

# Comparision of Kalman Filter and Unscented Kalman Filter to Estimate a Non-Linear Systems Position

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**Abstract:**

Estimating a real time location of an Autonomous Underwater Vehicle (AUV)'s is always of great interest. This paper discusses the use of Kalman filters to control AUV's. The Kalman filter (KF) is an ideal linear estimator when white Gaussian noise can be used to model the system noise and the measuring noise. But Unscented Kalman Filter has been proven to be superior alternative to Kalman Filter. So in order to cope up with the non-linearity of the system Unscented Kalman Filter has been adopted. There are two facets to the protocol: 1) first, tracking of AUV using a kalman filter. 2) second ,tracking using UKF. The paper offers a contrast for a nonlinear model between Kalman Filter and Unscented Kalman Filter. The results of the simulations demonstrate the efficacy of the proposed tracking scheme, error the estimated displacement and the displacement measured.

Keywords- Autonomous underwater vehicle (AUV's), tracking, energy-efficiency, dutycycle, kalman filter, unscented kalman filter.

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## I. INTRODUCTION

Marine autonomous systems, including submarine gliders and Autonomous underwater vehicles are revolutioning our ability to map and monitor the marine environment. Although truly autonomous systems are typically deployed from a research vessel, they are not tethered to the vessels and do not require direct human control while controlling data. They therefore provides opportunities for data acquisition in parts of the ocean previously inaccessible to vessel-based instruments,

And boost the spatial and temporal resolutions of a wide range of marine environments. Marine autonomous systems also have a growing range of applications for oil, gas infrared structure related protection, industrial and geohazard assessment.

Due to rapid attenuation of radio frequency signals, The Global Positioning System (GPS)

cannot be used underwater. Acoustic waves are the standard choice for underwater communications. In recent years Tracking of AUV helps in pipeline inspection, seafloor mapping and Anti-submarine warfare. The main task is to track the vehicle's estimated position and displacement. The purpose is to track the vehicle as early as possible by using proper tracking scheme. Tracking of AUV systems rely on Dead Reckoning sensors and transponder/modems. The main drawback of DR is that the estimation errors are cumulative. Therefore, calibrations are usually performed using acoustic modem determine the time of arrival between AUV's and the beacon. The following difficulties were faced to achieve AUV

monitoring. **First** the energy constraints are present on the AUV and the sensor nodes. **Second**, the change of the sound propagation speed by the uneven underwater environment. **Third**, irregularities in the motion of the AUV caused by

the unknown ocean currents. To improve energy efficiency the vehicle is operated in Duty-cycle mode. There are also some underwater target detection

inquiries taking into account the sound frequency variability and unexplained ocean currents. In [17], The sound velocity variance is modeled as a linear depth function. Compared to a straight line propagation hypothesis, this results in a more

reliable array design. In [18], To counteract the unknown sound rate, an uncertain minimum square (ULS) positioning algorithm

is applied to jointly estimate the target location and sound speed. The underwater monitoring models proposed in [19] and [20] consider as state variables the uncertain sound speed and ocean currents.

In this paper, we learn how to use Kalman Filter and Unscented Kalman Filter to control Autonomous vehicle in order to address the challenging task of dealing with the extremely noisy and nonlinear real time process. And we will also see the non-linear system's divergence from its real and predicted values for both Kalman Filter and Unscented Kalman Filter. And we will compare both Kalman Filter and Unscented Kalman Filter's results.

The rest of the paper is as follows, structured. We give an overview of the process in Section II. We give a system overview. We define the tracking protocol methods in Section III. Section IV, introduces the tracking algorithms. In Section V, through simulations we evaluate our analysis and the tracking algorithms performance. Finally, the paper is concluded in Section VI.

