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Some Properties of G_δ-$e^\star$-locally Closed Sets via Intuitionistic Fuzzy Topological Spaces

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Abstract:

This paper is devoted to the study of new class of sets called an intuitionistic fuzzy e^⋆-locally closed sets, intuitionistic fuzzy e^⋆ G_δ-set, intuitionistic fuzzy e^⋆ G_δ-locally closed sets and intuitionistic fuzzy G_δ-locally closed sets are introduced and studied. Also the concepts of an intuitionistic fuzzy G_δ-e^⋆-locally closed set, intuitionistic fuzzy subspace, intuitonistic fuzzy G_δ-e^⋆-local-δ-semi (resp., δ-pre and β) spaces are introduced and interesting properties are established. In this connection, interelations are discussed. Examples are provided where necessary.

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I. Introduction The concept of fuzzy sets was introduced by Zadeh [15] and Atanassov [1] introduced and studied intuitionistic fuzzy sets (briefly, $\mathcal{IFS}$). On the other hand, Coker [4] introduced the notions of an intuitionistic fuzzy topological spaces (briefly, $\mathcal{IFTS}$), intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy α-closed set was introduced by Biljana Krsteshka and Erdal Ekici [9]. The first step of locally closedness was done by Bourbaki [3]. Ganster and Relly used locally closed sets in [8] to define LC-continuity and LC-irresoluteness. Roja, Uma and Balasubramanian [2] discussed fuzzy G_δ continuous functions. The initiations of $e^\star$-open sets, $e^\star$-continuity and $e^\star$-compactness in topological spaces are due to Ekici [5, 6, 7]. In fuzzy topology, $e^\star$-open sets were introduced by seenivasan in 2014 [12]. Sobana et.al [13] were introduced the concept of fuzzy $e^\star$-open sets, fuzzy $e^\star$-continuity and fuzzy $e^\star$-compactness in intuitionistic fuzzy topological spaces (briefly., IFTS's). In this paper we introduce the concepts of an intuitionistic fuzzy $e^\star$-locally closed sets, intuitionistic fuzzy $e^\star G_\delta$-sets, intuitionistic fuzzy $e^\star G_\delta$-locally closed sets and intuitionistic fuzzy G_δ - $e^\star$-locally closed sets in IFTS's. Also the concepts of an intuitionistic fuzzy G_δ - $e^\star$-locally closed intuitionistic fuzzy subsapce, intuitionistic fuzzy G_δ - $e^\star$-local $T_{\frac{1}{2}}$ space, intuitionistic fuzzy G_δ-$e^\star$-local δ-semi (resp., δ-pre and β) spaces are introduced and studied. Some interesting properties and interrelations among sets and spaces are discussed with necessary examples.

II. Preliminaries

Definition 2.1 *[1] Let X be a nonempty fixed set and I the closed interval $[0,1]$. An $\mathcal{IFS}$ $\mathfrak{A}$ is an object of the following form $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$, where the mapping $\mu_{\mathfrak{A}}: X \to I$ and $\gamma_{\mathfrak{A}}: X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_{\mathfrak{A}}(x)$) for each element $x \in X$ to the set $\mathfrak{A}$, respectively, and $0 \leq \mu_{\mathfrak{A}}(x) + \gamma_{\mathfrak{A}}(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set $\mathfrak{A}$ on a nonempty set X is an IFS of the following form,$\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), 1 - \mu_{\mathfrak{A}}(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle$ for the $\mathcal{IFS}$ $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$.*

Definition 2.2 *[1] Let $\mathfrak{A}$ and $\mathfrak{B}$ be $\mathcal{IFS}$'s of the form $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$ and $\mathfrak{B} = \{\langle x, \mu_{\mathfrak{B}}(x), \gamma_{\mathfrak{B}}(x)\rangle : x \in X\}$. Then*

1. $\mathfrak{A} \subseteq \mathfrak{B}$ if and only if $\mu_{\mathfrak{A}}(x) \leq \mu_{\mathfrak{B}}(x)$ and $\gamma_{\mathfrak{A}}(x) \geq \gamma_{\mathfrak{B}}(x)$;
2. $\overline{\mathfrak{A}} = \{\langle x, \gamma_{\mathfrak{A}}(x), \mu_{\mathfrak{A}}(x)\rangle : x \in X\}$;

