

# A Cloudy Fuzzy Inventory System to Deteriorate Price-Dependent Demands

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## Abstract

In this paper, we have developed both fuzzy models, like cloudy fuzzy models, and crisp inventory models for deteriorating goods where demand takes the form of selling price in strength. Unlike all the fuzzy models, when we're doing cloudy fuzzy model defuzzification, we see the fuzziness can only be removed as time goes by. Here, either using Yager's index method or extension of Yager's ranking index method for different fuzzy models, we defuzzify the performance. For each model, numerical examples are given, and sensitivity analysis is performed to know the changes occur by adjusting the values of different parameters. Graphical diagrams are created in order to help explain the utility of the models and ultimately, we establish a hypothesis and scope of potential research.

## Article History

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## I. Introduction

Inventory concerns for aging products have been extensively invested in the past few years. Deterioration is described as decomposing, altering or spoiling, so that the objects are not in a condition to be used for their original purpose. Examples of decaying objects include medications, electronic goods, agri-cultural products, blood, oil, and turpentine. Throughout the course of time, numerous researchers have explored this issue.

Ghare and Scharder were the first to incorporate the concept of degradation in their proposed work as regards exponential decay. In some real-life circumstances Covert and Philip and Mishra introduced variable rate of deterioration. Sana et al. developed a manufacturing inventory model to deteriorate products with trendy demand and shortfalls. Mondal et al. implemented a variable cost production inventory model involving faulty products and marketing decisions. Manna and Chiang developed a model for economic quantity production to deteriorate products with a demand rate of the ramp kind. Ghosh et al. explored an optimal verification of price and lot size in the event of finite output, loss

of sales and partial backlogging for a perishable commodity [1]–[4]. Sahoo et al. analyzed an EOQ / EPQ model of three prices for declining goods under shortages.

The scale factor and unit sales price are known in the crisp model, and have a definite value. Though some of the business circumstances suit this circumstance, most of the circumstances in the real world and the conditions in the constantly evolving market environment cannot be taken as fixed values. Those parameters are defined in fuzzy sense in these cases.

Several researchers have suggested different fuzzy inventory models to cope with the uncertain business situation. All the cost factors involved in overall cost are known in crisp inventory models, and have definite values. But disruptions occur in the case of cost parameters in real-life problems due to uncertain demand. Thus fuzzy model of inventory fulfills the void [5]–[7]. Specific fuzzy inventory models are used in the case of cost parameters found in overall cost, due to various fuzzy numbers. Researchers in this field include: Zimmermann, Bellman and Zadeh, Yao and Su, Mahata and Goswamy,

Vijayan and Kumaran etc.

In the de-fuzzification study, in particular, Yager's benefit put a definitive termination on ranking fuzzy numbers. A good number of researchers took the initiative, after several years, to study the ranking methods and finally extracted several simulated formulae on the subject. Cheng was addressing a new approach to ranking fuzzy numbers, known as distance method [8]–[11]. Ezatti and Saneifard have suggested continuously weighted quasi-arithmetic defuzzification methods. Wang et al., Kumakar et al, Hajjari and Abbasbandy, Xu et al. suggested various methods of defuzzification based on the Fuzzy numbers rating. Zhang et al explored and applied a new approach for rating Fuzzy numbers in decision-making issues. De and Mahata used blurry, fuzzy numbers to operate on back-ordered inventory models. Karmaka et al. studied under a blurry, fuzzy demand rate on an EOQ model. De and Mahata recently suggested a vague, fuzzy EOQ model for poor quality products with appropriate proportionate discounts[7], [12]–[15]. Novelty behind the formulation of this production inventory model:

- (i) Production model for deteriorating items;
- (ii) Demand is taken as the power function of the sales price;
- (iii) Fuzzy model is provided using triangular fuzzy for the sale price;
- (iv) Fuzzy total cost is defuzzified using signed distance method via the Yager ranking index system;
- (v) Defuzzification of total cost of the blurry fuzzy process

