

# An Analytical Approach for Optimal Capacitor Placement in a Distribution System to Minimize total Annual Cost

M.J. Tahir<sup>1,2</sup><sup>1</sup>Dept of Electrical Engineering  
UniKL, BMISeleangor, Malaysia<sup>2</sup>Dept of  
Electrical Technology  
UOL

Lahore, Pakistan

muhammad.junaid@s.unikl.edu.  
myBadri Abu Bakar<sup>1,\*</sup><sup>1</sup>Dept of Electrical Technology  
UniKL, BMISeleangor, Malaysia  
badri@unikl.edu.myM. S. Mazliham<sup>4</sup><sup>4</sup>Chancellery, UniKL  
Kuala Lumpur, Malaysia  
mazliham@unikl.edu.myM. Alam<sup>3</sup><sup>3</sup>Dept of Computer Science and  
MIS

IoBM

Karachi, Pakistan

malam@iobm.edu.pk

<sup>3</sup>Dept of Information Technology  
UniKL, MiiTKuala Lumpur Malaysia  
mansoor@unikl.edu.my

## Article Info

Volume 83

Page Number: 2096 - 2102

Publication Issue:

May - June 2020

## Article History

Article Received: 11 August 2019

Revised: 18 November 2019

Accepted: 23 January 2020

Publication: 10 May 2020

## Abstract:

In this paper, an analytical approach with a self-sorting algorithm is proposed for the optimal capacitor placement and sizing (OCPS) in a radial distribution system. Initially, Loss sensitivity factor (LSF) method is used to nominate the potential buses for the capacitor placement and then the self-sorting algorithm is applied to find the optimal rating of the capacitors. The objective is to minimize the total annual cost and power losses in a distribution system along with enhancement of voltage profile. For this purpose, IEEE 15, 33 and 69 buses are considered to show the effectiveness of the proposed algorithm and the results are compared with the existing techniques in terms of voltage improvement, power losses minimization and total annual cost reduction.

**Keywords:** Distribution system, Loss sensitivity factor, Optimal capacitor placement, Total annual cost, Power losses minimization, Self-sorting algorithm

## I. INTRODUCTION

A power system consists of three basic sides i.e generation, transmission, and load side. With the passage of time load demand has been increased due to this reactive element participation also increased, which increases the power losses in the system. To minimize these power losses network reconfiguration, capacitor placement, and generator placement like conventional methods are used. Capacitor placement is a method in which reactive power in the system is injected, which cancels the effect of existing reactive power, as a result, power losses are minimized in the system. Capacitors are preferably installed at the distribution side because they show more influence when they are installed closer to the load side. In the last few decades, several heuristic and analytical approaches have been considered for the optimal capacitor placement and its sizing problem and their main objective was to minimize the power losses with certain system

limitations. Researchers used analytical, heuristic and their combinational techniques to resolve this issue as reported in [1].

Sarma and Rafi proposed Plant growth simulation algorithm for optimal capacitor placement (OCP) with cost minimization objective function [2]. An extended dynamic programming based algorithm reported in [3], to resolve capacitor placement problem with maximum savings objective. In [4] direct search algorithm is implemented, to optimally place and size the capacitors. Singh, Rao proposed particle swarm optimization technique [5]. A modified cultural algorithm is suggested for power losses minimization [6]. Muhtazaruddin et, al. [7] used the artificial bee colony technique to improve the system voltage. For the annual cost reduction of the power system, a fuzzy discrete harmony search algorithm is adopted in [8]. Gnanasekaran et, al. [9] presents a shark smell optimization method for