May – June 2020
ISSN: 0193-4120 Page No. 7450 - 7454

3. $\mathfrak{A} \cap \mathfrak{B} = \{\langle x, \mu_{\mathfrak{A}}(x) \wedge \mu_{\mathfrak{B}}(x), \gamma_{\mathfrak{A}}(x) \vee \gamma_{\mathfrak{B}}(x)\rangle : x \in X\}$;
4. $\mathfrak{A} \cup \mathfrak{B} = \{\langle x, \mu_{\mathfrak{A}}(x) \vee \mu_{\mathfrak{B}}(x), \gamma_{\mathfrak{A}}(x) \wedge \gamma_{\mathfrak{B}}(x)\rangle : x \in X\}$;

Definition 2.3 *[1] The $\mathcal{IFS}$'s $0_\sim$ and $1_\sim$ are defined by , $0_\sim = \{< x, 0, 1 >: x \in X\}$ and $1_\sim = \{< x, 1, 0 >: x \in X\}$.*

Definition 2.4 *[4] An intuitionistic fuzzy topology ($\mathcal{IFT}$) in Coker's sense on a nonempty set X is a family Γ of $\mathcal{IFS}$'s in X satisfying the following axioms:*

1. $0_\sim, 1_\sim \in \Gamma$;
2. $\mathfrak{G}_1 \cap \mathfrak{G}_2 \in \Gamma$, for any $\mathfrak{G}_1, \mathfrak{G}_2 \in \Gamma$;
3. $\cup\, \mathfrak{G}_i \in \Gamma$ for any arbitrary family $\{\mathfrak{G}_i : i \in \mathfrak{J}\} \subseteq \Gamma$.

In this paper by (X, Γ) or simply by X we will denote the $\mathcal{IFTS}$. Each $\mathcal{IFS}$ which belongs to Γ is called an $\mathcal{IF}$ open set ($\mathcal{IFOS}$) in X. The complement $\overline{\mathfrak{A}}$ of an $\mathcal{IFOS}$ $\mathfrak{A}$ in X is called an $\mathcal{IF}$ closed set ($\mathcal{IFCS}$) in X.

Definition 2.5 *[4] Let (X, Γ) be an $\mathcal{IFTS}$ and $\mathfrak{A} = \{< x, \mu_{\mathfrak{A}}, \nu_{\mathfrak{A}} >: x \in X\}$ be an $\mathcal{IFS}$ in X. Then the $\mathcal{IF}$ closure and $\mathcal{IF}$ interior of $\mathfrak{A}$ are defined by*

1. $cl(\mathfrak{A}) = \cap\ \{\mathfrak{C} : \mathfrak{C}$ isan IFCS $in\ X$and $\mathfrak{C} \supseteq \mathfrak{A}\}$;
2. $int(\mathfrak{A}) = \cup\ \{\mathfrak{D} : \mathfrak{D}$ isan IFOS $in\ X$and $\mathfrak{D} \subseteq \mathfrak{A}\}$;

It can be also shown that $cl(\mathfrak{A})$ is an $\mathcal{IFCS}$, $int(\mathfrak{A})$ is an $\mathcal{IFOS}$ in X and $\mathfrak{A}$ is and $\mathcal{IFCS}$ in X if and only if $cl(\mathfrak{A}) = \mathfrak{A}$; $\mathfrak{A}$ is an $\mathcal{IFOS}$ in X if and only if $int(\mathfrak{A}) = \mathfrak{A}$

Definition 2.6 *[14] Let $\mathfrak{A}$ be $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (X, Γ). $\mathfrak{A}$ is called an*

1. $\mathcal{IF}$ regular open set (briefly $IFROS$) if $\mathfrak{A} = intcl(\mathfrak{A})$
2. $\mathcal{IF}$ regular closed set (briefly $IFRCS$) if $\mathfrak{A} = clint(\mathfrak{A})$

Definition 2.7 *[2] Let (X, Γ) be a fuzzy topological space and $\mathfrak{A}$ be a fuzzy set in X. $\mathfrak{A}$ is called G_δ set if $\mathfrak{A} = \cap_{i=1}^{\infty} \mathfrak{A}_i$ where each $\mathfrak{A}_i \in T$. The complement of fuzzy G_λ is fuzzy F_σ*