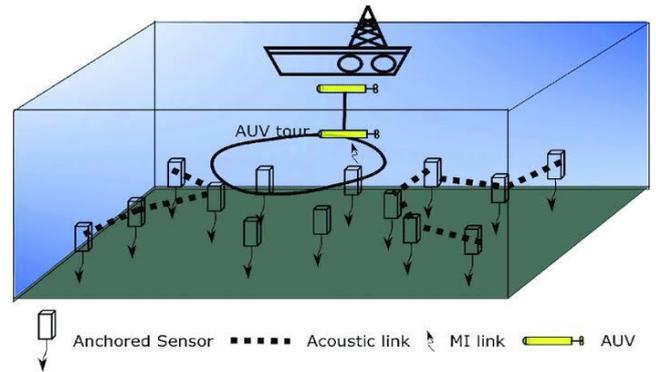


Fig1: AUV (Autonomous underwater vehicle) aided data gathering.

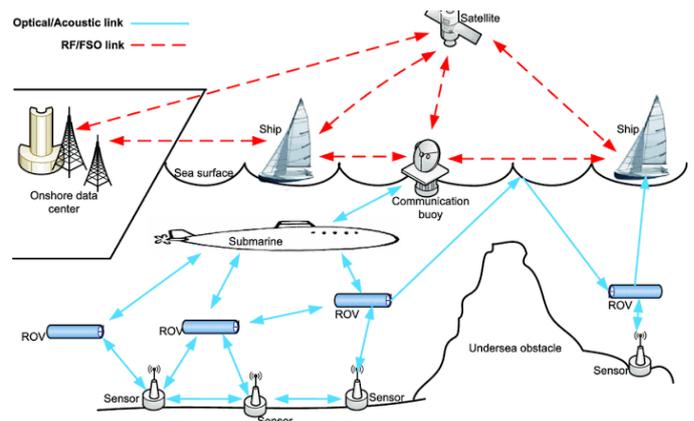


Fig 2: An aviation and terrestrial communication of underwater wireless sensor Network

## II. SYSTEM OVERVIEW

The proposed AUV tracking system setup by UWSNs is shown in Fig. 3. To track the AUV SNs are required. The AUV works in a duty-cycle mode which is symbolized as  $p$  and  $q$ , and thereby saves energy. The SN's sends packets which are being received in the listening period  $q$ . The receiver of AUV stops receiving data in the sleep period, which is given by  $[p-q]$ . On the other hand, in Active mode AUV detects the data. Only the triggered SNs regularly transmit packets, where the frequency and the length of the packet are  $T_b$  and  $T_p$ , respectively. The packets received are decoded from which the TOA measurements are extracted by measuring the differentiated time of delivery with the time of transmission contained in the packets. At the end of each duty cycle, the AUV uses the tracking algorithm to change its own location.

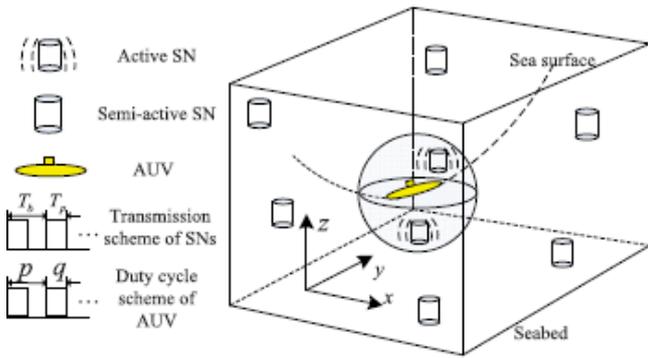


Fig 3: The architecture of AUV tracking using UWSNs.

