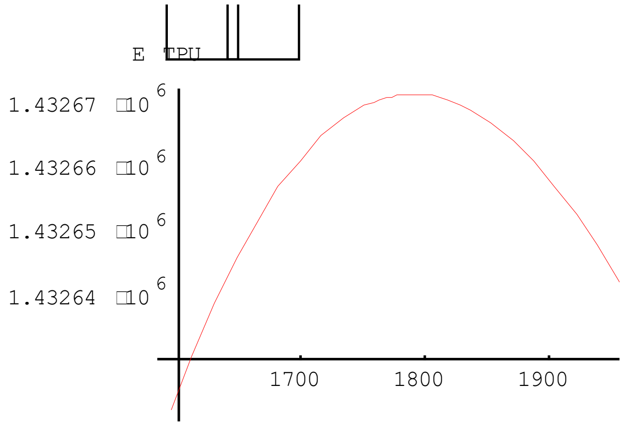
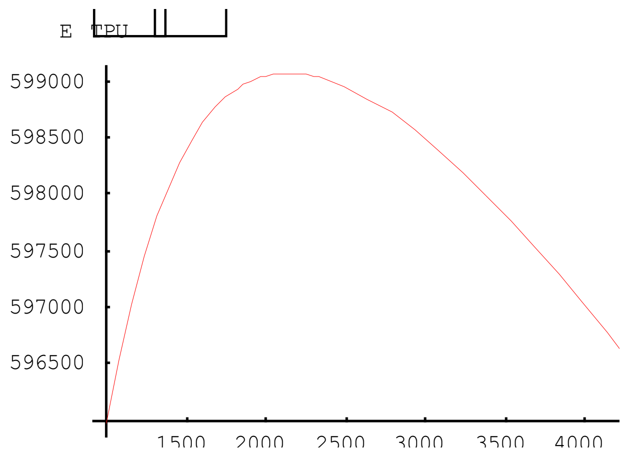
are: $f(P_S)=\begin{cases}4, & 0\le P_S\le 0.25\\ 0 & otherwise\end{cases}$ and $f(P_R)=\begin{cases}12.5, & 0\le P_R\le 0.08\\ 0 & otherwise\end{cases}$. For uniform distribution with $a=0$,the expected values present in the model are: $E(p)=\frac{b}{2}, E(p^2)=\frac{b^2}{3}$ and $E(1-p)=1-\frac{b}{2}$ .Optimal solution is obtained when there is discount (in AP) for the scrap items. The optimal value of z that is, $z^*=1681.81$ units and its corresponding $E(TPU(z))=1304940.126$.



[Fig. 4 $E(TPU)$ vs. z (AP)]

Optimal solution is obtained when there is discount (in GP) for the scrap items. The optimal value of z that is, $z^*=2140.26$ andit's corresponding $E(TPU(z))=599067$.



[Fig. 5 $E(TPU)$ vs. z (GP)]

From the classical EOQ model, the optimal value of z, which is given by Eq. (4.1.21); is $z=1414$. The 3D concavity graphs are shownin Fig. 6 to Fig. 8. Now, we study the effect of variations in the fraction of the scrap and rework items on z^* and the observed results are shown in the following tables. Here, we assume the scrap and rework fractions are uniformly distributed with $P_S \sim U(0,b_1)$ and $P_R \sim U(0,b_2)$ respectively.

Table 2: z^* and the ratio of z^* to z with variation inscrap and rework items.

	$E[P_S]$	$E(1-P_S)$	$E(1-P_S)^2$	$b_2=$ .04(.04).2	$E[P_R]$	$(E[P_R])^2$	z^*	$\frac{z^*}{z}$

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$b_1 = 0.08$	0.04	0.96	0.9216		0.02 0.04 0.06 0.08 0.1	0.0004 0.0016 0.0036 0.0064 0.01	1541 1543 1546 1552 1558	1.09 1.09 1.09 1.10 1.10
$b_1 = 0.2$	0.1	0.9	0.81		0.02 0.04 0.06 0.08 0.1	0.0004 0.0016 0.0036 0.0064 0.01	1606 1608 1612 1618 1626	1.14 1.14 1.14 1.14 1.15
$b_1 = 0.32$	0.16	0.84	0.7056		0.02 0.04 0.06 0.08 0.1	0.0004 0.0016 0.0036 0.0064 0.01	1672 1675 1680 1686 1695	1.18 1.18 1.19 1.19 1.20
$b_1 = 0.44$	0.22	0.78	0.6084		0.02 0.04 0.06 0.08 0.1	0.0004 0.0016 0.0036 0.0064 0.01	1740 1743 1749 1756 1766	1.23 1.23 1.24 1.24 1.25
$b_1 = 0.56$	0.28	0.72	0.5184		0.02 0.04 0.06 0.08 0.1	0.0004 0.0016 0.0036 0.0064 0.01	1808 1812 1818 1826 1837	1.28 1.28 1.29 1.29 1.30