Definition 2.8 *[14] Let (X, Γ) be an $\mathcal{IFTS}$ and $\mathfrak{A} = \langle x, \mu_{\mathfrak{A}}(x), \nu_{\mathfrak{A}}(x)\rangle$ be a $\mathcal{IFS}$ in X. Then the fuzzy δ closure of $\mathfrak{A}$ are denoted and defined by $cl_\delta(\mathfrak{A}) = \cap\{\mathfrak{K} : \mathfrak{K}$ is an $\mathcal{IFRCS}$ in X and $\mathfrak{A} \subseteq \mathfrak{K}\}$ and $int_\delta(\mathfrak{A}) = \cup\{\mathfrak{G} : \mathfrak{G}$ is an $\mathcal{IFROS}$ in X and $\mathfrak{G} \subseteq \mathfrak{A}\}$.*

Definition 2.9 *[13] Let $\mathfrak{A}$ be an $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (X, Γ). $\mathfrak{A}$ is called an $\mathcal{IF}$ δ-semiopen (resp. δ-preopen, β-open) set ($\mathcal{IF}\delta SO$ (resp. $\mathcal{IF}\delta PO$, $\mathcal{IF}\beta O$), for short), if $\mathfrak{A} \subseteq cl(int_\delta(\mathfrak{A}))$ (resp. $\mathfrak{A} \subseteq int(cl_\delta(\mathfrak{A}))$, $\mathfrak{A} \subseteq cl(int(cl(\mathfrak{A}))))$. $\mathfrak{A}$ is called an $\mathcal{IF}$ δ-semiclosed (resp. δ-preclosed, β-closed) set ($\mathcal{IF}\delta SC$ (resp. $\mathcal{IF}\delta PC$, $\mathcal{IF}\beta C$) (for short)) if $\mathfrak{A} \supseteq int(cl_\delta(\mathfrak{A}))$ (resp. $\mathfrak{A} \supseteq cl(int_\delta(\mathfrak{A}))$, $\mathfrak{A} \supseteq int(cl(int(\mathfrak{A}))))$.*

Definition 2.10 *[13] Let $\mathfrak{A}$ be an $\mathcal{IFS}$ in an $\mathcal{IFTS}(X, \Gamma)$. $\mathfrak{A}$ is called an $\mathcal{IF}$ $e^\star$-open set ($\mathcal{IF}e^\star$ OS, for short) in X if $\mathfrak{A} \subseteq clintcl_\delta(\mathfrak{A})$*

Definition 2.11 *[10] Let (X, Γ) be an $\mathcal{IFTS}$ and Y be any $\mathcal{IF}$ subset of X. Then $\Gamma_Y = (\mathfrak{A}/Y | \mathfrak{A} \in \Gamma)$ is an $\mathcal{IFT}$ on Y and is called the induced or relative $\mathcal{IFT}$. The pair (Y, Γ_Y) is called an $\mathcal{IF}$ subspace of (X, Γ): (Y, Γ_Y) is called an $\mathcal{IF}$ open/$\mathcal{IF}$ closed subspace if the $\mathcal{IF}$ characteristic function of (Y, Γ_Y) viz χ_Y is $\mathcal{IF}$ open/$\mathcal{IF}$ closed.*

III. Intuitionistic fuzzy G_δ - $e^\star$ -locally closed sets in an intuitionistic fuzzy topological spaces

Definition 3.1 *Let (X, Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ (X, Γ). Then $\mathfrak{A}$ is said to be $\mathcal{IF}$ $e^\star$- locally closed set (in short, $\mathcal{IF}$-$e^\star$-lcs) if $\mathfrak{A} = \mathfrak{C} \cap \mathfrak{D}$, where $\mathfrak{C} = \{\langle x, \mu_{\mathfrak{C}}(x), \gamma_{\mathfrak{C}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star$-open set and $\mathfrak{D} = \{\langle x, \mu_{\mathfrak{D}}(x), \gamma_{\mathfrak{D}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star$-closed set in (X, Γ).*

Definition 3.2 *Let (X, Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ X. Then $\mathfrak{A}$ is said to be an $\mathcal{IF}$ $e^\star G_\delta$- set if $\mathfrak{A} = \cap_{i=1}^{\infty} \mathfrak{A}_i$, where $\mathfrak{A}_i = \{\langle x, \mu_{\mathfrak{A}_i}(x), \gamma_{\mathfrak{A}_i}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star$-open set in an $\mathcal{IFTS}$ (X, Γ).*

