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On Bipolar Fuzzy Sub Ordered Γ-Near Rings

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Abstract:

In this paper, we introduce and study the concept of Bipolar Fuzzy Sub Ordered Γ-Nearring.we establish a one-one correspondence between Bipolar Fuzzy sub Ordered Γ-Nearring and crisp sub Ordered Γ-Nearring. Later, we define homomorphism on Bipolar-Fuzzy Ordered Γ-Nearring and we verified that the homomorphic pre image of a Bipolar Fuzzy sub Ordered Γ- Nearring is also a Bipolar Fuzzy sub Ordered Γ- Nearring . Moreover we prove that homomorphic image of a Bipolar Fuzzy sub Ordered Γ- Nearring possessing both Supremum property and infimum property is a Bipolar Fuzzy sub Ordered Γ-Nearring.

***Keywords:** Fuzzy sub set, Ordered Γ-Near ring, Bipolar Fuzzy sub Ordered Γ- Nearring*

I. INTRODUCTION

Zadeh [6] introduced fuzzy subset in 1965. Later, it has been studied by several authors and applied on different algebraic structures like semi groups, semi rings, Γ -semi rings..... etc. Later, the concept of ideal theory in Γ- Nearring and fuzzy ideals in Γ-Nearring were introduced and studied by Bh. Satyanarayana[1] and G.L.Booth[3]. Further, S.Ragamayi([8],[17]) has introduced and studied the structures of Lattice fuzzy sub Γ-Nearrings in her doctorial thesis.

Furthermore K.Balakoteswara rao([2][18]) studied and originated the above ideas on Ordered Γ-Nearring and introduced "Lattice-Fuzzy Sub Ordered Γ-Nearrings" and " vague Ordered Γ-Nearrings". As a sequel of above work, now we introduce Bipolar Fuzzy subset of an ordered Γ-Nearring which is an extension of a fuzzy subset of an ordered Γ-Nearring. We establish a one-one correspondence between Bipolar Fuzzy sub Ordered Γ-Nearring and crisp sub Ordered Γ-Nearring. Also, we prove that the intersection of two Bipolar Fuzzy sub ordered Γ-Nearring is a Bipolar Fuzzy sub ordered Γ-Nearring . Later we verified that the homomorphic pre image of a Bipolar Fuzzy sub Ordered Γ- Nearring is also a Bipolar Fuzzy sub Ordered Γ- Nearring . Moreover we prove that homomorphic image of a Bipolar Fuzzy sub Ordered Γ- Nearring possessing both Supremum and infimum properties is also a Bipolar Fuzzy sub Ordered Γ- Nearring.

II. Preliminaries

Definition 2.1 : A zero – symmetric GNR is a trip[le $(\widetilde{K},+,\Gamma)$, where

1. $(\widetilde{K},+)$ is a group
2. $(\widetilde{K},+,\delta)$ is a nearring where $\Gamma\neq\Phi$ with binary operators on $\widetilde{K}$, $\forall\ \delta\in\Gamma$.
3. $f\,\delta(g\,\eta\,h) = (f\,\delta g)\eta h\ \ \forall f,g,h\epsilon\,\widetilde{K}\,;\delta,\eta\in\Gamma$.
4. $f\,\delta\,g = 0\ ,\forall\,f\ \in\ \widetilde{K},\ \delta\in\Gamma$.

Definition 2.2 : $E\neq\Phi$, $A\subseteq\widetilde{K}$, where $\widetilde{K}$ be a GNR is said to be a SGNR if

1. $f-g\ \in E$
2. $f\,\delta\,g\in E,\forall\delta\in\Gamma;f,g\in\widetilde{K}$

Definition 2.3: A Fuzzy subset $\mathcal{W}$ of $\widetilde{K}$ is a Fuzzy SGNR , if

$$\mathcal{W}(f-g)\geq\min\{\ \mathcal{W}(f),\mathcal{W}(g)\}$$
$$\mathcal{W}(f\,\delta\,g)\geq\min\{\ \mathcal{W}(f),\mathcal{W}(g)\},\forall\delta\in\Gamma;f,g\in\widetilde{K}$$

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Definition 2.4 : A GNR, $\widetilde{K}$ is called as an OGNR if it admits a compatible relation "$\le$ ", If $a \le b$ and $c \le d$ then

$$a + c \le b + d$$
$$a\delta c \le b\delta d$$

$c\delta a \le d\delta b;\ \forall\ a, b, c, d \in \widetilde{K};\ \delta \in \Gamma.$

Definition 2.5: Let $\widetilde{K}$ be a partially OGNR . $\emptyset \neq \mathbb{N}$ of $\widetilde{K}$ is said to be SOGNR,if

$$f - g \in \mathbb{N}$$
$$f\delta g \in \mathbb{N}$$
$$\text{if } a \le b \text{ then } a + f \le b + g$$

if $a \le b$ and $c \ge 0$ then $c\,\delta a \le c\delta b$ and $a\delta c \le b\delta c$;

$\forall\ a, b, c, f, g \in \widetilde{K};\ \delta \in \Gamma.$

Definition 2.6: Let $\widetilde{K}$ be an OGNR .A fuzzy subset $\mathcal{W}$ of $\widetilde{K}$ is said to FSOGNR Of $\widetilde{K}$ if

$$\mathcal{W}(f - g) \ge \min\{\ \mathcal{W}(f), \mathcal{W}(g)\}$$
$$\mathcal{W}(f\,\delta\,g) \ge \min\{\ \mathcal{W}(f), \mathcal{W}(g)\},$$
$$f \le g \Longrightarrow \mathcal{W}(f) \ge \mathcal{W}(g);\ \forall\delta \in \Gamma;$$