capacitor placement with energy cost minimization of the system. Tabatabaei, Vahidi suggested fuzzy logic with bacteria foraging algorithm (BFA) to find the optimal location of the capacitor and its rating for the system cost minimization, power losses reduction and voltage improvement [10]. Ant colony algorithm is considered for capacitor location problem in [11] to highlight the benefits of the technique in terms of power loss and cost minimization. Reddy and Manoj reported a Fuzzy logic with bat algorithm solution technique for the capacitor allocation and its rating with power losses minimization objective [12]. A combinational approach power loss index with Cuckoo search algorithm is implemented for optimal capacitor placement and rating with power losses minimization and voltage profile improvement [13]. A new analytical technique is considered in [14] for optimal capacitor placement and its rating with voltage improvement and minimization of power losses. In [15], loss sensitivity factor along with a new analytical technique is suggested for capacitor allocation and its rating along with maximization of energy savings. AbulWafa presented a loss voltage sensitivity approach with an analytical method for capacitor location and is sizing along with annual cost savings [16].

In this paper, an analytical approach with a self-sorting algorithm is proposed for the optimal capacitor placement and sizing in a radial distribution system. The objective is to minimize the total annual cost and power losses along with enhancement of voltage profile. In the upcoming section 2 problem statement is expressed, section 3 explains the research methodology, and results are tabulated in section 4 and in the end conclusion is compiled in section 5.

## II. PROBLEM STATEMENT

The objective of this paper is to find the optimal placement and sizing for the fixed capacitors in the distribution system to minimize the total annual cost and power losses along with enhancement in the voltage profile of the power system.

$$Cost_{min} = K_l * AP_{loss} + \sum_{n=1}^m K_c * QC_n \quad (1)$$

For this purpose, a classical cost equation is considered, which is a sum of active power losses cost ( $K_l * AP_{loss}$ ) and capacitors cost ( $\sum_{n=1}^m K_c * QC_n$ ). Whereas:

' $K_l$ ' ~ is co-efficient for power loss in 168 \$/kWh,

' $AP_{loss}$ ' ~ is the active power losses in kW,

' $K_c$ ' ~ is co-efficient for reactive power in \$/kVAR,

' $QC_n$ ' ~ is the reactive compensation size in kVAR,

' $m$ ' ~ number of capacitors,

' $n$ ' ~ number of buses,

## III. METHODOLOGY

To minimize the annual cost of the power system, load flow understanding is essential. Power supplied by the source and absorbed by the load at each branch can be expressed as:

$$P_{supply / i} = P_{load / j} + P_{line - loss / k} \quad (2)$$

$$Q_{supply / i} = Q_{load / j} + Q_{line - loss / k} \quad (3)$$

Where  $P_{supply / i}$ ,  $Q_{supply / i}$  are the active and reactive power supplied to a certain bus of the system,  $P_{load / j}$  and  $Q_{load / j}$  are the active and reactive power absorbed by a certain load,  $P_{line - loss / k}$  and  $Q_{line - loss / k}$  are the active and reactive power line losses.

$$I_{line / k} = \frac{P_{supply / i} - jQ_{supply / i}}{V_{supply / i} \cos \delta_{supply / i}} \quad (4)$$

Where  $I_{line / k}$  is the line losses for a certain branch.

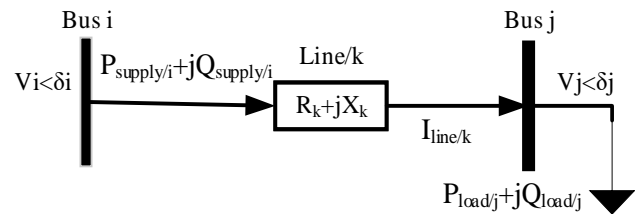


Fig. 1. Power flow in a k-branches of system.

$$P_{line - loss / k} = I_{line / k}^2 * R_{line / k} \quad (5)$$

$$Q_{line - loss / k} = I_{line / k}^2 * X_{line / k} \quad (6)$$

$$P_{line - loss / k} = \left( \frac{P_{load / j}^2 + Q_{load / j}^2}{|V_j|^2} \right) * R_{line / k} \quad (7)$$

$$Q_{line - loss / k} = \left( \frac{P_{load / j}^2 + Q_{load / j}^2}{|V_j|^2} \right) * X_{line / k} \quad (8)$$