Definition 3.3 *Let (X, Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ (X, Γ). Then $\mathfrak{A}$ is said to be an $\mathcal{IF}$ $e^\star G_\delta$-locally closed set (in short,$\mathcal{IF}$ -$e^\star G_\delta$-lcs) if $\mathfrak{A} = \mathfrak{C} \cap \mathfrak{D}$, where $\mathfrak{C} = \{\langle x, \mu_{\mathfrak{C}}(x), \gamma_{\mathfrak{C}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star G_\delta$ set and $\mathfrak{D} = \{\langle x, \mu_{\mathfrak{D}}(x), \gamma_{\mathfrak{D}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star$-closed set in (X, Γ).*

Definition 3.4 *Let (X, Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A} = \{\langle x, \mu_{\mathfrak{A}}(x), \gamma_{\mathfrak{A}}(x)\rangle : x \in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ (X, Γ). Then $\mathfrak{A}$ is said to be an $\mathcal{IF}$ G_δ-$e^\star$-locally closed set (in short,$\mathcal{IF}$ G_δ-$e^\star$-lcs) if $\mathfrak{A} = \mathfrak{B} \cap \mathfrak{C}$, where $\mathfrak{B} = \{\langle x, \mu_{\mathfrak{B}}(x), \gamma_{\mathfrak{B}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ G_δ set and $\mathfrak{C} = \{\langle x, \mu_{\mathfrak{C}}(x), \gamma_{\mathfrak{C}}(x)\rangle : x \in X\}$ is an $\mathcal{IF}$ $e^\star$-closed set in (X, Γ).*

The complement of an $\mathcal{IF}$ G_δ-$e^\star$-locally

May – June 2020
ISSN: 0193-4120 Page No. 7450 - 7454

closed set is said to be an $\mathcal{IF}$ G_δ-$e^\star$-locally open set (in short, $\mathcal{IF}$ G_δ-$e^\star$-los).

Definition 3.5 *Let (X,Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A}=\{\langle x,\mu_{\mathfrak{A}}(x),\gamma_{\mathfrak{A}}(x)\rangle: x\in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ (X,Γ). The $\mathcal{IF}$ G_δ-$e^\star$-locally closure of $\mathfrak{A}$ is denoted and defined by IFG_δ - $e^\star$ - $lcl(\mathfrak{A})=\cap\ \{\mathfrak{B}:\mathfrak{B}=\{\langle x,\mu_{\mathfrak{B}}(x),\gamma_{\mathfrak{B}}(x)\rangle: x\in X\}$ is an $\mathcal{IF}$ G_δ-$e^\star$-locally closed set in X and $\mathfrak{A}\subseteq\mathfrak{B}$.*

Proposition 3.1 *Let (X,Γ) be an $\mathcal{IFTS}$. For any two $\mathcal{IFS}$'s $\mathfrak{A}=\{\langle x,\mu_{\mathfrak{A}}(x),\gamma_{\mathfrak{A}}(x)\rangle: x\in X\}$ and $\mathfrak{B}=\{\langle x,\mu_{\mathfrak{B}}(x),\gamma_{\mathfrak{B}}(x)\rangle: x\in X\}$ of an $\mathcal{IFTS}$ (X,Γ) then the following statements are true.*

1. IFG_δ-$e^\star$-$lcl(0_\sim)=0_\sim$
2. $\mathfrak{A}\subseteq\mathfrak{B}\Rightarrow IFG_\delta$ - $e^\star$ - $lcl(\mathfrak{A})\subseteq IFG_\delta$-$e^\star$-$lcl(\mathfrak{B})$
3. IFG_δ - $e^\star$ - $lcl(IFG_\delta$ - $e^\star$ - $lcl(\mathfrak{A}))=IFG_\delta$-$e^\star$-$lcl(\mathfrak{A})$
4. IFG_δ - $e^\star$ - $lcl(\mathfrak{A}\cup\mathfrak{B})=(IFG_\delta$-$e^\star$-$lcl(\mathfrak{A}))\cup(IFG_\delta$-$e^\star$-$lcl(\mathfrak{B}))$