### III. METHODS OF TRACKING

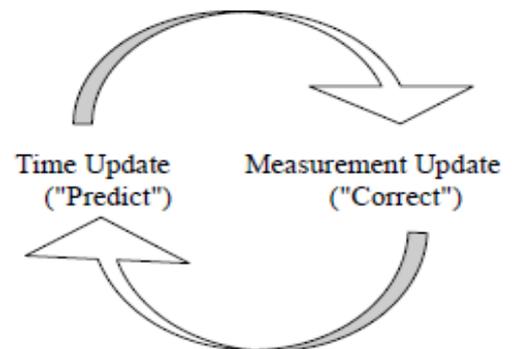
#### A. Working of Kalman Filter

The Kalman Filter is a very unusual algorithm, being one of the few that is known to be optimal. It's a new Approach to Linear Filtering and Prediction Problems. Object tracking is done by projecting the object's position from the previous information and checking the presence of the object in the sequences of the predicted location before tracking. It is used in areas as varied as aeronautics, signal processing, and trading futures. At its heart this propagates in an optimal manner a condition characterized by a Gaussian distribution using linear transition functions. Since it is optimal, it has remained relatively unchanged since it was first introduced, but has received many extensions to apply it to more than just linear Gaussian systems. The Kalman filter has a number of mathematical equations which provides an effective (recursive) of estimating the state of a system in various ways: it supports estimates of past, present, and even future states, and it can do the same even if the exact nature of the model system is unknown. The Kalman Filter uses a form of feedback control to estimate a process. At some point the filter estimates the system state and then provides feedback in the form of noisy measurements.

The equations for Kalman filters has two groups: equation for time update and equations for measurement update.

The condition is projected forwards in the time update process. We must propagate the uncertainty in the state forward also. The state are in Gaussian distributions so we can update mean and covariance. The measurement update is the correction step of Kalman filter. The measurable variable is calculated and fused with the prior distribution to estimate the posterior variable. equations is responsible for the feedback. After measurement Kalman gain is calculated. And it adds a new measurement into the a priori estimate and helps obtaining an improved posteriori estimate. The equation of the time update can also be known as predictor equations, while the equation of the measurement update can be considered as corrector equations.

Figure 4, numerical problems can be solved by the predictor-corrector which resembles final estimation algorithm. The time update projects the current state estimate ahead in time. At that time the calculations updates the estimation predicted by an actual measurement.



Time update equation,

State Prediction –

$$\text{-----}1$$

Error Covariance Prediction -

$$\text{-----}2$$

In eq.(1)  $X_{predk}$  vector representing predicted process state at time  $k$ .  $X$  is a 4-dimensional vector  $[x, y, dx, dy]$ , where  $x$  and  $y$  represent the coordinates of the object's centre, and  $dx$  and  $dy$  represent its velocity.  $X_{k-1}$  is vector representing process state at time  $k-1$ . Here  $A$  is a 4\*4 Matrix form is a control vector and  $B$  relates optional control vector  $U_k$  into statespace.  $W_{k-1}$  is a process noise. In eq.(2) we calculate is predict error covariance at time  $k$ . The first term is a matrix representing error covariance in the state prediction at time  $k-1$ , and  $Q$  is the process noise covariance [4]. Measurement update equations [3,4]: After predicting the state and its error covariance at time  $k$  using the time update steps, the Kalman filter next uses measurement to correct its prediction during the measurement update steps [4].

Kalman Gain 
$$K_k = P_{predk} * H^T (H * P_{predk} * H^T + R)^{-1}$$

State Update 
$$X_k = X_{predk} + K_k * (Z_k - H * X_{predk})$$

Error Covariance Update 
$$P_k = (1 - K_k * H) P_{predk}$$

In eq. (3) is Kalman gain.  $H$  is matrix converting state space into measurement space and  $R$  is measurement noise covariance. Determining  $R_k$  for set of measurement is difficult, many Kalman implementations statistically analyse training data to determine fixed  $R$  for all future time updates. In eq. (4)  $X_k$  is a process actual state. Using Kalman gain  $K_k$  and measurement  $Z_k$  process state  $X_k$  can be updated. Here  $Z_k$  is the most likely  $x$  and  $y$  coordinates of the target objects in the frame. The final step in Kalman filter is to update the error covariance  $P_{predk}$  into  $P_k$  as given in eq. (5). The process is repeated after each pair of time and measurement updates with previous posteriori estimates used to forecast or predict the current priori estimate.