Where  $P_{line - loss / k}$  and  $Q_{line - loss / k}$  are the active power and reactive power losses for a certain branch.

$$\begin{aligned} & \text{Total } AP_{loss} \\ &= \sum_{load / j=2}^{N_b} \sum_{line / k=1}^{N_b-1} \left( \frac{P_{load / j}^2 + Q_{load / j}^2}{|V_j|^2} \right) * R_{line / k} \end{aligned} \quad (9)$$

$$Total QP_{loss} = \sum_{load/j=2}^{N_b} \sum_{line/k=1}^{N_{b-1}} \left( \frac{P_{load/j}^2 + Q_{load/j}^2}{|V_j|^2} \right) \quad (10)$$

\*  $Q_{line/k}$

Where  $Total AP_{loss}$  and  $Total QP_{loss}$  are the total sum of active power losses and reactive power losses for all bus  $N_b$  and branches  $N_{b-1}$ .

#### A. Loss Sensitivity Factor

The first part of this research is to nominate the candidate buses for the OCP. Several methods are available which are considered for this purpose. In this paper, the LSF method is adopted for the nomination of candidate buses which is done with the help of load flow method[17]. LSF operates in four steps, in the first step derivate of equation 9 with respect to reactive power is taken, LSF of all buses is indexed in descending order in step two, then normalized voltage of all buses are gathered, and in the end on those buses are selected as candidate buses which are having normalized voltage less than 1.01p.u.

$$LSF = \left[ \left( \frac{2Q_{load/j}}{|V_j|^2} \right) * R_{line/k} \right] \quad (11)$$

TABLE I. LSF FOR 15 BUS SYSTEM

LSI Descending Order	Bus Number	$V_{norm(i)} = V[i]/0.95$	Base Voltage
1.6496820315892	8	1.007319221183	0.9569532601239
1.5586326770132	3	1.007019102285	0.9566681471712
0.8540390736972	11	0.999948371525	0.9499509529487
0.6189006191040	4	1.000951231761	0.9509036701729
0.5267957594869	12	0.995607761946	0.9458273738486
0.4219665288960	6	1.008663586550	0.9582304072225
0.3141558340553	15	0.998356096076	0.9484382912720
0.2925910636931	14	0.998533614131	0.9486069334240
0.1678227272254	13	0.994227234009	0.9445158723086
0.1372483725467	7	1.006323063327	0.9560069101603
0.1255609046273	5	0.999912455519	0.9499168327432

TABLE II. LSF FOR 33 BUS SYSTEM

LSI Descending Order	Bus Number	$V_{norm(i)} = V[i]/0.95$	Base Voltage
1.335674132005	27	1.001728918482	0.951642472558
1.328750361884	28	0.989772230055	0.940283618553
1.003233419793	7	1.002742074220	0.952604970509
0.780425314550	29	0.981182711075	0.932123575521
0.780357100285	8	0.988473065268	0.939049412004
0.591782471685	12	0.973246553269	0.924584225606
0.468551392201	9	0.981856570625	0.932763742094
0.462088077388	26	1.004408560357	0.954188132339
0.443349368627	30	0.977464559060	0.928591331107
0.296530830144	13	0.966794538647	0.918454811714
0.276774153114	6	1.006425075192	0.956103821432
0.262059774523	11	0.974829254791	0.926087792052
0.260620865313	16	0.961467686832	0.913394302491
0.236376286669	15	0.962911466105	0.914765892799
0.227187432069	14	0.964402096563	0.916181991735
0.136941171279	10	0.975736925611	0.926950079330
0.135057305294	31	0.973115529394	0.924459752924
0.132922576680	17	0.959328042006	0.911361639906

0.115772465979	18	0.958687307460	0.910752942087
0.089465186990	32	0.972158782658	0.923550843525
0.079597764499	33	0.971862335105	0.923269218350

