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Modeling Electrification of Nanoparticles in Free Convective Nanofluid Flow

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Abstract

Free convection boundary layer nanofluid flow past a vertical flat plate with electrification is investigated. Electrification phenomenon of nanoparticles is considered in two component Buongiorno's model.The PDEs of flow field are changed into a system of coupled ODEs through similarity analysis and solved with respect to transformed boundary conditions using MATLAB bvp4c solver. The effects of governing dimensionless parameters such as Thermophoresis parameter, Electrification parameter and Brownian motion parameter on concentration and temperature distributions are examined graphically. It is observed that the dimensionless heat and masstransfer coefficients are enhanced whereas the normalized temperature of nanofluid and non-dimensional nanoparticle concentration are reduced for larger values of Electrification parameter.

Keywords: *Nanofluid, Brownian motion, Electrification, Thermophoresis, Free convection*

1 Introduction

The problem of free convectiveflow with heat and mass transfer have been gained more attention due to its practical importance in industrial sectors such as solar collectors,nuclear reactors and heat exchangers. In most of the engineering and industrial heat exchanger processes, the used liquidslike oil, kerosene and water have less thermal conductivities, which limits the enhancement performance of heat exchangers. Choi [1] first introduced the term "nanofluid'' and he observed nanofluids have higher thermal conductivity relative to the conventional fluids. The study of nanofluid is gaining more interest because of its promising heat transfer applications in thermal management and mass transfer applications in drug delivery and cancer therapy.

Various numerical investigations have been conducted on free convectivenanofluidflow to enhance heat transfer efficiency. Thethermophoresis and Brownian motion are main mechanisms in convective flow examined by Buongiorno [2]. Kuznetsov and Nield [3] solved a problem on free convectionnanofluid flow past a vertical plate. Khan and Aziz [4] examined the same problem with uniform surface heat fluxeffect. Gorla and Chamkha [5] have also studiedthe same paper by taking nonisothermal vertical plateand porous medium. Dastmalchi et al. [6] have analysed the effect of double diffusion on free convectivenanofluid flow in a porous square enclosure. Recently, Pandey and Kumar [7] have presented the effect ofviscous dissipationandthermal radiationon the natural convective nanofluid flow embedded in a porous media over a stretching cylinder.

MHD free convectionnanofluid flow past a vertical plate has been investigated by several researchers. Hamad and Pop [8] examined the free convective unsteady MHDflow with constant heat source. Das and Jana [9] have examined the thermal radiationeffectson MHD flow. Recently, Goyal and Bhargabha [10] have investigated the transient magneto hydro dynamics free convectivenanofluid flow over an inclined plate.

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Past literature indicates that, free convective nanofluid flow past a vertical flat platewith electrification mechanism effectsis not examined. Loeb [11] and Soo [12] concluded that a small charge on the solid particles due to static electrification have a pronounced effect on the solid-fluid flow system. So, it is essential to include this phenomenon in nanofluid.

Assumption of non-homogeneity of nanofluid is well justified when nanoparticle migration phenomena occur, has been analyzed by Wen et al. [14]. Cu-Water nanofluid has more heat transfer efficiency compared with TiO_2-Water and Al_2O_3-Water nanofluids has been examined by Hemalatha et al. [13]. So, the non-homogeneous Cu-Water nanofluid model has been taken in the present analysis.

This study presents the effects of electrification, thermophoresis and Brownian motionmechanism onheat and mass transferin free convectivenanofluid flowover a vertical platewhichis an extension of Pati et al. [14].

2 Problem Analysis

Alaminar steady incompressible two-dimensional free convectionnanofluid flow is considered. The co-ordinate axes is selected in whichvertical plate along$x$axis. The flat plate is assumed at constant concentration andtemperatureC_w and $T_w$respectively.The ambient values of concentration and temperature are takenas $C_\infty$and T_∞ respectively. WhereT_w and $C_w$values are greater than T_∞ and C_∞values. The schematic diagram is represented in Figure 1.