Definition 3.6 *Let (X,Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A}=\{\langle x,\mu_{\mathfrak{A}}(x),\gamma_{\mathfrak{A}}(x)\rangle: x\in X\}$ be an $\mathcal{IFS}$ on an $\mathcal{IFTS}$ (X,Γ). The $\mathcal{IF}$ G_δ-$e^\star$-locally interior of $\mathfrak{A}$ is denoted and defined by IFG_δ - $e^\star$ - $lint(\mathfrak{A})=\cup\ \{\mathfrak{B}:\mathfrak{B}=\{\langle x,\mu_{\mathfrak{B}}(x),\gamma_{\mathfrak{B}}(x)\rangle: x\in X\}$ is an $\mathcal{IF}$ G_δ-$e^\star$-los in X and $\mathfrak{B}\subseteq\mathfrak{A}$.*

Proposition 3.2 *Let (X,Γ) be an $\mathcal{IFTS}$. Let $\mathfrak{A}=\{\langle x,\mu_{\mathfrak{A}}(x),\gamma_{\mathfrak{A}}(x)\rangle: x\in X\}$ and $\mathfrak{B}=\{\langle x,\mu_{\mathfrak{B}}(x),\gamma_{\mathfrak{B}}(x)\rangle: x\in X\}$ are $\mathcal{IFS}$'s in an $\mathcal{IFTS}$ (X,Γ). Then the following statements are true.*

1. IFG_δ-$e^\star$-$lcl(\mathfrak{A})$ is the largest $\mathcal{IF}$ G_δ-$e^\star$-los contained in $\mathfrak{A}$
2. If $\mathfrak{A}$ is an $\mathcal{IF}$ G_δ-$e^\star$-los then $\mathfrak{A}=IFG_\delta$-$e^\star$-$lint(\mathfrak{A})$
3. IF $\mathfrak{A}$ is an $\mathcal{IF}$ G_δ-$e^\star$-los then IFG_δ - $e^\star$ - $lint(IFG_\delta$ - $e^\star$ - $lint(\mathfrak{A}))=IFG_\delta$-$e^\star$-$lint(\mathfrak{A})$
4. IFG_δ-$e^\star$-$lint(\mathfrak{A})=IFG_\delta$-$e^\star$-$lcl(\overline{\mathfrak{A}})$
5. IFG_δ-$e^\star$-$lcl(\mathfrak{A})=IFG_\delta$-$e^\star$-$lint(\overline{\mathfrak{A}})$
6. If $\mathfrak{A}\subseteq\mathfrak{B}$ then IFG_δ-$e^\star$-$lint(\mathfrak{A})\subseteq IFG_\delta$-$e^\star$-$lint(\mathfrak{B})$
7. $(IFG_\delta$ - $e^\star$ - $lint(\mathfrak{A}))\cap(IFG_\delta$-$e^\star$-$lint(\mathfrak{B}))\supseteq IFG_\delta$-$e^\star$-$lint(\mathfrak{A}\cap\mathfrak{B})$.

Remark 3.1

1. IFG_δ-$e^\star$-$lcl(\mathfrak{A})=\mathfrak{A}$ if and only if $\mathfrak{A}$ is an $\mathcal{IF}$ G_δ-$e^\star$-lcs
2. IFG_δ - $e^\star$ - $lint(\mathfrak{A})\subseteq A\subseteq IFG_\delta$-$e^\star$-$lcl(\mathfrak{A})$
3. IFG_δ-$e^\star$-$lint(1_\sim)=1_\sim$
4. IFG_δ-$e^\star$-$lint(0_\sim)=0_\sim$
5. IFG_δ-$e^\star$-$lcl(1_\sim)=1_\sim$

Proposition 3.3 *Every $\mathcal{IF}$ $e^\star$-lcs is an $\mathcal{IF}$ $e^\star G_\delta$-lcs.*

Remark 3.2 *The converse of the Proposition 3.3 need not be true as show in Example 3.1.*