TABLE III. LSF FOR 69 BUS SYSTEM

LSI Descending Order	Bus Number	$V_{norm(i)} = V[i]/0.95$	Base Voltage
1.329130989996	57	0.989574426398	0.940095705078
0.835202511153	60	0.968138244672	0.919731332439
0.632551341236	59	0.973427867309	0.924756473944
0.526888856840	58	0.977931270517	0.929034706991
0.217666502738	63	0.959637755678	0.911655867894
0.099178373358	16	1.009449089168	0.958976634709
0.065927291995	64	0.957637285928	0.909755421631
0.057917854967	20	1.007710784032	0.957325244830
0.055748611483	18	1.008513460178	0.958087787168
0.044457242798	62	0.960045942114	0.912043645008
0.035866880710	19	1.008024542699	0.957623315564
0.033212953237	61	0.960350835265	0.912333293501
0.020425012882	24	1.006956339298	0.956608522333
0.018886369883	23	1.007120969991	0.956764921491
0.008676430708	22	1.007196605604	0.956836775324
0.008428495473	25	1.006778338689	0.956439421755
0.007897965631	65	0.957032678205	0.909181044294
0.002363256634	26	1.006704909617	0.956369664136
0.001023663638	17	1.008522831667	0.958096690084
0.000830296380	21	1.007203844400	0.956843652180
0.000391532270	27	1.006684326921	0.956350110575

#### B. System Limitations

There are certain limitations needed to be considered for capacitor sizing once candidate buses are decided. i-e voltage, maximum reactive power, reactive rating.

##### 1) Voltage constraints:

$$V_i^{min} \leq V_i \leq V_i^{max} \quad (12)$$

While performing the capacitor sizing, system voltage should remain between 0.95p.u and 1.05p.u voltage

##### 2) Maximum Reactive power compensation:

$$\sum_{i=1}^n Q_{cap}(i) \leq \sum_{j=1}^n Q_{load}(j) \quad (13)$$

While compensating the reactive power it is to be considered that total reactive power injection should not be greater than the total load reactive power.

##### 3) Reactive compensation:

$$Q_{min} \leq Q_c \leq Q_{max} \quad (14)$$

It is essential to maintain reactive compensation within the available limits, i-e  $Q_{min}$  is 150KVar and  $Q_{max}$  is 1500 KVar.

TABLE IV. CAPACITOR COST PER KVAR

Capacitor size	150	300	450	600	750
Cost in \$/kVar	0.5	0.35	0.253	0.220	0.276
Capacitor size	900	1050	1200	1350	1500
Cost in \$/kVar	0.183	0.228	0.170	0.207	0.201

#### C. Self-Sorting Algorithm

It can be seen from the flow chart, that algorithm starts with loading the case data and performing the initial load flow. In step 2 LSF method is applied to nominate the potential candidate buses for the capacitor placement. Once candidate buses for OCP and system constraints are defined, the self-sorting algorithm (SSA) will start playing his role to outline the sizing of the fixed capacitors for total annual cost, and power losses minimization along with

enhancement in voltage profile of the power system. The flow chart expresses the flow of the algorithm, to achieve the stated objective considering the candidate buses with system constraints in step 3 the capacitors with minimum limit inserted on all candidate buses one by one and load flow are performed for each case. The bus index with capacitor rating, which gives minimum total annual cost along with power losses is stored in step 4. The algorithm will repeat the step 3 and 4 until  $Q_{cap} \leq Q_{load}$  and other system constraints are satisfied. On the other side if a certain bus index with certain capacitor rating, power losses are minimized but doesn't satisfy any of the constraints the loop will not store the bus index and capacitor rating and move towards the next candidate bus and capacitor sizing. In the end results will be gathered in step 5 in the form of buses index and capacitors rating.

#### IV. RESULTS AND DISCUSSION

Fig. 2. Flow chart of Self sorting algorithm.