$f, g \in \widetilde{K}.$

Definition 2.7: The union and intersection of any family $\{\mu_i / i \in \Delta\}$ of Fuzzy subsets of a set X are defined as

$$\underset{i\in\Delta}{\cup}\mu_i(x) = \underset{i\in\Delta}{\text{Sup}}\,\mu_i(x), \forall x \in X$$
$$\underset{i\in\Delta}{\cap}\mu_i(x) = \underset{i\in\Delta}{\text{Inf}}\,\mu_i(x), \forall x \in X$$

Definition 2.8: Let X be the universe of discourse. A bipolar-valued fuzzy set φ in X is an object having the form

$\varphi=\{< x, \mu_\varphi^+\ (x), \mu_\varphi^-\ (x)>/x \in X\}$ where , μ_φ^+: $X\rightarrow[0, 1]$ and μ_φ^- : $X\rightarrow[-1, 0]$ are mappings. The positive membership degree μ_φ^+ denoted the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set $\varphi=\{< x, , \mu_\varphi^+\ (x), \mu_\varphi^-\ (x)>/x \in X\}$ and the negative membership degree μ_φ^- denotes the satisfaction degree of x to some implicit counter-property of $\varphi=\{< x, , \mu_\varphi^+\ (x), \mu_\varphi^-\ (x)>/x \in X\}$.

Definition 2.9: For a bipolar fuzzy set

$\varphi = (X; , \mu_\varphi^+\ (x), \mu_\varphi^-\ (x))$ and $(s,t)\in[-1,0]$ X $[0,1]$ we define level subset of , μ_φ^+ is

$\varphi_t^P = \{x \in X;\ \mu_\varphi^+\ (x) \ge t\}$ and $\varphi_s^N = \{x \in X;\ \mu_\varphi^-\ (x) \le s\}$ which are called the positive t-cut of $\varphi = (X;\ \mu_\varphi^+\ (x), \mu_\varphi^-\ (x))$ and the negative s-cut of $\varphi = (X;\ \mu_\varphi^+\ (x), \mu_\varphi^-\ (x))$ respectively. For $(s,t)\in[-1,0]$ X $[0,1]$, the set $\varphi_{(t,s)} = \varphi_t^P \cap \varphi_s^N$ is called the (t, s)-cut of $\varphi = (X;\ \mu_\varphi^+\ (x), \mu_\varphi^-\ (x))$.

Definition 2.10: Let $\mu_i = \{(\mu_i^+, \mu_i^-)/\ i \in \Delta\}$ be a family bipolar Fuzzy subsets of a set X then the union and intersection are defined by

$$\underset{i\in\Delta}{\cup}\mu_i^+\ (x) = \underset{i\in\Delta}{\text{Sup}}\,\mu_i^+(x)\ ,\ \forall\ x \in X$$
$$\underset{i\in\Delta}{\cap}\mu_i^-\ (x) = \underset{i\in\Delta}{\text{Inf}}\,\mu_i^-(x)\ ,\ \forall\ x \in X$$

Notations :

K̃ represents "zero-symmetric Ordered Γ-Nearring".

GNR represents " Gamma Nearring".

OGNR represents " Ordered Gamma Nearring".

SGNR represents " Sub Gamma Nearring".

SOGNR represents " Sub Ordered Gamma Nearring".

FSOGNR represents "Fuzzy Sub Ordered Gamma Nearring".

BFSOGNR represents " Bipolar Fuzzy Sub Ordered Gamma Nearring".

III. Main results on Bipolar Fuzzy Sub Ordered Γ-Nearring

Definition 3.1: Let $\widetilde{K}$ be an OGNR and φ be a bipolar Fuzzy subset of $\widetilde{K}$. We say that φ is a BFSOGNR of $\widetilde{K}$ if

1) $\mu_\varphi^+\ (p - q) \ge \min\{\mu_\varphi^+\ (p), \mu_\varphi^+\ (q)\}$ and $\mu_\varphi^-\ (p - q) \le \max\{\mu_\varphi^-\ (p), \mu_\varphi^-\ (q)\}$

2) $\mu_\varphi^+\ (p\ \gamma\ q) \ge \min\{\mu_\varphi^+\ (p), \mu_\varphi^+\ (y)\}$ and $\mu_\varphi^-\ (p\ \gamma\ q) \le \max\{\mu_\varphi^-\ (p), \mu_\varphi^-\ (y)\}$

3) $p \le q \rightarrow \mu_\varphi^+\ (p) \ge \mu_\varphi^+\ (q)$ and $\mu_\varphi^-\ (p) \le \mu_\varphi^-\ (q)$ $\forall p,q \in \widetilde{K}, \gamma \in \Gamma$.

Example 3.2: Consider the additive group $\widetilde{K}=\mathbb{I}$,Set of integers and $\Gamma = \{0,1,2\}$.

Then $\widetilde{K}$ is a GNR with zero symmetry with the mapping $\mathbb{I} \times \Gamma \times \mathbb{I} \rightarrow \mathbb{I}$.