Table 1: Authors beneficence to the literature

Reference paper	EPQ	Deterioration	Price dependent demand	Fuzzy	Cloudy fuzzy
Forghani et al. (2012)	...	✓	✓	...	...
Alfares et al. (2015)	...			✓	✓
De and Mahata (2016)	...	...	✓		
Shah et al. (2017)	✓	✓	✓	...	...
Saha and Chakrabati(2017)				✓	✓
Karmakar et al. (2018)	...	...	...		✓
De and Mahata (2019)	✓	✓	✓	✓	✓
This article (2019)					

## II. Preliminaries

### II.1. Normalized General Triangular Fuzzynumber(NGTFN)[4]

Let  $D$  be a NGTFN having the form  $\tilde{D} = (C_1, C_2, C_3)$ . Then its membership function is defined by

$$\lambda(\tilde{D}) = \begin{cases} \frac{D - C_1}{C_2 - C_1} & \text{if } C_1 \leq D \leq C_2 \\ \frac{C_3 - D}{C_3 - C_2} & \text{if } C_2 \leq D \leq C_3 \\ 0 & \text{if } D < C_1 \text{ and } D > C_3 \end{cases} \quad (1)$$

Now, the right and left  $\alpha$ -cuts  $\lambda(\tilde{D})$  are given by

$$L(\alpha) = C_1 + \alpha(C_2 - C_1) \text{ and } R(\alpha) = C_3 - \alpha(C_3 - C_2)$$

The measures of fuzziness are obtained from the following formula.

### II.2. Yager's Ranking Index

If the  $\alpha$ -cuts of a Fuzzy Number  $D$  are left and right, then the defuzzification rule under the Yager ranking index is given by

$$\omega(x, t) = \begin{cases} 0 & \text{if } x < c_2 \left(1 - \frac{\tau}{1+t}\right) \text{ and } x > c_2 \left(1 + \frac{\kappa}{1+t}\right) \\ \frac{x - c_2 \left(1 - \frac{\tau}{1+t}\right)}{\frac{\tau c_2}{1+t}} & \text{if } c_2 \left(1 - \frac{\tau}{1+t}\right) \leq x \leq c_2 \\ \frac{c_2 \left(1 + \frac{\kappa}{1+t}\right) - x}{\frac{\kappa c_2}{1+t}} & \text{if } c_2 \leq x \leq c_2 \left(1 + \frac{\kappa}{1+t}\right) \end{cases} \quad (5)$$

$$I(\tilde{D}) = \frac{1}{2} \int_0^1 [L(\alpha) + R(\alpha)] d\alpha = \frac{1}{4} (C_1 + 2C_2 + C_3)$$

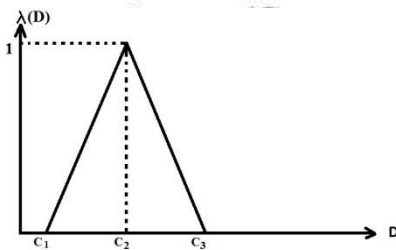


Fig. 1: Membership function of NGTFN

Here the measures of fuzziness or degree of fuzziness  $d_f$  can be obtained from the formula

$$d_f = \frac{U_b - L_b}{2m} \text{ where } L_b \text{ and } U_b \text{ are the lower}$$

boundaries and upper boundaries of the respective Fuzzy numbers and  $m$  shall be their respective mode.

### II.3. Cloudy Normalized Triangular Fuzzy Number (CNTFN)[4]

**Definition 1:** A fuzzy form number  $\tilde{D} = (c_1, c_2, c_3)$  is said to be cloudy triangular fuzzy number if the set itself converges to a crisp singleton after an infinite time. This implies, since  $t$  appears to be infinite, both  $c_1, c_3 \rightarrow c_2$ .

$$\tilde{D} = \left( c_2 \left(1 - \frac{\tau}{1+t}\right), c_2, c_2 \left(1 + \frac{\kappa}{1+t}\right) \right) \quad (4)$$