To show the effectiveness of the proposed algorithm IEEE 15,33 and 69 bus systems are considered and Matlab 2015® with CPU @ 3.20GHz 64 bit is used. IEEE 15 bus system operates at 11kV and 1.2264MW, 1.2511MVar load with 61.7926kW initial power loss and 0.9445p.u. minimum voltage at bus 13. With the help of LSF [8, 3, 11, 4, 12, 6, 15, 14, 13, 7, 5] nominated as candidate buses for the OCP. Using the proposed SSA algorithm, 5 capacitors

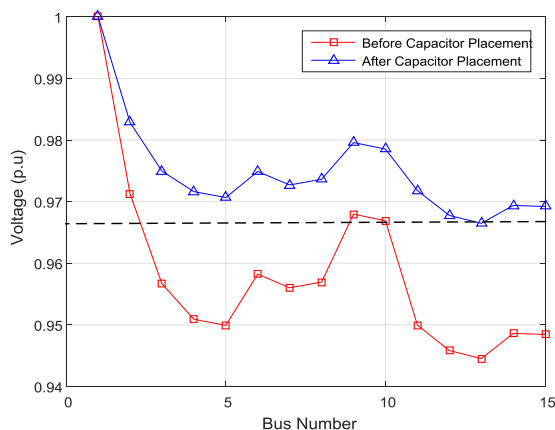
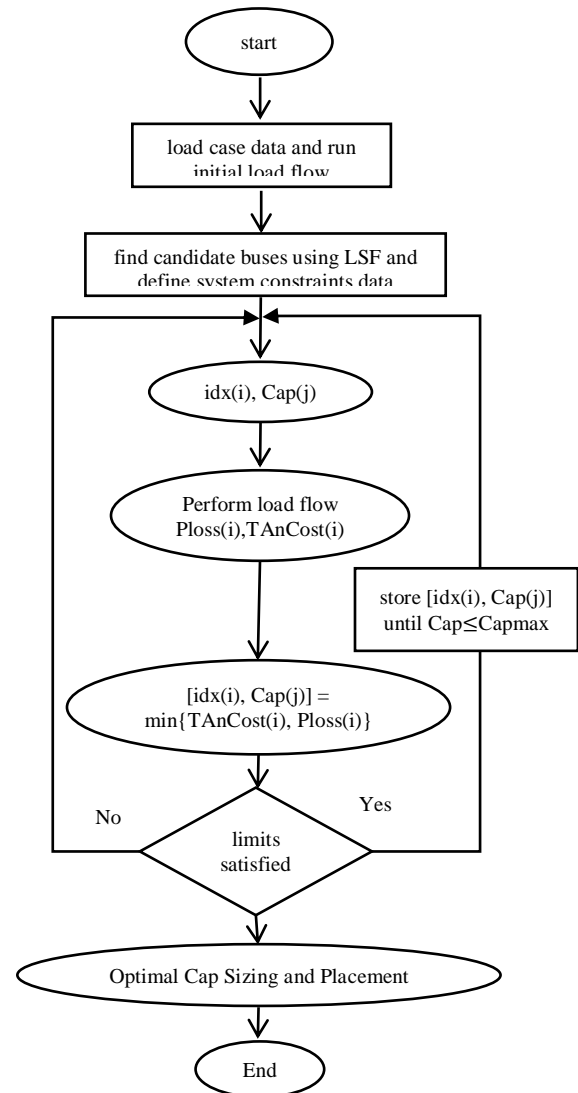


Fig. 3. IEEE 15 bus voltage profile.

are placed having a total 1200kVar rating which decreased the power losses up to 30.71KW and enhanced the minimum voltage profile to 0.9702p.u. along with total annual cost 5,594 \$/year decreased up to 47.58%.