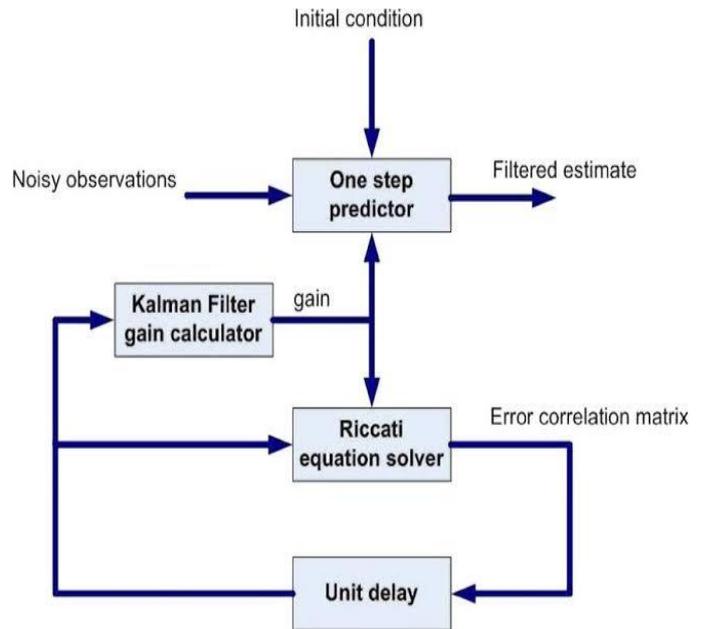
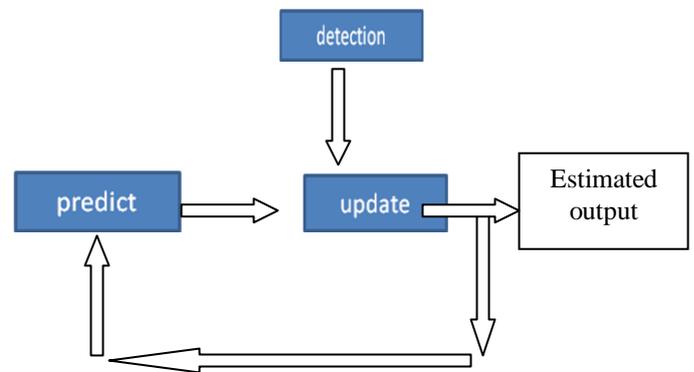


Fig 5: Block diagram of Kalman Filter

**B. Block diagram of UKF**



The Unscented Kalman Filter approximates the probability density resulting from the nonlinear transformation of a random variable. It is done by evaluating the nonlinear function with a minimal set of carefully chosen sample points, known as sigma points. The posterior mean and covariance estimated from the sample points are accurate to the second order for any nonlinearity.

UKF is used to estimate the state of the nonlinear system. Object tracking is done by predicting the object position from the previous information and we verify the object position at the estimated state. It has a feedback control to correct the result that we get from the predicted state.

Inputs- position of the object at time instant k-1

Output- estimated displacement after certain time interval.

Random variable- displacement

### Working of UKF

UKF fixes the EKF's approximation problems. A GRV again represents the state distribution, but is now defined using a limited set of carefully selected sample points.

The UKF is based on the idea that it is simpler to approximate a probability distribution to an arbitrary nonlinear function or

transformation. The sigma points are chosen to ensure their mean and covariance. Each sigma point is then propagated by the nonlinearity that ultimately results in a cloud of transformed points. Depending on their numbers, the new estimated mean and covariance are determined. This process is called as Unscented Transformation. The unscented transformation is a method of computing a random variable's

statistics that undergoes a nonlinear transformation. Consider the following nonlinear system, described by the difference equation and the observation model with additive noise.