For  $0 < \tau, \kappa < 1$ .

$$\text{Note that, } \lim_{t \rightarrow \infty} c_2 \left(1 - \frac{\tau}{1+t}\right) \rightarrow c_2 \text{ and } \lim_{t \rightarrow \infty} c_2 \left(1 + \frac{\kappa}{1+t}\right) \rightarrow c_2, \text{ so } \tilde{D} \rightarrow c_2$$

Then the membership functions  $0 \leq t$  as follows:

Let's assume the fuzzynumber

Defuzzification method of CNTFN[4]

Let us consider  $L(\alpha, t)$  and  $R(\alpha, t)$  to be the left  $\alpha$ -cuts and right  $\alpha$ -cut of  $\omega(x, t)$  as mentioned in (5). Now the addition of Yager's ranking index formula used in case of defuzzification under time parameter is given by

$$I(\tilde{B}) = \frac{1}{2T} \int_{\alpha=0, t=0}^{\alpha=0, t=T} [L(\alpha, t) + R(\alpha, t)] d\alpha dt \quad (6)$$

where,  $t$  and  $\alpha$  are two independent variables. Let  $\tilde{B}$  be a CNTFN given as (4). Then the connection function is  $\omega(x, t)$  as mentioned in (5). Using (5) and (6) we have

$$I(\tilde{B}) = \frac{c_2}{2T} \left[ 2T + \frac{\kappa - \tau}{2} \text{Log}(1+T) \right] \quad (7)$$

$$I(\tilde{B}) = c_2 \left[ 1 + \frac{\kappa - \tau}{4} \text{Log}\left(\frac{1+T}{T}\right) \right] \quad (8)$$

And for  $T \rightarrow \infty$ ,  $\log \frac{1+T}{T} \rightarrow 0$  and it implies

$I(\tilde{B}) \rightarrow c_2$ . Now, the factor  $\log\left(\frac{1+T}{T}\right)$  is addressed as a cloudy index (CI).

In real life situation the time horizon cannot be infinite therefore, the defuzzification never give any crisp value in its result.

### III. Notations and Assumptions

This section gives notations and assumptions for the mathematical model.

#### III.1. Assumptions

The following assumptions are made to develop this model.

- (1) The inventory structure involves production of single product.
- (2) Unavailability are not allowed and lead time is zero.
- (3) Inventory deteriorates at a constant rate.
- (4) Demand is a power function of selling price; i.e.  $= ap^{-b}$  where  $a (>) 0$  is the scale factor,  $b (>) 0$  is index of

priceelasticity.

- (5) Replenishment is instantaneous.
- (6) Price is taken to be as fuzzy.

### III.1.1. Notations.

- $O_c$ : Ordering cost per unit per unit time.
- $C_h$ : Holding cost per unit per unit time.
- $C_p$ : Rate of production.
- $\theta_d$ : Rate of deterioration.
- $D_c$ : Cost incurred due to deterioration per unit per unit time.
- $T$ : Cycle length.
- $T_1$ : Cycle length when signed distance method of defuzzification is used.
- $T_2$ : Cycle length when graded mean integration method of defuzzification is used.
- $T_3$ : Cycle length for Cloudy fuzzy model.
- $t_1$ : Production duration.
- $q_1(t)$ : Level of inventory at time  $t$ ,  $0 \leq t \leq t_1$ .
- $q_2(t)$ : Level of inventory at time  $t$ ,  $t_1 \leq t \leq T$ .
- $TC$ : Per production cycle wise total average cost.
- $\bar{TC}$ : Per production wise total average cost when signed distance method is used for defuzzification.
- $\tilde{TC}$ : Per production wise total average cost for the cloudy fuzzy model.

## IV. Mathematical model

In this section, a mathematical model is developed under the consideration of  $q = DT$  and a complete solution of it has been presented.