And clearly $\mathbb{I}$ is admiting a compatible relation "≤" which is a partial ordering on $\mathbb{I}$ satisfies the following conditions.

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If $a \le b$ and $c \le d$ then
(i) $a + c \le b + d$
(ii) $a\, \gamma\, c \le b\, \gamma\, d$
(iii) $c\, \gamma\, a \le d\, \gamma\, b$; $\forall a, b, c, d \in \mathbb{I}$; $\gamma \in \Gamma$.
Hence $\mathbb{I}$ is an OGNR.
Define $\mu_\varphi^+ : \mathbb{I} \to [0, 1]$ by

$$\mu_\varphi^+ (p) = \begin{cases} 0.8, & \text{if } p = 0 \\ 0.5, & \text{if } p \text{ is positive integer} \\ 0.4, & \text{if } p \text{ is negitive integer} \end{cases}$$

Then μ_φ^+ is a FSOGNR of $\mathbb{I}$.
Define $\mu_\varphi^- : \mathbb{I} \to [-1,0]$ by

$$\mu_\varphi^- (p) == \begin{cases} -0.5, & \text{if } p = 0 \\ -0.3, & \text{if } p \text{ is positive integer} \\ -0.1, & \text{if } p \text{ is negitive integer} \end{cases}$$

Then μ_φ^- is a FSOGNR of $\mathbb{I}$.
Therefore $\varphi=\{< p, \mu_\varphi^+ (p), \mu_\varphi^- (p)>/p \in \mathbb{I}\}$ is a BFSOGNR of $\mathbb{I}$.

Theorem 3.3: A Bipolar valued Fuzzy subset φ of an OGNR, $\tilde{K}$ is a BFSOGNR, $\tilde{K}$ if and only if it's (t,s)-cut is a SOGNR, $\tilde{K}$.
proof : Suppose φ is BFSOGNR, $\tilde{K}$.
case(1): Let the positive membership degree of φ,i.e, μ_φ^+ is a FSOGNR, $\tilde{K}$.
Now, we have to prove that
$\varphi_t^P = \{p \in M / \mu_\varphi^+ (p) \ge t\}$ is SOGNR, $\tilde{K}$.
Let $p,q \in \varphi_t^P$.
$\to \mu_\varphi^+ (p) \ge t$ and $\mu_\varphi^+ (q) \ge t$
$\to \min\{ \mu_\varphi^+ (p), \mu_\varphi^+ (q)\} \ge t$
Now, 1. $\mu_\varphi^+ (p\text{-}q) \ge \min\{ \mu_\varphi^+ (p), \mu_\varphi^+ (q)\} \ge t$.
$\to p\text{-}q \in \varphi_t^P$.
2. $\mu_\varphi^+ (p\, \gamma\, q) \ge \min\{ \mu_\varphi^+ (p), \mu_\varphi^+ (q)\} \ge t$.
$\to p\, \gamma\, q \in \varphi_t^P$.
Then φ_t^P is SOGNR of $\tilde{K}$.
on the contrary suppose that φ_t^P is a SOGNR of $\tilde{K}$.
Now, we prove that μ_φ^+ is FSOGNR of $\tilde{K}$.
Let $p,q \in \tilde{K}$.
Let $\mu_\varphi^+ (p)=t1$; $\mu_\varphi^+ (q)=t2$
put $t=\min(t_1, t_2)$.
Then $\mu_\varphi^+ (p)= t_1 \ge t$; $\mu_\varphi^+ (q)= t_2 \ge t$.
$\to p,q \in \varphi_t^P$.
$\to p\text{-}q \in \varphi_t^P$ and $p\, \gamma\, q \in \varphi_t^P$
$\to \mu_\varphi^+ (p\text{-}q) \ge t$ and $\mu_\varphi^+ (p\, \gamma\, q) \ge t$
$\to \mu_\varphi^+ (p\text{-}q) \ge \min(\mu_\varphi^+ (p), \mu_\varphi^+ (q))$ and
$\to \mu_\varphi^+ (p\, \gamma\, q) \ge \min(\mu_\varphi^+ (p), \mu_\varphi^+ (q))$
Let $p \le q$.
presume if $\mu_\varphi^+ (p) < \mu_\varphi^+ (q)$.
Then $\exists t_1 \in [0,1] \ni \mu_\varphi^+ (p) < t_1 < \mu_\varphi^+ (q)$.
Then $q \in \varphi_{t1}^P$ and $p \notin \varphi_{t1}^P$.
This is an inconsistency.
thus $\mu_\varphi^+ (p) \ge \mu_\varphi^+ (q)$.
Then μ_φ^+ is a FSOGNR of $\tilde{K}$.
case(2): Let the negative membership degree of φ,i.e, μ_φ^- is a FSOGNR of $\tilde{K}$.
Now, we have to verify that
$\varphi_s^N = \{p \in M / \mu_\varphi^- (p) \le s\}$ is SOGNR of $\tilde{K}$.
Let $k,m \in \varphi_s^N$.
$\to \mu_\varphi^- (k) \le s$ and $\mu_\varphi^- (m) \le s$
$\to \max\{ \mu_\varphi^- (k), \mu_\varphi^- (m)\} \le s$
Now, $\mu_\varphi^- (k\text{-}m) \le \max\{ \mu_\varphi^- (k), \mu_\varphi^- (m)\} \le s$.
$\to k\text{-}m \in \mu_\varphi^-$. and
Also, we have
$\mu_\varphi^- (k\, \gamma\, m) \le \max\{ \mu_\varphi^- (k), \mu_\varphi^- (m)\} \le s$.
$\to k\, \gamma\, m \in \varphi_s^N$
Then φ_s^N is a SOGNR of $\tilde{K}$.
on the contrary presume that φ_s^N is a SOGNR of $\tilde{K}$.
Now, we prove that μ_φ^- is a FSOGNR of $\tilde{K}$.
Let $k,m \in \tilde{K}$.
Let $\mu_\varphi^- (k)=s_1$ $\mu_\varphi^- (m)= s_2$.
put $s=\max(s_1, s_2)$.
Then $\mu_\varphi^- (k)= s_1 \le s$; $\mu_\varphi^- (m)= s_2 \le s$.
$\to k,m \in \varphi_s^N$
$\to k\text{-}m \in \varphi_s^N$ and $\to k\, \gamma\, m \in \varphi_s^N$
$\to \mu_\varphi^- (k\text{-}m) \le s$ and
$\to \mu_\varphi^- (k\, \gamma\, m) \le s$.
$\to \mu_\varphi^- (k\text{-}m) \le \max(\mu_\varphi^- (k), \mu_\varphi^- (m))$
and
$\to \mu_\varphi^- (k\, \gamma\, m) \le \max(\mu_\varphi^- (k), \mu_\varphi^- (m))$
Let $k \le m$.