### Steps of UKF :

1. selection of sigma points –based on the previous initial measurements made.

$$X_{K-1} = \{X_{k-1}^j, W^j\} | j = 0 \dots 2n$$

Consider the following selection of sigma points, selection that incorporates higher order information in the selected points,

where the weights must obey the condition

$$\sum_{j=0}^{2n} W^j = 1$$

$$\text{and } \sqrt{\frac{m}{1 - W^0} p_{k-1}}$$

**2. Model forecast-** The random variables are given through a non-linear system.

$$X_k^{f,j} = f(X_{K-1}^j)$$

The transformed points are used to compute the mean and covariance of the forecast value of  $x_k$

$$x_k^f = \sum_{j=0}^{2n} W^j X_k^{f,j}$$

$$p_k^f = \sum_{j=0}^{2n} W^j (X_k^{f,j} - x_k^f)(X_k^{f,j} - x_k^f)^T + Q_{k-1}$$

We propagate then the sigma points through the nonlinear observation model:

The resulting transformed observations measure their mean

and covariance (covariance of innovation):

The cross covariance ,

$$\text{Cov}(Z_{k-1}^f) = \sum_{j=0}^{2n} W^j (Z_{k-1}^{f,j} - Z_{k-1}^f)(Z_{k-1}^{f,j} - Z_{k-1}^f)^T$$

**3. Data Assimilation-** The measurements taken till now is added with a new value and finally the state and covariance equations are updated .

$$X_k^a = X_k^f + K_k(Z_k - Z_{k-1}^f)$$

The gain  $K_k$  is given by,

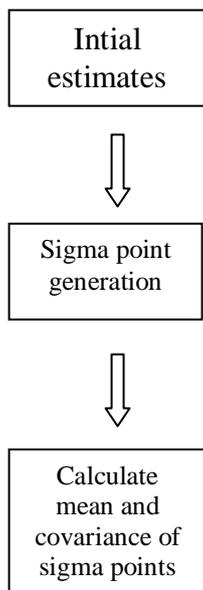
$$K_k = \text{cov}(X_k^{\sim f}, Z_{k-1}^{\sim f}) \text{cov}^{-1}(Z_{k-1}^{\sim f})$$

After the following formula, the posterior covariance is

Modified,

$$p_k = p_k^f - k_k \text{cov}(Z_{k-1}^{\sim f}) K_k^T$$

**Flowchart**



The initial points represents the mean and covariace of the random variable at time interval  $k-1$ . Then from this initial estimates we generate the selected points known as sigma points for the variables. And calculate mean and covariance of the sigma points. After that the nonlinear system undergoes Unscented Transformation to process and measure the estimated future values. And finally we add the previous estimated calculation with the new measured value and update the measurement state and covariance state. Here estimated output is the displacement at some time interval  $t$ .

**IV. RESULTS AND SIMULATIONS**

**Trajectory of the Path**

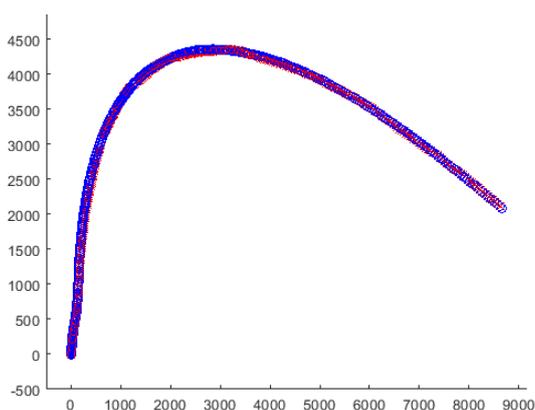


Figure 6: trajectory of the system

This graph shows the trajectory in which the system is being tracked. It depicts the distance the it has travelled .Here the vehicle moves in a plane .Motion is defined by 3 variables transition  $x$  axis ,transition  $y$  axis and rotation by angle  $\theta$  around  $z$  axis. Here  $x$  axis and  $y$  axis motion are uncorrelated.  $X$  axis denotes the distance travelled and  $Y$  axis denotes the angle of rotation.

**Results of Kalman Filter**

If we want to track the movement of this vehicle in a specified time interval  $T$  in the plane, we must know its pose  $(x,y,\theta)$  at every moment of time within the time interval  $T$ . We can measure the pose of this object at every instant of time. However; sensor's readings are usually noisy, and they can't give us an accurate value of the object's pose. One way to solve this problem is to use a Kalman filter to estimate the pose of the object at every time step in the time interval  $T$ . First graph shows the displacement in  $x$ -axis where we see the variation between estimated displacement, measured displacement and the true value. Second graph shows the  $y$ -axis displacement between the estimated, measured, and true displacement. And finally the third graph shows the angular variation between the estimated ,measured and true values.