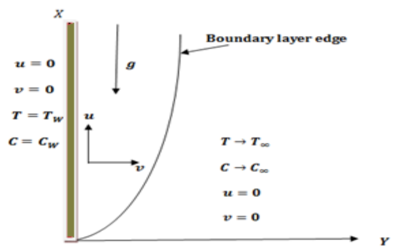

Figure 1. Schematic diagram of Flow Field

The boundary layer continuity, motion, energy and concentration including dynamic effects ofthermophoresis,Brownian motion and electrification ([15], [16], [17])applying the Oberbeck-Boussinesq approximation can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\rho_{nf}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right] = \mu_{nf}\left[\frac{\partial^2 u}{\partial y^2}\right]+\rho_s\left(\frac{q}{m}\right)(C-C_\infty)E_x + \beta_{f\infty}\rho_{f_\infty}(1-C_\infty)(T-T_\infty)g - (\rho_s-\rho_{f\infty})(C-C_\infty)g \quad (2)$$

$$(\rho c)_{nf}\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right] = k_{nf}\left[\frac{\partial^2 T}{\partial y^2}\right]+(\rho c)_s D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{(\rho c)_s D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{q}{m}\right)\frac{C(\rho c)_s}{F}\left(E_x\frac{\partial T}{\partial x}+E_y\frac{\partial T}{\partial y}\right) \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+\left(\frac{q}{m}\right)\frac{1}{F}\left[\frac{\partial(C\mathrm{E_x})}{\partial x}+\frac{\partial(C\mathrm{E_y})}{\partial y}\right] \quad (4)$$

and the governing boundary conditions:

$$\left.\begin{matrix} y=0, u=0, & v=0, & T=T_W, & C=C_w \\ y\to\infty, & u=0, & v=0, & T\to T_\infty, C\to C_\infty \end{matrix}\right\} \quad (5)$$

where$u$ and v are the velocity components along the$x$ and $y$axes, respectively. $T$and$C$ are the local temperature and local concentration respectively. Thethermophysical properties of the base fluid, nanoparticle andnanofluidare denoted by the subscripts$f$, $s$andnf respectively. ρ, k, μ and(ρc) denotes the density,thermal conductivity, viscosity and heat capacity, respectively. $g$is the gravitational acceleration. β_f,

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D_B and D_T are the volumetric thermal expansion coefficient, Brownian diffusion coefficientand thermophoretic diffusion coefficient respectively. q is the electric charge of the nanoparticles. m is the mass of the nanoparticles. E_x and E_y are the components of electric intensity in x and y axes, respectively. F is the time constant for momentum transfer between the fluid and nanoparticles. The free stream values are denoted by the subscript ∞.

The following variables are used to reduce the equations (2) to (4) into the equations (7) to (9).

$$\eta = \frac{y}{x}(Ra)^{\frac{1}{4}},\ \psi = \alpha_f (Ra)^{\frac{1}{4}} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$s(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad (6)$$

where ψ denotes the stream function with $u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$ and η denotes thesimilarity variable.

$Ra = \frac{(1-C_\infty)\beta_f g (T_w - T_\infty) x^3}{\nu_f \alpha_f}$, is the local Rayleigh number and α_f denotes the thermal diffusivity. The continuity equation (1) is thus identically satisfied.

$$f''' + \frac{\varphi_1}{4Pr}[3ff'' - 2(f')^2] + \varphi_1\varphi_2 \frac{M\, Sc\, Nb}{Pr\, N_F} s + \frac{1}{\varphi_5}(\theta - Nrs) = 0 \qquad (7)$$

$$\theta'' + \frac{3}{4}\frac{1}{\varphi_3\varphi_4} f\theta' + \frac{1}{\varphi_4}\Pr Nbs'\theta' + \frac{1}{\varphi_4}\Pr Nt(\theta')^2 + \frac{1}{\varphi_4} Sc\, Nb \left[\frac{N_F}{N_{Re}} - \frac{1}{4}M\right](s + Nc)\eta\theta' = 0 \qquad (8)$$

$$s'' + \frac{3}{4}\frac{Sc}{Pr} fs' + \frac{Nt}{Nb}\theta'' - \frac{1}{4}\frac{M\, Sc}{Pr}\eta s' + \frac{N_F Sc}{Pr\, N_{Re}}(\eta s' + s + Nc) = 0 \qquad (9)$$