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Infer that if possible $\mu_\varphi^-(k) > \mu_\varphi^-(m)$.
Then $\exists s_1 \in [-1,0] \ni \mu_\varphi^-(k) > s_1 > \mu_\varphi^-(m)$.
Then $m \in \varphi_{s1}^N$ and $k \notin \varphi_{s1}^N$.
Which is a contradiction.
Therefore $\mu_\varphi^-(k) \leq \mu_\varphi^-(m)$.
Then μ_φ^- is a FSOGNR of $\widetilde{K}$.

Theorem 3.4: Let $\varphi = (\widetilde{K};\ \mu_\varphi^+,\ \mu_\varphi^-)$ be a BFSOGNR of $\widetilde{K}$. Then the sets
$M_{\mu_\varphi^+} = \{ p \in \widetilde{K} / \mu_\varphi^+(p) = \mu_\varphi^+(0)\}$ and
$M_{\mu_\varphi^-} = \{ p \in \widetilde{K} / \mu_\varphi^-(p) = \mu_\varphi^-(0)\}$ are SOGNRs of $\widetilde{K}$.
proof:
case(1): Let the positive membership degree of φ,i.e, μ_φ^+
be a FSOGNR of M and let $p,q \in M_{\mu_\varphi^+}$.
Then 1) $\mu_\varphi^+(p\text{-}q) \geq \min\{\mu_\varphi^+(p), \mu_\varphi^+(q)\} = \mu_\varphi^+(0)$
$\rightarrow \mu_\varphi^+(p\text{-}q) = \mu_\varphi^+(0)$
Hence $p\text{-}q \in M_{\mu_\varphi^+}$.
2) $\mu_\varphi^+(p\,\gamma q) \geq \min\{\mu_\varphi^+(p), \mu_\varphi^+(q)\} = \mu_\varphi^+(0)$
$\rightarrow \mu_\varphi^+(p\,\gamma\, q) = \mu_\varphi^+(0)$
Hence $p\,\gamma\, q \in M_{\mu_\varphi^+}$.
Thus $M_{\mu_\varphi^+}$ is a SOGNR of $\widetilde{K}$.
case(2): Let the negative membership degree of φ,i.e, μ_φ^-
be a FSOGNR of $\widetilde{K}$ and let $p,q \in M_{\mu_\varphi^-}$.
Then
1) $\mu_\varphi^-(p\text{-}q) \leq \max\{\mu_\varphi^-(p), \mu_\varphi^-(q)\} = \mu_\varphi^-(0)$
$\rightarrow \mu_\varphi^-(p\text{-}q) = \mu_\varphi^-(0)$
Hence $p\text{-}q \in M_{\mu_\varphi^-}$..
2) $\mu_\varphi^-(p\,\gamma\, q) \leq \max\{\mu_\varphi^-(p), \mu_\varphi^-(q)\} = \mu_\varphi^-(0)$
$\rightarrow \mu_\varphi^-(p\,\gamma\, q) = \mu_\varphi^-(0)$
Hence $p\gamma\, q \in M_{\mu_\varphi^-}$.
Thus $M_{\mu_\varphi^-}$ is a SOGNR of $\widetilde{K}$.