Example 3.1 *Let $X=\{a,b\}$ be a nonempty set. Let $\mathfrak{A}=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$, $\mathfrak{B}=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$, $\mathfrak{A}\cup\mathfrak{B}=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$ and $\mathfrak{A}\cap\mathfrak{B}=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$ be $\mathcal{IFS}$'s of X. Then the family $\Gamma=\{0_\sim,1_\sim,\mathfrak{A},\mathfrak{B},\mathfrak{A}\cup\mathfrak{B},\mathfrak{A}\cap\mathfrak{B}\}$ is an $\mathcal{IFT}$ on X. Now $\mathfrak{C}=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.9},\frac{b}{0.9})\right\rangle$ be $\mathcal{IF}$ $e^\star G_\delta$- set let $\mathfrak{D}=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{1},\frac{b}{1})\right\rangle$ be an $\mathcal{IF}$ $e^\star$-closed set. Hence $\mathfrak{E}=\mathfrak{C}\cap\mathfrak{D}=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{1},\frac{b}{1})\right\rangle$ is $\mathcal{IF}$ $e^\star G_\delta$-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ $e^\star$-lcs. Hence, $\mathcal{IF}$ $e^\star G_\delta$-set need not be an $\mathcal{IF}$ $e^\star$-lcs.*

Proposition 3.4 *Every $\mathcal{IF}$ G_δ-lcs is an $\mathcal{IF}$ G_δ-$e^\star$-lcs.*

Remark 3.3 *The converse of the Propositionn 3.4 need not be true as shown in Example 3.2.*

Example 3.2 *In Example 3.1 $\mathfrak{A}\cap\mathfrak{B}$ is an $\mathcal{IF}$ G_δ-set. Let $\mathfrak{F}=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.7},\frac{b}{0.7})\right\rangle$ be an $\mathcal{IF}$ $e^\star$-closed set. Hence $\mathfrak{E}=(\mathfrak{A}\cap\mathfrak{B})\cap\mathfrak{F}=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.7},\frac{b}{0.7})\right\rangle$ is G_δ-$e^\star$-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ G_δ-lcs. Hence, $\mathcal{IF}$ G_δ-$e^\star$-lcs need not be an $\mathcal{IF}$ G_δ-lcs.*

Remark 3.4 *$\mathcal{IF}$ $e^\star$ - lcs and $\mathcal{IF}$ G_δ-$e^\star$-lcs's are independent of each other as shown by the following Example 3.3.*

Example 3.3 *In Example 3.1 $\mathfrak{A}\cap\mathfrak{B}$ is an $\mathcal{IF}$ G_δ-set. Let $\mathfrak{G}=\left\langle x,(\frac{a}{0.1},\frac{b}{0.2}),(\frac{a}{0.5},\frac{b}{0.5})\right\rangle$ be an $\mathcal{IF}$ $e^\star$-closed set. Hence $\mathfrak{E}=(\mathfrak{A}\cap\mathfrak{B})\cap\mathfrak{G}=\left\langle x,(\frac{a}{0.1},\frac{b}{0.1}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$ is G_δ-$e^\star$-lcs. But, $\mathfrak{E}$ is not an*

May – June 2020
ISSN: 0193-4120 Page No. 7450 - 7454

$\mathcal{IF}$ $e^{\star}$-lcs and $\mathfrak{H} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ be an $\mathcal{IF}$ $e^{\star}$ -open set Hence $\mathfrak{E} = \mathfrak{G} \cap \mathfrak{H} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ is $\mathcal{IF}$ $e^{\star}$-lcs. But, $\mathfrak{E}$ is not G_{δ}-$e^{\star}$-lcs.

Proposition 3.5 *Every $\mathcal{IF}$ G_{δ}-$e^{\star}$-lcs is an $\mathcal{IF}$ $e^{\star}G_{\delta}$-lcs.*

Remark 3.5 *The converse of the Proposition 3.5 need not be true as shown in Example 3.4.*

Example 3.4 *In Example 3.1 $\mathfrak{C} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ be $\mathcal{IF}$ $e^{\star}G_{\delta}$- set and $\mathfrak{G} = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an $\mathcal{IF}$ $e^{\star}$ -closed set. Hence $\mathfrak{E} = \mathfrak{C} \cap \mathfrak{G} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ is $e^{\star}G_{\delta}$-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ G_{δ}-$e^{\star}$-lcs. Hence, $\mathcal{IF}$ $e^{\star}G_{\delta}$-lcs need not be an $\mathcal{IF}$ G_{δ}-$e^{\star}$- lcs.*