IEEE 33 bus system operates at 12.66kV, and 3.7150MW, 2.3000MVar load with 211.02kW power losses with minimum voltage 0.9038p.u. at bus 18, before capacitor placement. Candidate buses [27,28,7,29,8,12,9,26,30,13,6,11,16,15,14,10,31,17, 18,32,33] recommended by LSF method are considered, and 6 capacitors having total 2100kVar rating are installed using the proposed algorithm, which results in the reduction of power losses and the total annual cost to 139.4971kW and 24066 \$/year with minimum voltage is enhanced up to 0.9361. Total power losses cost is decreased up to 32.12 %.

IEEE 69 bus operates at 12.66kV and drives a 3.7919MW, 2.6936MkVar load and 225kW early power without capacitor placement. The candidate buses [57, 60,59, 58, 63, 16, 64, 20, 18, 62, 19, 61, 24, 23, 22, 25, 65, 26, 17, 21, 27] suggested by LSF method are considered by the



proposed algorithm SSA and 5 capacitors with 1800kVar rating finalized, results in minimization of power losses up to 146.0159kW with total annual cost is reduction of 25039 \$/year which is 33.76 %alongwith 0.9305p.u minimum voltage.

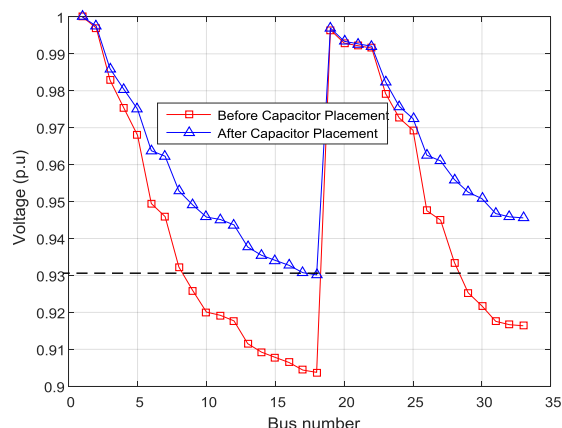


Fig. 4. IEEE 33 bus voltage profile.

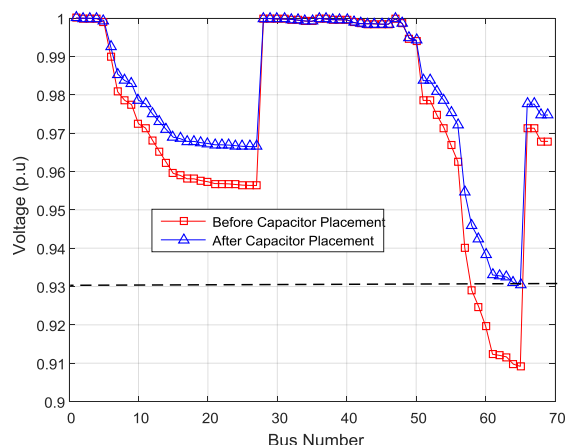


Fig. 5. IEEE 69 bus voltage profile.

TABLE V. RESULTS IEEE 15 BUS

Items/15 Bus	BaseCase	PSO [18]	TSM[19]	MAPSO [20]	Proposed Method (SSA)
Total Power Losses (kW)	61.7926	32.7	32.426	30.9534	31.3181
Loss Reduction (%)	---	47.22	47.51	50.04	49.32
Minimum Bus Voltage(p.u)	0.9445 (13)	---	0.9695	0.978	0.9665
Optimal Buses	---	6,11	3,6,4	4,6,7,11,15	4, 6, 11
Optimal Capacitor Rating	---	871,321	175,375, 750	345,264,143,300,143	350,350, 350
Total kVar	---	1192	1300	1195	1050
Annualpower loss Cost	10381	5494	5448	5200	5261
Total Annual Cost (\$)	10381	5,735	5811	5628	5527
Total Annual Cost Reduction (%)	---	44.76	44.02	45.79	46.76