### IV.1. Mathematical Model formulation

For the market selling price-dependent demand for deteriorating goods the inventory model is created. Here at the beginning of time interval the inventory is at zero point. With the rise in output the inventory level increases. Production rate is assumed to be constant i.e.,  $C_p$ . Regardless of demand and depletion the inventory amount is zero at  $t =$

$$\begin{aligned} \frac{dI_1(t)}{dt} + \theta_d I_1(t) &= C_p - (ap^{-b}), 0 \leq t \leq t_1 \\ \frac{dI_2(t)}{dt} + \theta_d I_2(t) &= -(ap^{-b}), 0 \leq t \leq T \end{aligned} \quad (10)$$

with boundary conditions  $I_1(0) = 0$ ,  $I_1(t_1) = I_2(t_1)$  and  $I_2(T) = 0$ . Solving these equations and using boundary conditions we have

$$I_1(t) = \frac{1}{\theta_d} [C_p - ap^{-b} (1 - e^{-\theta_d t})] \quad (11)$$

$$I_2(t) = \frac{ap^{-b}}{\theta_d} [e^{-\theta_d (T-t)} - 1] \quad (12)$$

For finding  $t_1$  by using  $I_1(t_1) = I_2(t_1)$

$$\begin{aligned} \frac{1}{\theta_d} [C_p - ap^{-b} (1 - e^{-\theta_d t_1})] &= \frac{ap^{-b}}{\theta_d} [e^{-\theta_d (T-t_1)} - 1] \\ \Rightarrow C_p - C_p e^{-\theta_d t_1} + ap^{-b} e^{-\theta_d t_1} &= ap^{-b} e^{-\theta_d (T-t_1)} \end{aligned}$$

T. Based on the above conditions, the inventory level is given as the following differential equations at any time in  $[0, T]$ :

$$\begin{aligned} \Rightarrow C_p e^{\theta_d t_1} &= [C_p - ap^{-b}] + ap^{-b} (e^{\theta_d T} - 1) \\ \Rightarrow t_1 &= \frac{1}{\theta_d} \ln \left[ 1 + \frac{ap^{-b}}{C_p} (e^{\theta_d T} - 1) \right] \end{aligned} \quad (13)$$

The holding cost per cycle

$$\begin{aligned} &= C_h \left[ \int_0^{t_1} I_1(t) dt + \int_{t_1}^T I_2(t) dt \right] \\ &= C_h \left[ \int_0^{t_1} \frac{1}{\theta_d} [C_p - ap^{-b}] (1 - e^{-\theta_d t}) dt + \int_{t_1}^T \frac{ap^{-b}}{\theta_d} [e^{\theta_d (T-t)} - 1] dt \right] \\ &= \frac{C_h}{\theta_d^2} \left[ C_p (\theta_d t_1 + e^{-\theta_d t_1} - 1) + (ap^{-b}) [e^{\theta_d (T-t_1)} - e^{-\theta_d t_1}] - ap^{-b} \theta_d T \right] \\ &= \frac{C_h}{\theta_d^2} \left[ C_p (\theta_d t_1 + e^{-\theta_d t_1} - 1) + (C_p - C_p e^{\theta_d t_1}) - ap^{-b} \theta_d T \right] \end{aligned}$$

(Using (13))

$$= \frac{C_h}{\theta_d} [C_p t_1 - ap^{-b} T] \quad (14)$$

The deterioration cost per cycle

$$\begin{aligned} &= D_c \left[ \int_0^{t_1} I_1(t) dt + \int_{t_1}^T I_2(t) dt \right] \\ &= D_c [C_p t_1 - ap^{-b} T] \end{aligned} \quad (15)$$

Total Cost incurred per unit time

TC = ordering cost + holding cost + deteriorating cost

$$\begin{aligned} &= \frac{O_c}{T} + \frac{C_h}{\theta_d T} [C_p t_1 - ap^{-b} T] + \frac{D_c}{T} [C_p t_1 - ap^{-b} T] \\ &= \frac{O_c}{T} + [C_p t_1 - ap^{-b} T] \frac{(C_h + \theta_d D_c)}{\theta_d T} \end{aligned}$$

$$= \frac{O_c}{T} + \frac{(C_h + \theta_d D_c)}{\theta_d T} \left[ \frac{C_p}{\theta_d} \ln \left( 1 + \frac{ap^{-b}}{C_p} (e^{\theta_d T} - 1) \right) - ap^{-b} T \right]$$