Remark 3.6 *$\mathcal{IF}$ G_{δ} - lcs and $\mathcal{IF}$ $e^{\star}G_{\delta}$-lcs's are independent of each other as shown by the following Example 3.5.*

Example 3.5 *In Example 3.1 $\mathfrak{C} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ be $\mathcal{IF}$ $e^{\star}G_{\delta}$- set and $\mathfrak{G} = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an $\mathcal{IF}$ $e^{\star}$ -closed set. Hence $\mathfrak{E} = \mathfrak{C} \cap \mathfrak{G} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ is $e^{\star}G_{\delta}$-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ G_{δ}-lcs and $\mathfrak{A} \wedge \mathfrak{B}$ is an $\mathcal{IF}$ G_{δ}-set let $\mathfrak{I} = \left\langle x, (\frac{a}{0.6}, \frac{b}{0.6}), (\frac{a}{0.3}, \frac{b}{0.1}) \right\rangle$ be an $\mathcal{IF}$ closed set. Hence $\mathfrak{E} = (\mathfrak{A} \cap \mathfrak{B}) \cap \mathfrak{I} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is G_{δ}-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ $e^{\star}G_{\delta}$-lcs.*

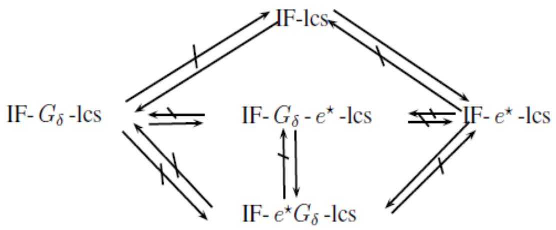
Proposition 3.6 *Every $\mathcal{IF}$ lcs is an $\mathcal{IF}$ $e^{\star}$-lcs.*

Remark 3.7 *The converse of the Proposition 3.6 need not be true as shown in Example 3.6*

Example 3.6 *In Example 3.1 Let $\mathfrak{I} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ be $\mathcal{IF}$ $e^{\star}$-open set and $\mathfrak{H} = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.4}, \frac{b}{0.4}) \right\rangle$ be an $\mathcal{IF}$ $e^{\star}$ -closed set. Hence $\mathfrak{E} = \mathfrak{I} \cap \mathfrak{H} = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ is $e^{\star}$-lcs. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ lcs.*

Remark 3.8 *Every $\mathcal{IF}$ lcs is an $\mathcal{IF}$ G_{δ}-lcs but the converse need not be true as shown in [10].*

Remark 3.9 *Clearly the followinng diagram holds.*



Definition 3.7 *Let (X, Γ) be an $\mathcal{IFTS}$ and Y be any $\mathcal{IF}$ subset of X. Then $\Gamma_Y = (\mathfrak{A}/Y | \mathfrak{A} \in \Gamma)$ is an $\mathcal{IFT}$ on Y and is called the induced or relative $\mathcal{IFT}$. The pair (Y, Γ_Y) is called an $\mathcal{IF}$ subspace of (X, Γ): (Y, Γ_Y) is called an $\mathcal{IF}$ G_{δ}-$e^{\star}$-lc $\mathcal{IF}$ subspace if the $\mathcal{IF}$ characteristic function of (Y, Γ_Y) viz χ_Y is $\mathcal{IF}$ G_{δ}-$e^{\star}$-lcs.*

Proposition 3.7 *Let (X, Γ) be an $\mathcal{IFTS}$. Suppose $Z \subseteq Y \subseteq X$ and (Y, Γ_Y) is an $\mathcal{IF}$ G_{δ}-$e^{\star}$-locally closed $\mathcal{IF}$ subspace of an $\mathcal{IFTS}$ (X, Γ). If (Z, Γ_Z) is an $\mathcal{IF}$ G_{δ}-$e^{\star}$-lc $\mathcal{IF}$ subspace in an $\mathcal{IFTS}$ $(X, \Gamma) \Leftrightarrow (Z, \Gamma_Z)$ is an $\mathcal{IF}$ G_{δ}-$e^{\star}$-lc $\mathcal{IF}$ subspace in an $\mathcal{IFTS}$ (Y, Γ_Y).*