TABLE VI. RESULTS IEEE 33 BUS

Items/33 Bus	BaseCase	GA[17]	PA-FRCGA [21]	TSM[19]	Proposed Method(SSA)
Total Power Losses (kW)	211.02	150.03	148.6951	144.04	141.9986
Loss Reduction (%)	---	28.8654	29.52	31.73	32.71
Minimum Bus Voltage(p.u)	0.9038 (18)	0.93	0.9665	0.9251	0.9302
Optimal Buses	---	2,4,5,7,8,9,10,11,12,13,14,23,29,30,31,33	28,6,29,8,30,9	7,29,30	11, 30
Optimal Capacitor Rating	---	100,100,100,100,100,100,100,100,100,100,100,100, 100,700,100,100	100,325,425,350,675,375	850,25,900	550, 1100
Total kVar	---	2200	2250	1775	1650
Annualpower loss Cost	35,451	25205	24981	24199	23856
Total Annual Cost (\$)	35,451	26098	25590	24532	24164
Total Annual Cost Reduction (%)	---	26.38	27.82	30.80	31.84

TABLE VII. RESULTS IEEE 69 BUS

Items/69 Bus	BaseCase	PSO [18]	TSM[19]	CSA[22]	Proposed Method (SSA)
Total Power Losses (kW)	224.95	152.48	149.2920	147.95146	146.3522
Loss Reduction (%)	---	32.2	33.59	34.2434	34.94
Minimum Bus Voltage(p.u)	0.9092(65)	---	0.9284	---	0.9312
Optimal Buses	---	46,47,50	19,63	21,62	17, 61, 64
Optimal Capacitor Rating	---	241,365,1015	225,1100	250,1200	350, 1050, 350
Total kVar	---	1621	1325	1450	1750
Annualpower loss Cost	37,792	25616.64	25081.06	24855.6	24587
Total Annual Cost (\$)	37, 792	26024.76	25346.81	25147.1	25004
Total Annual Cost Reduction (%)	---	31.15	32.94	33.47	33.84

## V. CONCLUSION

LSF with self-sorting, an analytical approach has been considered in this paper for the optimal placement and sizing of fixed capacitors with the objective is to minimize the total annual cost and power losses along with enhancement of voltage profile of the power system. Potential buses have been nominated by LSF method and self-sorting algorithm (SSA) decided the sizing of the capacitors. Comparative results have been tabulated and it can be stated that the proposed approach performs well and healthily tackles the optimization problem. In terms of power losses and total annual cost minimization along with voltage enhancement for the considered IEEE 15,33,65 bus system.

## ACKNOWLEDGMENT

I would like to acknowledge UniKL, BMI for supporting me through research grant.