(using (13))

$$= \frac{O_c}{T} + \frac{(C_h + D_c \theta_d)}{\theta_d T} \left[ \frac{C_p}{\theta_d} \ln \left( 1 + \frac{ap^{-b}}{C_p} \left( \theta_d T + \frac{\theta_d^2 T^2}{2} \right) \right) - ap^{-b} T \right]$$

(Neglecting higher power of  $\theta$ )

$$= \frac{O_c}{T} + \frac{(C_h + D_c \theta_d)}{\theta_d T} \left[ \frac{C_p}{\theta_d} \left( \frac{ap^{-b}}{C_p} \left( \theta_d T + \frac{\theta_d^2 T^2}{2} \right) - \frac{(ap^{-b})^2}{2C_p^2} \left( \theta_d T + \frac{\theta_d^2 T^2}{2} \right)^2 \right) - (ap^{-b}) T \right]$$

Neglecting higher power of  $\theta$ )

$$= \frac{O_c}{T} + \frac{(C_h + D_c \theta_d)}{\theta_d T} \left[ \frac{C_p}{\theta_d} \left( \frac{(ap^{-b}) \theta_d T^2}{2} - \frac{(ap^{-b}) \theta_d T^2}{2C_p} \right) \right]$$

$$= \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b})^2 \frac{T^2}{C_p} \right]$$

(16)

So our objectives is to

$$\left\{ \begin{array}{l} \text{Minimize } TC = \left( \frac{1}{2} \right) \left[ \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b})^2 \frac{T^2}{C_p} \right] \right] \\ \text{Subject to} \\ q = (ap^{-b}) T \end{array} \right.$$

$$(17)$$

## 5. Formulation of Fuzzy Models (NGTFN/CNTFN)

Under the consideration of unstable market price of the product, we develop two different kinds of fuzzy models. In the present case we consider the unit selling price  $p$  as a fuzzy parameter and is denoted by  $\tilde{p}$ . Introducing  $\tilde{p}$  in (2) we obtain the following problem.

$$\left\{ \begin{array}{l} \text{Minimize } TC = \left( \frac{1}{2} \right) \left[ \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap^{-b})^2 \frac{T^2}{C_p} \right] \right] \\ \text{Subject to} \\ q = (ap^{-b}) T \end{array} \right.$$

$$(18)$$

Now the fuzzy number  $\tilde{p}^{-b}$  is used in the following form:

$$\tilde{p} = \begin{cases} \langle p_1, p_2, p_3 \rangle & \text{for NGTFN} \\ \langle p(1 - \frac{\tau}{1+T}), p, p(1 + \frac{\kappa}{1+T}) \rangle & \text{for CNTFN} \\ \text{for } 0 < T, \kappa < 1 \text{ and } T > 0 \end{cases}$$

(19)

Hence, using (1), the connection function for the fuzzy objective and order quantity under NGTFN are given by

$$\lambda_1(TC) = \begin{cases} TC_1 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap_1^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap_1^{-b})^2 \frac{T^2}{C_p} \right] \\ TC_2 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap_2^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap_2^{-b})^2 \frac{T^2}{C_p} \right] \\ TC_3 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) (ap_3^{-b}) T^2 - \frac{1}{2} (C_h + D_c \theta_d) (ap_3^{-b})^2 \frac{T^2}{C_p} \right] \end{cases}$$

(20)

$$\lambda_2(q) = \begin{cases} q_1 = ap_1^{-b} \\ q_2 = ap_2^{-b} \\ q_3 = ap_3^{-b} \end{cases}$$

(21)