Table 3: *Lot size and profit with learning effect on holding cost and ordering cost*

n	$C_h(n)$	$C_k(n)$	z_n	$TPU(z_n)$
1	5	100	1791.34	1432670
2	4.87055	98.7055	1804.35	1432810
3	4.80274	98.0274	1811.41	1432880
4	4.75786	97.5786	1861.18	1432930
5	4.72478	97.2478	1819.74	1432960
6	4.69883	96.9883	1822.57	1432990
7	4.67761	96.7761	1824.9	1433020
8	4.65975	96.5975	1826.87	1433030
9	4.64439	96.4439	1828.58	1433050
10	4.63096	96.3096	1830.08	1433070
11	4.61904	96.1904	1831.42	1433080
12	4.60836	96.0836	1832.63	1433090
13	4.5987	95.987	1833.72	1433100

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14	4.58989	95.8989	1834.73	1433110
15	4.58181	95.8181	1835.65	1433120
16	4.57435	95.7435	1836.5	1433130
17	4.56743	95.6743	1837.29	1433130
18	4.56098	95.6098	1838.04	1433140
19	4.55494	95.5494	1838.73	1433150
20	4.54928	95.4928	1839.39	1433150
100	4.39811	93.9811	1857.38	1433320
1000	4.25119	92.5119	1875.91	1433470
100000	4.1	91	1896.16	1433640

Table 4: *Lot size and profit with learning effect on holding cost*

n	$C_h(n)$	z_n	$TPU(z_n)$
1	5	1708.59	1433010
2	4.87055	1731.15	1433100
3	4.80274	1743.32	1433150
4	4.75786	1751.53	1433180
5	4.72478	1757.65	1433200
6	4.69883	1762.49	1433220
7	4.67761	1766.49	1433240
8	4.65975	1769.87	1433250
9	4.64439	1772.79	1433260
10	4.63096	1775.36	1433270
11	4.61904	1777.65	1433280
12	4.60836	1779.71	1433290
13	4.5987	1781.58	1433300
14	4.58989	1783.29	1433300
15	4.58181	1784.86	1433300
16	4.57435	1786.31	1433310
17	4.56743	1787.67	1433320
18	4.56098	1788.93	1433320
19	4.55494	1790.12	1433330
20	4.54928	1791.23	1433330
100	4.39811	1821.75	1433440
1000	4.25119	1852.97	1433550
100000	4.1	1886.82	1433670

Table 5: *Lot size and profit with learning effect on ordering cost*

n	$C_k(n)$	z_n	$TPU(z_n)$
1	100	2002.78	1433440
2	98.7055	1991.04	1433480
3	98.0274	1984.87	1433500
4	97.5786	1980.77	1433520
5	97.2478	1977.74	1433530
6	96.9883	1975.37	1433530
7	96.7761	1973.42	1433540
8	96.5975	1971.78	1433550

9	96.4439	1970.37	1433550
10	96.3096	1969.14	1433550
11	96.1904	1968.04	1433560
12	96.0836	1967.06	1433560
13	95.987	1966.17	1433560
14	95.8989	1965.36	1433570
15	95.8181	1964.62	1433570
16	95.7435	1963.93	1433570
17	95.6743	1963.29	1433570
18	95.6098	1962.7	1433580
19	95.5494	1962.14	1433580
20	95.4928	1961.67	1433580
100	93.9811	1947.62	1433630
1000	92.5119	1933.92	1433670
100000	91	1919.71	1433720

5.1.Observations

- From Table 2, shows that with anincrease in b_2 and a fixed value of b_1, z^* and the ratio of z^* to z both increase with respect to different fractions of scrap and rework items.
- From Table 3, when learning is introduced for $C_h(n)$ and $C_k(n)$ with an increase in the number of sub-intervals it is observed that, $C_h(n)$ and $C_k(n)$ decrease and z_n and $TPU(z_n)$ increase.
- From Table 4, when learning is introduced in $C_h(n)$, keeping $C_k(n)$ fixed,and withan increase in the number of sub-intervalsit is observed that, $C_h(n)$ decreases and z_n and $TPU(z_n)$ both increase.
- From Table 5, when learning is introduced in $C_k(n)$, keeping $C_h(n)$ fixed, and with an increase in the number of sub-intervals it is observed that, $C_k(n)$ and z_n bothdecrease and that $TPU(z_n)$ increases.