Theorem 3.5: Let $\varphi_1 = (\widetilde{K};\ \mu_\varphi^+, \mu_\varphi^-)$ and $\varphi_2 = (\widetilde{K};\ \sigma_\varphi^+, \sigma_\varphi^-)$ be two BFSOGNR of $\widetilde{K}$.
Then $\varphi_1 \cap \varphi_2$ is a BFSOGNR of $\widetilde{K}$.
Proof:
Case-1: Let the positive membership degree of φ_1 and φ_2 be
μ_φ^+ and σ_φ^+ are two FSOGNR of $\widetilde{K}$.
Let $p,q \in \widetilde{K}$, Take
1)
$(\mu_\varphi^+ \cap \sigma_\varphi^+)(p\text{-}q) = \min\{\mu_\varphi^+(p\text{-}q), \sigma_\varphi^+(p\text{-}q)\}$
$\geq \min\{\min[\mu_\varphi^+(p), \mu_\varphi^+(q)], \min[\sigma_\varphi^+(p), \sigma_\varphi^+(q)]\}$
$= \min\{\min[\mu_\varphi^+(p), \sigma_\varphi^+(p)], \min[\mu_\varphi^+(q), \sigma_\varphi^+(q)]\}$
$= \min\{(\mu_\varphi^+ \cap \sigma_\varphi^+)(p), (\mu_\varphi^+ \cap \sigma_\varphi^+)(q)\}$
Therefore, $(\mu_\varphi^+ \cap \sigma_\varphi^+)(p\text{-}q) \geq \min\{(\mu_\varphi^+ \cap \sigma_\varphi^+)(p), (\mu_\varphi^+ \cap \sigma_\varphi^+)(q)\}$.
2)
$(\mu_\varphi^+ \cap \sigma_\varphi^+)(p\,\gamma\, q) = \min\{\mu_\varphi^+(p\,\gamma\, q), \sigma_\varphi^+(p\,\gamma\, q)\}$
$\geq \min\{\min[\mu_\varphi^+(p), \mu_\varphi^+(q)], \min[\sigma_\varphi^+(p), \sigma_\varphi^+(q)]\}$
$= \min\{\min[\mu_\varphi^+(p), \sigma_\varphi^+(p)], \min[\mu_\varphi^+(q), \sigma_\varphi^+(q)]\}$
$= \min\{(\mu_\varphi^+ \cap \sigma_\varphi^+)(p), (\mu_\varphi^+ \cap \sigma_\varphi^+)(q)\}$
Therefore, $(\mu_\varphi^+ \cap \sigma_\varphi^+)(p\,\gamma\, q) \geq \min\{(\mu_\varphi^+ \cap \sigma_\varphi^+)(p), (\mu_\varphi^+ \cap \sigma_\varphi^+)(q)\}$.
3)If $p \leq q$ then
$\mu_\varphi^+(p) \geq \mu_\varphi^+(q)$ and $\sigma_\varphi^+(p) \geq \sigma_\varphi^+(q)$
$(\mu_\varphi^+ \cap \sigma_\varphi^+)(p) = \min\{\mu_\varphi^+(p), \sigma_\varphi^+(p)\}$
$\geq \min\{\mu_\varphi^+(q), \sigma_\varphi^+(q)\}$
$= (\mu_\varphi^+ \cap \sigma_\varphi^+)(q)$
consequently we proved that the intersection of positive membership of φ_1 and φ_2 i.e,
$(\mu_\varphi^+ \cap \sigma_\varphi^+)$ is also FSOGNR of $\widetilde{K}$.
Case-2: Let the negative membership degree of φ_1 and φ_2 be μ_φ^- and σ_φ^- which are two FSOGNR of $\widetilde{K}$.
Let $p,q \in \widetilde{K}$, Take
1)
$(\mu_\varphi^- \cap \sigma_\varphi^-)(p\text{-}q) = \max\{\mu_\varphi^-(p\text{-}q), \sigma_\varphi^-(p\text{-}q)\}$
$\leq \max\{\max[\mu_\varphi^-(p), \mu_\varphi^-(q)], \max[\sigma_\varphi^-(p), \sigma_\varphi^-(q)]\}$
$= \max\{\max[\mu_\varphi^-(p), \sigma_\varphi^-(p)], \max[\mu_\varphi^-(q), \sigma_\varphi^-(q)]\}$
$= \max\{(\mu_\varphi^- \cap \sigma_\varphi^-)(p), (\mu_\varphi^- \cap \sigma_\varphi^-)(q)\}$
Therefore, $(\mu_\varphi^- \cap \sigma_\varphi^-)(p\text{-}q) \leq \max\{(\mu_\varphi^- \cap \sigma_\varphi^-)(p), (\mu_\varphi^- \cap \sigma_\varphi^-)(q)\}$.
2)

$(\mu_\varphi^- \cap \sigma_\varphi^-)(p\,\gamma\, q) = \max\{\mu_\varphi^-(p\,\gamma\, q),\ \sigma_\varphi^-(p\gamma q)\}$
$\leq \max\{\max[\mu_\varphi^-(p),\ \mu_\varphi^-(q)],\ \max[\sigma_\varphi^-(p),\ \sigma_\varphi^-(q)]\}$
$= \max\{\max[\mu_\varphi^-(p),\ \sigma_\varphi^-(p)], \max[\mu_\varphi^-(q),\ \sigma_\varphi^-(q)]\}$
$= \max\{(\mu_\varphi^- \cap \sigma_\varphi^-)(p), (\mu_\varphi^- \cap \sigma_\varphi^-)(q)\}$
Therefore, $(\mu_\varphi^- \cap \sigma_\varphi^-)(p\,\gamma q) \leq \max\{(\mu_\varphi^- \cap \sigma_\varphi^-)(p), (\mu_\varphi^- \cap \sigma_\varphi^-)(q)\}$.
3) If $p \leq q$ then $\mu_\varphi^-(p) \leq \mu_\varphi^-(q)$ and $\sigma_\varphi^-(p) \leq \sigma_\varphi^-(q)$
$(\mu_\varphi^- \cap \sigma_\varphi^-)(p) = \max\{\mu_\varphi^-(p),\ \sigma_\varphi^-(p)\}$
$\leq \max\{\mu_\varphi^-(q),\ \sigma_\varphi^-(q)\}$
$= (\mu_\varphi^- \cap \sigma_\varphi^-)(q)$
Therefore we proved that the intersection of negative membership of φ_1 and φ_2, i.e, $(\mu_\varphi^- \cap \sigma_\varphi^-)$ is also FSOGNR of $\widetilde{K}$.
Since we proved that $(\mu_\varphi^+ \cap \sigma_\varphi^+)$ and $(\mu_\varphi^- \cap \sigma_\varphi^-)$ are FSOGNR of $\widetilde{K}$ which implies that
$\varphi_1 \cap \varphi_2$ is a BFSOGNR of $\widetilde{K}$.