Subject to:

$$\left.\begin{array}{l} \eta = 0, f = 0, f' = 0, \theta = 1, s = 1 \\ \eta \to \infty, f' \to 0, \theta \to 0, s \to 0 \end{array}\right\} \qquad (10)$$

whereprime denotesthe derivative of variables with respect to η. The dimensionless parameters are given by

$$Sc = \frac{\nu_f}{D_B},\ Pr = \frac{\nu_f}{\alpha_f},\ Nt = \frac{(\rho c)_s D_T (T_w - T_\infty)}{(\rho c)_f \nu_f T_\infty},$$
$$Nb = \frac{(\rho c)_s D_B (C_w - C_\infty)}{(\rho c)_f \nu_f},$$
$$Nr = \frac{(\rho_s - \rho_f)(C_w - C_\infty)}{(1 - C_\infty)\rho_f \beta_f (T_w - T_\infty)},$$
$$M = \left(\frac{q}{m}\right)\frac{1}{F\left(\frac{\alpha_f (Ra_x)^{\frac{1}{2}}}{x}\right)} E_x,\ N_F = \frac{\left(\frac{\alpha_f (Ra_x)^{\frac{1}{2}}}{x}\right)}{Fx},$$
$$\frac{1}{N_{Re}} = \left(\frac{q}{m}\right)^2 \frac{\rho_s}{\epsilon_0} \frac{x^2}{\left(\frac{\alpha_f (Ra_x)^{\frac{1}{2}}}{x}\right)^2},\ Nc = \frac{C_\infty}{(C_w - C_\infty)}. \qquad (11)$$

Here $Sc, Pr, Nt, Nb, Nr, M, N_F, N_{Re}, Nc$ denote theSchmidt number, Prandtl number,thermophoresis parameter, Brownian motion parameter, buoyancy ratio, electrification parameter, momentum transfer number, electric Reynolds number, concentration ratio, respectively. The thermophysical constants $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and φ_5 are given by

$$\varphi_1 = \frac{\nu_f}{\nu_{nf}} = \frac{\mu_f}{\mu_{nf}}\frac{\rho_{nf}}{\rho_f} = (1 - C_\infty)^{2.5}\left[(1 - C_\infty) + C_\infty \frac{\rho_s}{\rho_f}\right]$$

$$\varphi_2 = \frac{c_f}{c_s}\frac{1}{\left[C_\infty \frac{\rho_s}{\rho_f} + (1 - C_\infty)\right]},\ \tau = \frac{(\rho c)_s}{(\rho c)_f},$$

$$\varphi_3 = \frac{(\rho c)_f}{(\rho c)_{nf}} = \frac{1}{C_\infty \tau + (1 - C_\infty)},$$

$$\varphi_4 = \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2C_\infty(k_f - k_s)}{(k_s + 2k_f) + C_\infty(k_f - k_s)}$$ (Maxwell-Garnett model [18]),

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$$\varphi_5 = \frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-C_\infty)^{2.5}}$$ (Brinkman model [19]), (12)

where C_∞ denotes the volume concentration of nanoparticles. The nanofluid is water based with $\Pr = 6.2$ (considering pure water) containing 1% of copper (Cu) nanoparticle. The thermo physical properties [20] of the Cu - water nanofluid are presented in Table 1.

Table 1. Thermophysical properties

	Properties		
	$\rho(kg/m^3)$	$c(J/kgK)$	$k(W/mK)$
Pure water	997.1	4179	0.613
Cu	8933	385	400

For practical applications, the dimensionless heat transfer coefficient local Nusselt numberand mass transfer coefficient local Sherwood number are denoted by Nu_x and Sh_x which are given as follows:

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (13)$$

where $q_w = -k_f\left(\frac{\partial T}{\partial y}\right)_{y=0}$ and $q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$ are denotes the wall heat flux and wall mass flux respectively . The reduced Nusselt number Nu_r and the reduced Sherwood number Sh_r can be written as:

$$Nu_r = Nu_x / {Ra_x}^{1/4} = -\theta'(0) \quad (14)$$

$$Sh_r = Sh_x / {Ra_x}^{1/4} = -s'(0) \quad (15)$$

3 ResultAnalysis

The highly non-linear coupled dimensionless equations (7) to (9) with transformed boundary conditions (10), have been tackled using MATLAB bvp4c solver. Results are obtained for the influence ofthermophoresis,electrification and Brownian motion mechanism. To check the numerical accuracy, the $(-\theta'(0))$valuesare compared with the numerical values obtained by Bejan [21] and Kuznetsov and Nield [3]for a regular fluid and it is found that both the results are similar as seen from Table 2.