Definition 3.8 *An $\mathcal{IFTS}$ (X, Γ) is said to be an $\mathcal{IF}$ G_{δ}-$e^{\star}$-local-$T_{\frac{1}{2}}$ space if for every $\mathcal{IF}$ G_{δ}-$e^{\star}$-lcs is an $\mathcal{IF}$ closed set in an $\mathcal{IFTS}$ (X, Γ).*

Definition 3.9 *An $\mathcal{IFTS}$ (X, Γ) is said to be an $\mathcal{IF}$ G_{δ}-$e^{\star}$-local-δ-semi (resp.,δ-pre and β) space if for every $\mathcal{IF}$ G_{δ}-$e^{\star}$-lcs is an $\mathcal{IF}$ δ-semi (resp.,δ-pre and β) closed set in an $\mathcal{IFTS}$ (X, Γ).*

Proposition 3.8 *Every $\mathcal{IF}$ G_{δ}-$e^{\star}$-local-$T_{\frac{1}{2}}$ space is an $\mathcal{IF}$ G_{δ} - $e^{\star}$ -local- δ -semi (resp.,G_{δ}-$e^{\star}$-local-δ-pre and G_{δ}-$e^{\star}$-local-β) space.*

Remark 3.10 *The converse of the Proposition 3.8 need not be true as shown in Examples 3.7 and 3.8.*

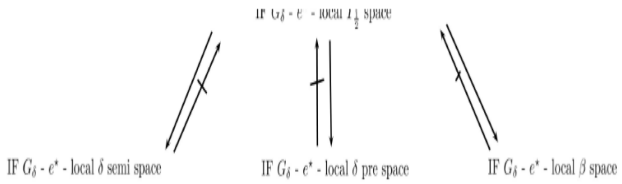
Example 3.7 *Let $X = \{a, b\}$ be a nonempty set. Let $\mathfrak{A} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$, $\mathfrak{B} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$, $\mathfrak{A} \cup \mathfrak{B} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ and $\mathfrak{A} \cap \mathfrak{B} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ be $\mathcal{IF}$ sets of X. Then the family $\Gamma = \{0_{\sim}, 1_{\sim}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \cap \mathfrak{B}, \mathfrak{A} \cup \mathfrak{B}\}$ is an*

May – June 2020
ISSN: 0193-4120 Page No. 7450 - 7454

$\mathcal{IFT}$ on X. Now,$\mathfrak{A} \cap \mathfrak{B}$ is an $\mathcal{IF}$ G_δ-set. Let $\mathfrak{C} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an $\mathcal{IF}$ $e^\star$- closed set. Hence $\mathfrak{E} = (\mathfrak{A} \cap \mathfrak{B}) \cap \mathfrak{C} = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is δ -semi-closed. Hence (X, Γ) is an $\mathcal{IF}$ G_δ-$e^\star$-local-δ-semi space. But, $\mathfrak{E}$ is not an $\mathcal{IF}$ closed set. Thus, (X, Γ) is not an $\mathcal{IF}$ G_δ-$e^\star$-local $T_{\frac{1}{2}}$ space. Hence, $\mathcal{IF}$ G_δ-$e^\star$-local-δ-semi space need not be an $\mathcal{IF}$ G_δ-$e^\star$-local $T_{\frac{1}{2}}$.

Example 3.8 *In Example 3.7 Let $\mathfrak{D} = \left\langle x, (\frac{a}{0}, \frac{b}{0}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an $\mathcal{IF}$ e- closed set. Hence $\mathfrak{E} = (\mathfrak{A} \cap \mathfrak{B}) \cap \mathfrak{D} = \left\langle x, (\frac{a}{0}, \frac{b}{0}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is δ-pre-closed(resp., β-closed). Hence (X, Γ) is an $\mathcal{IF}$ G_δ-$e^\star$-local-δ-pre (G_δ-$e^\star$-local-β-space). But, $\mathfrak{E}$ is not an $\mathcal{IF}$ closed set. Thus, (X, Γ) is not an $\mathcal{IF}$ G_δ - $e^\star$ -local $T_{\frac{1}{2}}$ space. Hence, $\mathcal{IF}$ G_δ-$e^\star$-local-δ-pre (G_δ-$e^\star$-local-β)space need not be an $\mathcal{IF}$ G_δ-$e^\star$-local $T_{\frac{1}{2}}$.*

Remark 3.11 *Clearly the following diagram holds.*



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