IV. Main results on Homomorphism of Bipolar Fuzzy Sub Ordered Γ-Nearring

Definition 4.1: Let $\mathbb{V}$ and $\mathbb{W}$ be two Ordered Γ-Nearrings. Then f: $\mathbb{V} \to \mathbb{W}$ is called a homomorphism if
(i) $f(x + y) = f(x) + f(y)$ and
(ii) $f(x\,\alpha y) = f(x)\,\alpha f(y)$, $\forall\, x, y \in \mathbb{V}$; $\alpha \in \Gamma$.

Definition 4.2: Let $\mathbb{V}$ and $\mathbb{W}$ be two OGNRs and $f : \mathbb{V} \to \mathbb{W}$. Let $\varphi_1=(\mu_\varphi^+,\ \mu_\varphi^-)$ be a bipolar Fuzzy subset. If the positive membership degree of φ_1,i.e., μ_φ^+ is a Fuzzy subset of $\mathbb{V}$ then the image of μ_φ^+, $f(\mu_\varphi^+)$ is the Fuzzy subset in $\mathbb{W}$ defined by

$$f(\mu_\varphi^+)(y) = \begin{cases} \sup\limits_{z\in f^{-1}(y)} \mu_\varphi^+(z), \text{if } f^{-1}(y) \neq \emptyset \\ 0, \text{ otherwise} \end{cases}$$

$\forall\, y \in \mathbb{W}$, where $f^{-1}(y) = \{x/f(x)=y\}$.
Also If negative membership degree of φ_1,i.e., μ_φ^- is a Fuzzy subset of $\mathbb{V}$ then the image of μ_φ^-, $f(\mu_\varphi^-)$ is the Fuzzy subset in $\mathbb{W}$ defined by

$$f(\mu_\varphi^-)(y) = \begin{cases} \sup\limits_{z\in f^{-1}(y)} \mu_\varphi^-(z), \text{if } f^{-1}(y) \neq \emptyset \\ 0, \text{ otherwise} \end{cases}$$

$\forall\, y \in \mathbb{W}$, where $f^{-1}(y) = \{x/f(x)=y\}$.
Let $\varphi_2 =(\sigma_\varphi^+, \sigma_\varphi^-)$ be a bipolar Fuzzy subset.
If positive membership degree of φ_2,i.e., σ_φ^+ is a Fuzzy subset in $\mathbb{W}$ then the inverse image$f^{-1}(\sigma_\varphi^+)$ of σ_φ^+ is the Fuzzy subset in $\mathbb{V}$ by $f^{-1}(\sigma_\varphi^+)(x) = \sigma_\varphi^+(f(x))$, $\forall\, x \in \mathbb{V}$.
If negative membership degree of φ_2,i.e., σ_φ^- is a Fuzzy subset in $\mathbb{W}$ then the inverse image $f^{-1}(\sigma_\varphi^-)$ of σ_φ^- is the Fuzzy subset in $\mathbb{V}$ by $f^{-1}(\sigma_\varphi^-)(x) = \sigma_\varphi^-(f(x))$, $\forall\, x \in \mathbb{V}$.

Definition 4.3: Let $\varphi_1=(\mu_\varphi^+,\ \mu_\varphi^-)$ be a bipolar Fuzzy subset. The Fuzzy subset μ_φ^+ of $\mathbb{V}$ is said to have the Sup. property if for any subset A of $\mathbb{V}$, $\exists x_0 \in A \ni \mu_\varphi^+(x_0)= \sup\limits_{x\in A} \mu_\varphi^+(x)$.
The Fuzzy subset μ_φ^- of M is said to have the Inf. property if for any subset A of $\mathbb{V}$, $\exists x_0 \in A \ni \mu_\varphi^-(x_0) = \inf\limits_{x\in A} \mu_\varphi^-(x)$.

Theorem 4.4: An OGNR homomorphic pre-image of a BFSOGNR is a BFSOGNR.
Proof:
Let f: $\mathbb{V} \to \mathbb{W}$ be an OGNR homomorphism.
Let $\varphi_1= (\mathbb{V};\ \mu_\varphi^+,\ \mu_\varphi^-)$ be a BFSOGNR of $\mathbb{V}$.
and $\varphi_2=(\mathbb{W};\ \sigma_\varphi^+,\ \sigma_\varphi^-)$ be a BFSOGNR of $\mathbb{W}$.
Case-1:
Let the positive membership degree of φ_2 i.e., σ_φ^+ be a BFSOGNR of $\mathbb{W}$
and Let the positive membership degree of φ_1 i.e., μ_φ^+ be the pe-image of σ_φ^+ under 'f'.
Now, we have to prove that μ_φ^+ is BFSOGNR of $\mathbb{V}$ under 'f'.
Then,
1)
$\mu_\varphi^+(p\text{-}q) = \sigma_\varphi^+(f(p\text{-}q))$
$= \sigma_\varphi^+(f(p)\text{-}f(q))$
$\geq \min[\sigma_\varphi^+(f(p)),\ \sigma_\varphi^+(f(q))]$
$= \min[\mu_\varphi^+(p),\ \mu_\varphi^+(q)]$.
Therefore $\mu_\varphi^+(p\text{-}q) \geq \min[\mu_\varphi^+(p),\ \mu_\varphi^+(q)]$
Again,