Table 2. Comparison of$(-\theta'(0))$with the numerical valuesobtained by Bejan [22], Kuznetsov and Nield [3].

Pr	$-\theta'(0)$ [Bejan]	$-\theta'(0)$ [Kuznetsov and Nield]	$-\theta'(0)$ [present]
1	0.401	0.401	0.401
10	0.465	0.463	0.463
100	0.490	0.481	0.480
1000	0.499	0.484	0.482

The influence ofelectrification parameter M,thermophoresis parameter $Nt$and Brownian motion parameter Nb on the dimensionless temperature$\theta(\eta)$ and concentration $s(\eta)$profiles against the similarity variable η were investigated numerically and the results are presented in graphical form. Also the effect of these parameters on $(-\theta'(0))$ and $(-s'(0))$ is shown in Table 3.

$\theta(\eta)$and$s(\eta)$ profiles for differentNb are plotted in figures 2 and 3 respectively. It is noticed that $\theta(\eta)$ increases whereas $s(\eta)$ decreases with the increasing values ofNb. Due to increase in Nb, the diffusion of nanoparticles increases, which helps to increase $\theta(\eta)$ and simultaneous nanoparticle deposition towards plate decreases the concentration.

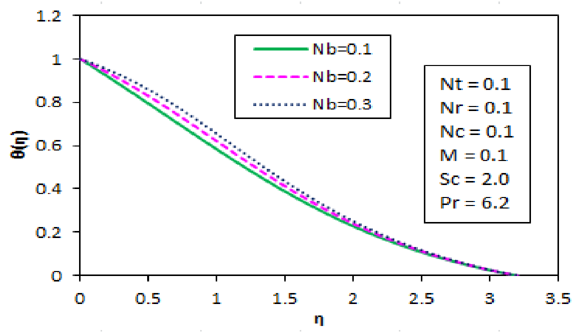

Figure 2. Variation of $\theta(\eta)$ with respect to $Nb$when $N_{Re} = 10, N_F = 0.01$.

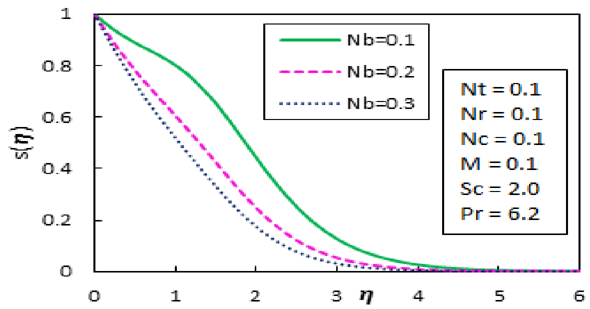

Figure 3. Variation of $s(\eta)$ with respect to Nb when $N_{Re} = 10, N_F = 0.01$.

The dimensionless temperature $\theta(\eta)$ and concentration $s(\eta)$ distributions for varied values of Nt are displaced in figures 4 and 5 respectively. Both $\theta(\eta)$ and $s(\eta)$ increase with the increase in Nt . For increasing values of Nt , the movement of nanoparticles occurs from hot region to cold region and consequently it increases the temperature profile. For higher values of Nt , the diffusion of nanoparticles increases towards fluid and consequently it increases the nanoparticle concentration.

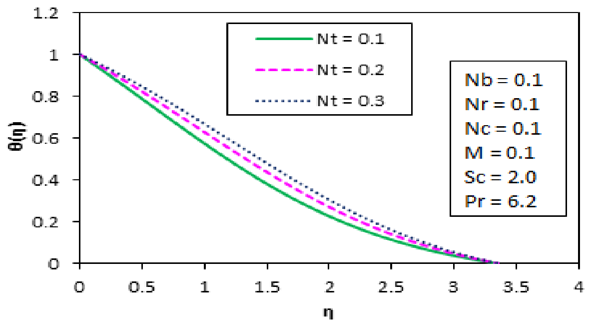

Figure 4. Variation of $\theta(\eta)$ with respect to Nt when $N_{Re} = 10, N_F = 0.01$.