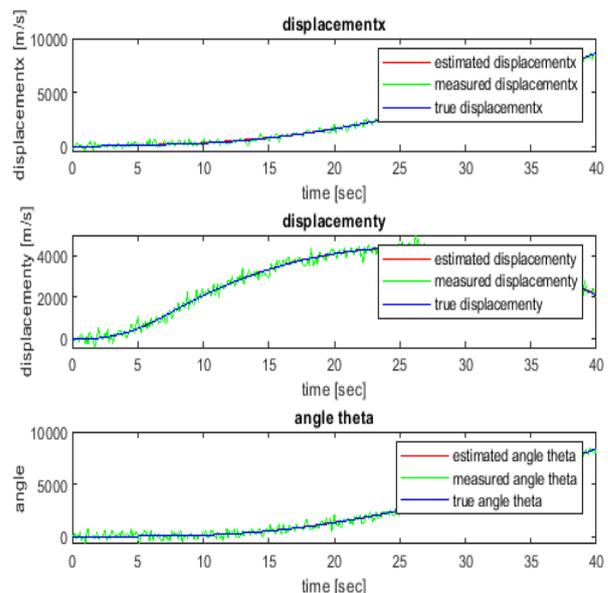


Figure 7- simulations using KF for estimated, and desired displacement

## Results of UKF

Here a non-additive measurement noise are considered because the measurement noise is not simply and it is added to a function of states.  $X_1$  and  $X_2$  both are estimated from the noisy measurements. Two functions are required for this purpose

First a function named (dvdp Measurement NonAdditive Noise Fcn.m). This function is given to the Unscented Kalman Filter during object construction. The measurement model is incorporated with non-additive noise, and it is specified through setting the Has Additive Measurement Noise property of the object as false. During object construction this is done. For non-additive process noise in the state transition function, the Has Additive Process Noise property is specified as false.

As it is non-linear system so we find noise in our graph. But the True and measured values are nearly same. In first graph it depicts the first order state of the random variable and in the second graph it depicts the second order state of the random variable.

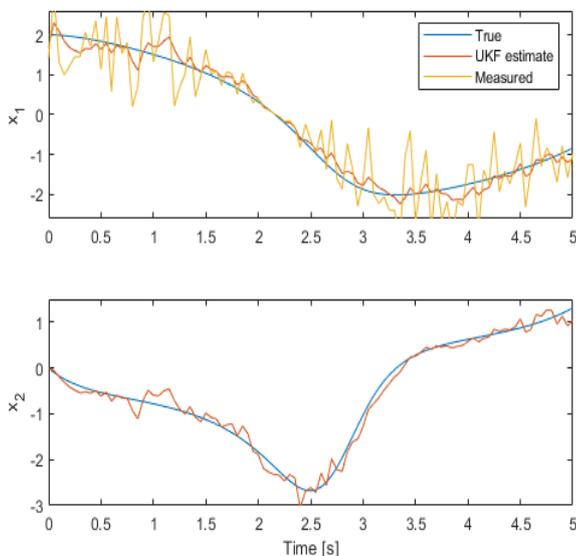


Fig7:Simulation using UKF

$x_1$ - first state of the random variable

$x_2$ -second state of the random variable

## V. DISCUSSION AND CONCLUSION

In this Paper, The UKF represents the extra uncertainty about a linearized function due to linearization errors by the covariance of the deviations in the regression points between the linear and the non-linear function. The approximations obtained with a sampling point of at least  $2n + 1$  are correct to the third order of Gaussian inputs for all

Nonlinearities and at least to the second for non Gaussian input The Reduced Sigma Point Filters only uses  $n + 1$  sampling points but this time the linearization errors are not taken into account. The results show the efficacy under the designed tracking system of the proposed tracking algorithms.

Simulations confirm the feasibility of the tracking system, but a field test is needed to show that the method is feasible in the real world.

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