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2) $\mu_\varphi^+ (p\, \gamma\, q) = \sigma_\varphi^+ (f(p\, \gamma\, q))$
$= \sigma_\varphi^+(f(p)\, \gamma\, f(q))$
$\geq \min[\sigma_\varphi^+ (f(p)),\ \sigma_\varphi^+ (f(q))]$
$= \min[\mu_\varphi^+(p),\ \mu_\varphi^+(q)]$.
Therefore $\mu_\varphi^+(p\, \gamma\, q) \geq \min[\mu_\varphi^+(p),\ \mu_\varphi^+ (q)]$

3) If $p \leq q$ and $f(p) \geq f(q)$ then
$\mu_\varphi^+ (p) = \sigma_\varphi^+ (f(p))$
$\geq \sigma_\varphi^+ (f(q))$
$= \mu_\varphi^+ (q)$
Therefore $\mu_\varphi^+ (p) \geq \mu_\varphi^+ (q)$.
Thus, μ_φ^+ is FSOGNR of $\mathbb{V}$ under 'f'.
case-2:
Let the negative membership degree of φ_2 i.e., σ_φ^- be a BFSOGNR of $\mathbb{W}$
and Let the negative membership degree of φ_1 i.e., μ_φ^- be the pe-image of σ_φ^- under 'f'.
Now, we have to prove that $\mu_\varphi^-$is BFSOGNR of $\mathbb{V}$ under 'f'.
Then,
1) $\mu_\varphi^- (p - q) = \sigma_\varphi^- (f(p-q))$
$= \sigma_\varphi^- (f(p)-f(q))$
$\leq \max[\sigma_\varphi^- (f(p)),\ \sigma_\varphi^- (f(q))]$
$= \max[\mu_\varphi^- (p),\ \mu_\varphi^- (q)]$.
Therefore $\mu_\varphi^- (p-q) \leq \min[\mu_\varphi^- (p),\ \mu_\varphi^- (q)]$
Again,
2) $\mu_\varphi^- (p\, \gamma\, q) = \sigma_\varphi^- (f(p\, \gamma\, q))$
$= \sigma_\varphi^- (f(p)\, \gamma\, f(q))$
$\leq \max[\sigma_\varphi^- (f(p)),\ \sigma_\varphi^- (f(q))]$
$= \max[\mu_\varphi^- (p),\ \mu_\varphi^- (q)]$.
Therefore $\mu_\varphi^- (p\, \gamma\, q) \leq \min[\mu_\varphi^- (p),\ \mu_\varphi^- (q)]$
3) If $p \leq q$ and $f(p) \leq f(q)$ then
$\mu_\varphi^- (p) = \sigma_\varphi^- (f(p))$
$\leq \sigma_\varphi^- (f(q))$
$= \mu_\varphi^- (q)$
Therefore $\mu_\varphi^- (p) \leq \mu_\varphi^- (q)$.
Thus, μ_φ^- is FSOGNR of $\mathbb{V}$ under 'f'.
as we proved that μ_φ^+ and μ_φ^- are FSOGNRs of $\mathbb{V}$ which implies that φ_1 is a BFSOGNR of $\mathbb{V}$. Hence the homomorphic pre-image of a BFSOGNR is a BFSOGNR.

Theorem 4.5: An OGNR homomorphic image of a BFSOGNR possessing both the Supremum and infimum properties is a BFSOGNR.
Proof: Let f: $\mathbb{V} \rightarrow \mathbb{W}$ be an OGNR homorphism.
Let $\varphi_1 = (\mathbb{V};\ \mu_\varphi^+,\ \mu_\varphi^-)$ be a BFSOGNR of $\mathbb{V}$.
and $\varphi_2 = (\mathbb{W};\ \sigma_\varphi^+, \sigma_\varphi^-)$ be a BFSOGNR of $\mathbb{W}$.
Case-1:
Let the positive membership degree of φ_1 i.e., μ_φ^+ be a BFSOGNR of $\mathbb{V}$ possessing Supremum Property and Let the positive membership degree of φ_2 i.e., σ_φ^+ be the image of μ_φ^+ under 'f'. Now, we have to prove that σ_φ^+ is BFSOGNR of $\mathbb{W}$.
Given $f(p), f(q) \in \mathbb{W}$.
We have
$p_0 \in f^{-1}(f(p)) \ni \mu_\varphi^+ (p_0) = \sup_{z \in f^{-1}(f(p))} \mu_\varphi^+(z)$