From Table 3 to 5, show that the values of comes with increasedrepetitionsand with learning effect for a single parameter than the results obtained with the effect of learning for more number of parameters.

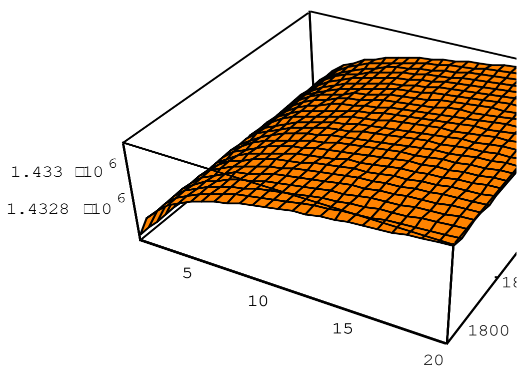

Fig.6. $TPU(z_n)$ w.r.t.. z_n

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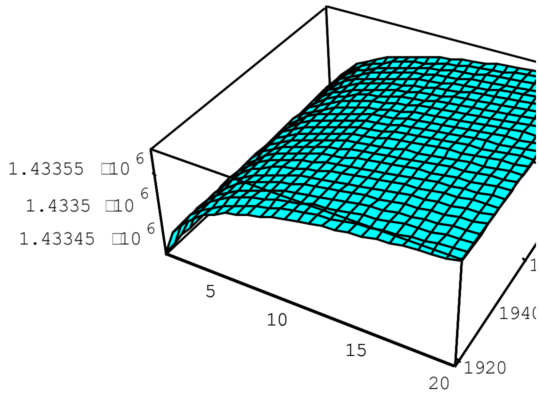

Fig.7. $TPU(z_n)$ w.r.t.. z_n

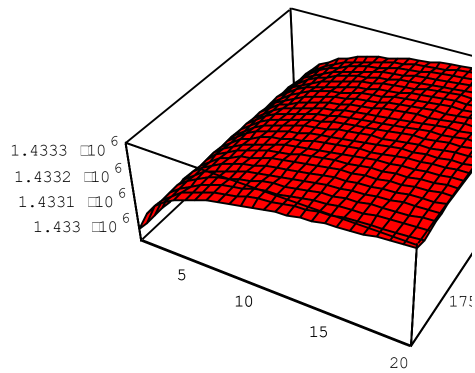

Fig.8. $TPU(z_n)$ w.r.t. z_n

From Fig.6 - Fig.8, we observed the following outcomes:

- When the range of z is from 1595 to 2010 the total profit function $E(TPU)$ gives strictly a concave function.
- When the range of z is from 1000 to 4600 the total cost function $E(TPU)$ gives strictly a concave function.
- From Fig.4 and Fig.5 we conclude that, the optimal replenishment chart is unique and $E(TPU)$ is a concave function for the proposed model.

5.2.Managerial implication

The main goal is to improve the profit. When the product is of good quality,there isless rework (as scrap items become less), and it reduces the raw material costs. It also increases customer satisfaction because of quality of product and thereby the demand for the company's products increases. For these reasons, higher inproducts qualitycan provide a better competitiveness in the market and it leads to profitability.

6. Conclusion

Two different EOQmodels (without and with learning) were developed for two kinds of imperfect

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products namely scrap and rework items. The objective of the models was to maximize the total profit by introducing a proportionate discount for the scrap items present in each lot underlearning effect. The modelsconsidered that the incoming lot has a fraction of both scrap and rework items. The rework items are returned to the supplier, re-worked, and received again as good quality items within the same cycle. The scrap items are sold in a single batch with different types of proportionate discounts (both in AP and in GP)and theconcavity graphs (profit graphs) are drawn with respect to the results obtained for both the cases. In these models, the total profits are calculated under a learning effect for different parameters as well as combinations of different parameters taken together and sensitivity analysis is conducted for the models. The results showed, when the learning effect is introduced either for holding cost or both for holding cost and ordering cost both the lot size and the total profit increase simultaneously but, when there is learning only for ordering cost the lot size decreases and profit increases. Results are illustrated graphically in sensitivity analysis section.

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