## REFERENCES

- [1] M. J. Tahir, Badri Abu Bakar, Muhammad Alam, and M. S. Mazliham, "A comprehensive review: Optimal capacitor placement and sizing in a distribution system," *Journal of Engineering Technology*, vol. 6, pp. 31-36, 2018.
- [2] A. K. Sarma and K. M. Rafi, "Optimal selection of capacitors for radial distribution systems using plant growth simulation algorithm," *International Journal of Advanced Science and Technology*, vol. 30, pp. 43-54, 2011.
- [3] J. F. V. González, C. Lyra, and F. L. Usberti, "A pseudo-polynomial algorithm for optimal capacitor placement on electric power distribution networks," *European journal of operational research*, vol. 222, pp. 149-156, 2012.
- [4] M. R. Raju, K. R. Murthy, and K. Ravindra, "Direct search algorithm for capacitive compensation in radial distribution systems," *International Journal of Electrical Power & Energy Systems*, vol. 42, pp. 24-30, 2012.
- [5] S. Singh and A. Rao, "Optimal allocation of capacitors in distribution systems using particle swarm optimization," *International Journal of Electrical Power & Energy Systems*, vol. 43, pp. 1267-1275, 2012.
- [6] V. Haldar and N. Chakraborty, "Power loss minimization by optimal capacitor placement in radial distribution system using modified cultural algorithm," *International Transactions on Electrical Energy Systems*, vol. 25, pp. 54-71, 2015.
- [7] M. N. Muhtazaruddin, J. J. Jamian, D. Nguyen, N. A. Jalalludin, and G. Fujita, "Optimal capacitor placement and sizing via artificial bee colony," *International Journal of Smart Grid and Clean Energy*, vol. 3, pp. 200-206, 2014.
- [8] K. Ameli, A. Alfi, and M. Aghaebrahimi, "A fuzzy discrete harmony search algorithm applied to annual cost reduction in radial distribution systems," *Engineering Optimization*, vol. 48, pp. 1529-1549, 2016.
- [9] N. Gnanasekaran, S. Chandramohan, P. S. Kumar, and A. M. Imran, "Optimal placement of capacitors in radial distribution system using shark smell optimization algorithm," *Ain Shams Engineering Journal*, vol. 7, pp. 907-916, 2016.
- [10] S. Tabatabaei and B. Vahidi, "Bacterial foraging solution based fuzzy logic decision for optimal capacitor allocation in radial distribution system," *Electric Power Systems Research*, vol. 81, pp. 1045-1050, 2011.
- [11] R. Sirjani and B. Hassanpour, "A new ant colony-based method for optimal capacitor placement and sizing in distribution systems," *Research Journal of Applied Sciences, Engineering and Technology*, vol. 4, pp. 888-891, 2012.
- [12] V. U. Reddy and A. Manoj, "Optimal capacitor placement for loss reduction in distribution systems using bat algorithm," *IOSR journal of Engineering*, vol. 2, pp. 23-27, 2012.
- [13] A. A. El-Fergany and A. Y. Abdelaziz, "Cuckoo search-based algorithm for optimal shunt capacitors allocations in distribution networks," *Electric Power Components and Systems*, vol. 41, pp. 1567-1581, 2013.
- [14] G. Mohan and P. Aravindhbabu, "Optimal locations and sizing of capacitors for voltage stability enhancement in distribution systems," 2010.
- [15] P. V. Babu and S. Singh, "Capacitor allocation in radial distribution system for maximal energy savings," in *2016 National Power Systems Conference (NPSC)*, 2016, pp. 1-6.
- [16] A. Abul'Wafa, "Novel loss-voltage sensitivity factor for capacitor placement in radial distribution system using analytical approach," *Journal of Electrical & Electronic Systems*, vol. 6, pp. 1-7, 2017.

- [17] M. J. Tahir, Badri Abu Bakar, Muhammad Alam, and M. S. Mazliham "Optimal Capacitor Placement in a Distribution System Using ETAP Software," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 15, 2019.
- [18] A. Abdelaziz, E. Ali, and S. A. Elazim, "Optimal sizing and locations of capacitors in radial distribution systems via flower pollination optimization algorithm and power loss index," *Engineering Science and Technology, an International Journal*, vol. 19, pp. 610-618, 2016.
- [19] A. R. Abul'Wafa, "Optimal capacitor allocation in radial distribution systems for loss reduction: A two stage method," *Electric Power Systems Research*, vol. 95, pp. 168-174, 2013.
- [20] S. Kannan, P. Renuga, S. Kalyani, and E. Muthukumaran, "Optimal capacitor placement and sizing using Fuzzy-DE and Fuzzy-MAPSO methods," *Applied Soft Computing*, vol. 11, pp. 4997-5005, 2011.
- [21] A. R. Abul'Wafa, "Optimal capacitor placement for enhancing voltage stability in distribution systems using analytical algorithm and Fuzzy-Real Coded GA," *International Journal of Electrical Power & Energy Systems*, vol. 55, pp. 246-252, 2014.
- [22] Y. M. Shuaib, M. S. Kalavathi, and C. C. A. Rajan, "Optimal capacitor placement in radial distribution system using gravitational search algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 64, pp. 384-397, 2015.