$q_0 \in f^{-1}(f(q)) \ni \mu_\varphi^+ (q_0) = \sup_{z \in f^{-1}(f(q))} \mu_\varphi^+(z)$
1) $\sigma_\varphi^+ (f(p) - f(q)) = \sup_{z \in f^{-1}(f(p)-f(q))} \mu_\varphi^+(z)$
$= \mu_\varphi^+ (p_0 - q_0)$
$\geq \min[\mu_\varphi^+ (p_0),\ \mu_\varphi^+ (q_0)]$
$= \min\{\sup_{z \in f^{-1}(f(p))} \mu_\varphi^+(z),\ \sup_{z \in f^{-1}(f(q))} \mu_\varphi^+(z)\}$
$= \min[\sigma_\varphi^+ (f(p)),\ \sigma_\varphi^+ (f(q))]$

2) $\sigma_\varphi^+ (f(p)\, \gamma\, f(q)) = \sup_{z \in f^{-1}(f(p)\gamma f(q))} \mu_\varphi^+(z)$
$= \mu_\varphi^+ (p_0 \gamma\, q_0)$
$\geq \min[\mu_\varphi^+ (p_0),\ \mu_\varphi^+ (q_0)]$
$= \min\{\sup_{z \in f^{-1}(f(p))} \mu_\varphi^+(z),\ \sup_{z \in f^{-1}(f(q))} \mu_\varphi^+(z)\}$
$= \min[\sigma_\varphi^+ (f(p)),\ \sigma_\varphi^+ (f(q))]$

3) If $p \leq q$ then
$\sigma_\varphi^+ (f(p)) = \sup_{z \in f^{-1}(f(p))} \mu_\varphi^+(z)$
$= \mu_\varphi^+ (p_0)$
$\geq \mu_\varphi^+ (q_0)$
$= \sup_{z \in f^{-1}(f(q))} \mu_\varphi^+(z)$
$= \sigma_\varphi^+ (f(q))$

Therefore $\sigma_\varphi^+ (f(p)) \geq \sigma_\varphi^+ (f(q))$.

This proves that σ_φ^+ is a Fuzzy SOGNR of $\mathbb{W}$.

Case-2:

Let the negative membership degree of φ_1 i.e., μ_φ^- be a FSOGNR of $\mathbb{V}$ possessing Infimum Property and

Let the negative membership degree of φ_2 i.e., σ_φ^- be the image of μ_φ^- under 'f'.

Now, we have to prove that σ_φ^- is FSOGNR of $\mathbb{W}$.

Given $f(p), f(q) \in \mathbb{W}$.

$p_0 \in f^{-1}(f(p))\ \ni\ \mu_\varphi^- (p_0) = \operatorname{Inf}_{z \in f^{-1}(f(p))} \mu_\varphi^-(z)$

$q_0 \in f^{-1}(f(q))\ \ni\ \mu_\varphi^- (q_0) = \operatorname{Inf}_{z \in f^{-1}(f(q))} \mu_\varphi^-(z)$

1) $\sigma_\varphi^- (f(p) - f(q)) = \operatorname{Inf}_{z \in f^{-1}(f(p)-f(q))} \mu_\varphi^-(z)$

$= \mu_\varphi^- (p_0 - q_0)$

$\leq \max[\mu_\varphi^- (p_0), \mu_\varphi^- (q_0)]$

$= \max \{\operatorname{Inf}_{z \in f^{-1}(f(p))} \mu_\varphi^-(z), \operatorname{Inf}_{z \in f^{-1}(f(q))} \mu_\varphi^-(z)\}$

$= \max [\sigma_\varphi^- (f(p)), \sigma_\varphi^- (f(q))]$

2) $\sigma_\varphi^- (f(p)\,\gamma\, f(q)) = \operatorname{Inf}_{z \in f^{-1}(f(p)\gamma f(q))} \mu_\varphi^-(z)$

$= \mu_\varphi^- (p_0 \gamma q_0)$

$\leq \max[\mu_\varphi^- (p_0), \mu_\varphi^- (q_0)]$

$= \max \{\operatorname{Inf}_{z \in f^{-1}(f(p))} \mu_\varphi^-(z), \operatorname{Inf}_{z \in f^{-1}(f(q))} \mu_\varphi^-(z)\}$

$= \max [\sigma_\varphi^- (f(p)), \sigma_\varphi^- (f(q))]$

3) If $p \leq q$ then

$\sigma_\varphi^- (f(p)) = \operatorname{Inf}_{z \in f^{-1}(f(p))} \mu_\varphi^-(z)$

$= \mu_\varphi^- (p_0)$

$\leq \mu_\varphi^- (q_0)$

$= \operatorname{Inf}_{z \in f^{-1}(f(q))} \mu_\varphi^-(z)$

$= \sigma_\varphi^- (f(q))$

Therefore $\sigma_\varphi^- (f(p)) \geq \sigma_\varphi^- (f(q))$.

This proves that σ_φ^- is a FSOGNR of $\mathbb{W}$.

Since σ_φ^+ and σ_φ^- are FSOGNR,N which implies that φ_2 is a BFSOGNR of $\mathbb{W}$.

Hence the homomorphic image of a BFSOGNR possessing both the Supremum and infimum properties is a BFSOGNR.

V. Scope and conclusion

In this paper, the concept of Bipolar Fuzzy SOGNR is introduced and we established a one-one correspondence between Bipolar Fuzzy SOGNR and crisp SOGNR. We are expecting that these structures are useful in developing Bipolar Fuzzy prime ideals, Bipolar Fuzzy maximal ideals and Bipolar Fuzzy semiprime ideals of an Ordered Γ-Nearring.

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