Using (2) and (3), the index values of fuzzy objective and order quantities are given as follows:

$$I(TC) = \left( \frac{1}{4T} \right) \left[ 4O_c + \frac{1}{2} (C_h + D_c \theta_d) a T^2 (p_1^{-b} + p_2^{-b} + p_3^{-b}) - \frac{1}{2} (C_h + D_c \theta_d) a^2 (p_1^{-b} + p_2^{-b} + p_3^{-b})^2 \frac{T^2}{C_p} \right]$$

$$I(\tilde{q}) = \frac{1}{4} (q_1 + 2q_2 + q_3) = \frac{aT}{4} (p_1 + 2p_2 + p_3)^{-b}$$

(22)

Again for the CNTFN case the membership function for the objective and the order quantity are given by

$$\omega_1(TC, T) = \begin{cases} TC_1 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) T^2 a \left( 1 - \frac{\tau}{1+T} \right) p^{-b} \left( 1 - \frac{a(1-\frac{\tau}{1+T})p^{-b}}{C_p} \right) \right] \\ TC_2 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) T^2 (ap^{-b}) \left( 1 - \frac{ap^{-b}}{C_p} \right) \right] \\ TC_3 = \left( \frac{1}{T} \right) \left[ O_c + \frac{1}{2} (C_h + D_c \theta_d) T^2 a \left( 1 + \frac{\tau}{1+T} \right) p^{-b} \left( 1 + \frac{a(1+\frac{\tau}{1+T})p^{-b}}{C_p} \right) \right] \end{cases}$$

(23)

$$\omega_2(q, T) = \begin{cases} q_1 = a \left( 1 - \frac{\tau}{1+T} \right) p^{-b} \\ q_2 = ap^{-b} \\ q_3 = a \left( 1 + \frac{\tau}{1+T} \right) p^{-b} \end{cases}$$

(24)

Using (6), the index values of fuzzy objective and order quantities are given as follows:



$$K(TC) = \frac{1}{2T} \left[ 2O_c \ell n \left| \frac{T}{\varepsilon} \right| + \frac{ap^{-b}(C_h + \theta_d D_c)}{4} \right] 2T^2 \left( 1 - \frac{ap^{-b}}{C_p} \right) + (\kappa - \tau) \left( \left( 1 - \frac{2ap^{-b}}{C_p} \right) \right)$$

$$(T - \ell n(1+T)) - \frac{ap^{-b}}{C_p} \left( \frac{\kappa + \tau}{1+T} + \ell n(1+T)(\tau^2 + \kappa^2) \right)$$

$$K(\tilde{q}) = \frac{Tap^{-b}}{2} + \frac{\kappa - \tau}{4} \left[ 1 - \ell n \left( \frac{1+T}{T} \right) \right] \quad (25)$$

Where  $\varepsilon$  is sufficiently small number, that indicates the objective converges to a finite value.

## 6. Numerical examples

In this section an extensive numerical study is performed to show the application of the suggested model. We have used MATLAB optimization tool (2015a) to find the optimum value of the model problem.

### 6.1. Input parameters for numerical study.

Table 2: Values of parameters for numerical experiments

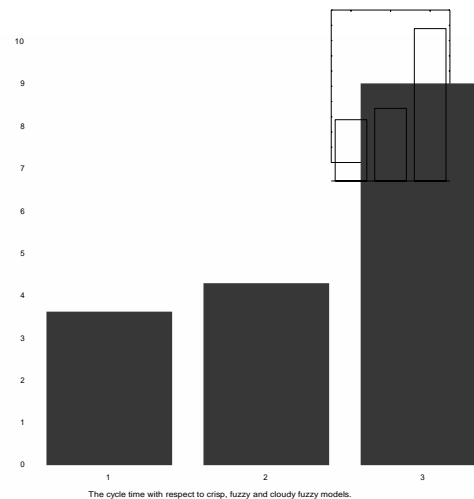
$O_c = 490$	$C_h = 5$	$D_c = 10$	$C_p = 150$	$a = 145$
$b = 0.5$	$p_1 = 125$	$p_2 = 126$	$p_3 = 127$	$\tau = 0.40$
$\kappa = 0.35$	$\theta_d = 0.01$	$\varepsilon = 0.05$		

Table 3: Optimal solution of the EPQ model

Model	Cycle time ( $T^*$ ) (Days)	Ordered quantity ( $q^*$ )	Minimum total cost ( $TC^*$ )	$D_f = \frac{U_s - L_s}{2m}$	$CI = \frac{(1+T)}{T} = \text{Log}$
Crisp	3.63	47.09	219.86	...	...
NGTFN	4.30	6.94	185.87	0.0079	...
CNTFN	9.02	64.87	291.59	...	0.1045

### Illustrations of the model: -

Here we draw the figures where the time varies with respect to crisp, general fuzzy and cloudy fuzzy model.



### 6.2. Sensitivity analysis

Table 4, 5 and 6 show that the optimal result of the prototypical is changing within a certain range when one of the parameters varies from 50% to +50% observance other parameters fixed. On the basis of the results given in table 4, 5, and 6, the following are the observations:

- In crisp model total cost is increasing with respect to increase in  $O_c$ ,  $C_h$ ,  $C_p$ ,  $D_c$ ,  $a$  and  $\theta_d$  independently. But the total cost is decreasing with respect to increase in  $p$  and  $b$  independently.
- Similarly, in case of both general fuzzy and cloudy fuzzy models, the total cost is increasing with respect to increase in  $O_c$ ,  $C_h$ ,  $C_p$ ,  $D_c$ ,  $a$  and  $\theta_d$  independently and the total cost is decreasing with respect to decrease in  $b$ .

In cloudy fuzzy model, with the increasing values of the fuzzy indicating parameters ( $\tau$ ,  $\kappa$ ) and converging parameter  $s$  the objective function does not show much difference in it.

## 7. Conclusion

Here we have established EPQ models under crisp, cloudy fuzzy and general fuzzy environments. Mostly, in this work the focus is on the application of the newly developed fuzzy (cloudy) number in the production inventory model. Though the idea of considering time to be infinite in case of cloudy fuzzy is vague but the eradication of total cloud concept is

also irrational. From the result of the cloud inventory model, it can be found that the solution of the model problem exists and which is at par with the results of general fuzzy and crisp models. Further, in future research this model can be extended by including one more realistic assumption that the product is deteriorating/defective.

#### Appendix

We have the price as fuzzy number  $(p_1, p_2, p_3) = (125, 126, 127)$ . The upper and lower bounds are  $L_b = 125$  and  $U_b = 127$  respectively. The mean is 125, the median is 126. The formula of  $Mode(m) = 3 \times \text{median} - 2 \times \text{mean} = 126$ . Therefore,

$$d_f = \frac{U_b - L_b}{2m} = \frac{1}{126} = 0.0079$$

$$\text{and } CT = \frac{\log(1+T)}{T} = \frac{\log(1+9.02)}{9.02} = 0.1045$$

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Table 4: Sensitivity study with respect to different parameters in Crisp sense

Parameter	Changes(%)	$T^*$	$q^*$	$TC^*$
$O_c$	-50%	2.57	33.36	155.46
	-25%	3.15	40.86	190.40
	+25%	4.06	52.76	245.81
	+50%	4.45	57.79	269.27
$C_h$	-50%	5.09	66.09	156.98
	-25%	4.18	54.31	191.02
	+25%	3.26	42.29	245.33
	+50%	2.98	38.65	268.39
$D_c$	-50%	3.65	47.42	218.78
	-25%	3.64	47.30	219.32
	+25%	3.62	47.07	220.40
	+50%	3.61	46.96	220.93
$C_p$	-50%	3.82	49.59	209.19
	-25%	3.69	47.95	216.36
	+25%	3.56	46.26	224.27
	+50%	3.58	46.46	223.30
$a$	-50%	5.02	32.60	159.10
	-25%	4.15	40.39	192.64
	+25%	3.29	53.39	242.88
	+50%	3.04	59.21	262.82

$p$	-50%	3.12	57.25	256.28
	-25%	3.41	51.08	234.52
	+25%	3.82	44.38	208.89
	+50%	3.99	42.27	200.38
$b$	-50%	2.25	97.81	354.65
	-25%	2.80	66.47	285.41
	+25%	4.81	34.17	166.03
	+50%	6.44	24.99	124.16
$\theta_d$	-50%	3.65	47.42	218.78
	-25%	3.64	47.30	219.32
	+25%	3.66	47.54	218.23
	+50%	3.62	46.96	220.93

Table 5: Sensitivity study with respect to different parameters in Fuzzy sense

Parameter	Changes(%)	$T^*$	$q^*$	$TC^*$
$O_c$	-50%	3.04	4.91	131.43
	-25%	3.72	6.01	160.97
	+25%	4.81	7.76	207.81
	+50%	5.27	8.51	227.64
$C_h$	-50%	6.02	9.73	132.71
	-25%	4.95	7.99	161.49
	+25%	3.85	6.22	207.40
	+50%	3.52	5.69	226.90
$D_c$	-50%	4.32	6.98	184.96
	-25%	4.31	6.96	185.41
	+25%	4.29	6.93	186.32
	+50%	4.28	6.91	186.78
$C_p$	-50%	6.24	10.08	128.03
	-25%	4.73	7.65	168.81
	+25%	4.09	6.61	195.39
	+50%	3.97	6.41	201.49
$a$	-50%	5.75	4.00	139.08
	-25%	4.91	5.13	162.74
	+25%	4.23	7.37	189.03
	+50%	4.11	8.59	194.40
$b$	-50%	...	...	...
	-25%	4.23	14.89	188.79
	+25%	5.23	3.88	152.88
	+50%	6.73	2.29	118.80
$\theta_d$	-50%	4.32	6.98	184.96
	-25%	4.31	6.96	185.41
	+25%	4.29	6.93	186.32
	+50%	4.28	6.91	186.78

$O_c$	-50%	8.04	62.94	140.57
	-25%	8.75	64.01	205.58
	+25%	8.83	66.76	245.84
	+50%	10.30	67.59	270.64
$C_h$	-50%	10.90	67.59	230.71
	-25%	9.95	66.92	280.50
	+25%	8.65	64.20	309.51
	+50%	8.32	63.60	330.91
$D_c$	-50%	9.02	64.90	290.90
	-25%	8.90	64.86	185.41
	+25%	8.82	64.50	186.32
	+50%	8.70	64.41	186.78
$C_p$	-50%	11.25	69.09	240.04
	-25%	9.74	65.40	266.71
	+25%	8.90	64.59	305.49
	+50%	7.97	63.39	311.30
$a$	-50%	10.74	62.02	237.09
	-25%	9.90	63.15	260.71
	+25%	8.93	66.30	295.04
	+50%	7.12	68.73	305.50
$b$	-50%	...	...	...
	-25%	8.90	72.60	295.70
	+25%	10.12	60.22	265.21
	+50%	11.72	58.10	211.25
$\theta_d$	-50%	9.05	64.90	289.90
	-25%	8.96	64.82	291.43
	+25%	8.90	64.70	291.87
	+50%	8.70	64.50	292.78
$s$	-50%	10.50	69.59	310.15
	-25%	10.20	66.62	297.20
	+25%	8.90	59.37	287.57
	+50%	8.35	55.80	285.24
$\tau$	-50%	...	...	...
	-25%	9.02	70.86	290.81
	+25%	9.05	71.46	292.19
	+50%	9.07	73.50	292.65
$\kappa$	-50%	9.01	72.94	291.94
	-25%	9.04	72.42	290.30
	+25%	9.08	71.79	290.63
	+50%	9.10	70.59	290.06

Table 6: Sensitivity study with respect to different parameters in Cloudy fuzzy sense

Parameter	Changes(%)	$T^*$	$q^*$	$TC^*$
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