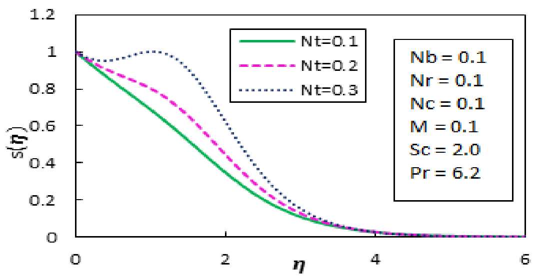

Figure 5. Variation of $s(\eta)$ with respect to Nt when $N_{Re} = 10, N_F = 0.01$.

The non-dimensional temperature $\theta(\eta)$ and concentration $s(\eta)$ distributions for various values of M are drawnin figures 6 and 7 respectively. By increasing the electrification parameter M, the temperature profile decreases as shown in figure 7. For higher values of M, the hotter fluid particle moves away causing the cooling of fluid and ultimately the temperature of the fluid reduces. Figure 8 shows the concentration profile reduces with increase in M. This is because of the increase in M, the movement of nanoparticles increases from fluid region towards plate and consequently the nanoparticle concentration decreases.

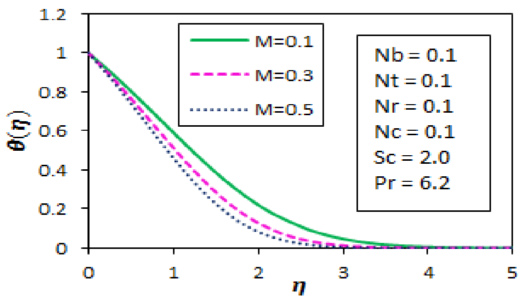

Figure 6. Variation of $\theta(\eta)$ with respect to $M$when $N_{Re} = 10, N_F = 0.01$.

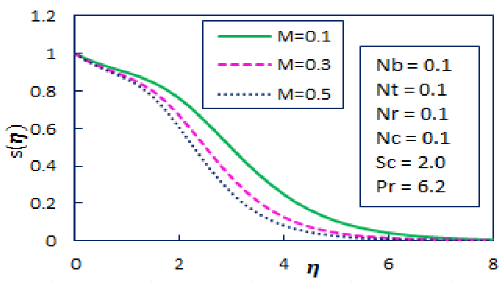

Figure 7. Variation of $s(\eta)$ with respect to $M$when $N_{Re} = 10, N_F = 0.01$.

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Table 3 representsthe results for the effects of Nb , Nt and M on dimensionless heat and mass transfer coefficients. It is noticed that, $(-\theta'(0))$ decreases with increase in Nb and Nt but increases with increase in M whereas $(-s'(0))$ increases with increasing Nb and M but decreases with increasing values of Nt .

Table 3. The effects of Nb ,Nt and Mon $(-\theta'(0))$and $(-s'(0))$ when $Pr = 6.2, Sc = 2, Nr = N_F = Nc = 0.1, N_{Re} = 2.0$

Nb	Nt	M	$-\theta'(0)$	$-s'(0)$
0.1	**0.1**	**0.1**	**0.36235**	**0.14057**
0.2			0.31097	0.28447
0.3			0.2559	0.34458
0.1	**0.1**		**0.36235**	**0.14057**
	0.2		0.31685	0.11657
	0.3		0.27444	0.09031
		0.1	**0.36235**	**0.14057**
		0.2	0.39808	0.16407
		0.3	0.42627	0.17926

4 Conclusions

The influenceof Nb , Nt and M infree convection Cu -Water nanofluid flowis examined. The numerical values obtained using MATLAB bvp4c solver.The following findings are derived from the numerical investigation:

1. The increasing values of both Nb and Nt enhance the temperature whereas the temperature decaysfor higher M.
2. For larger values of Nt , the nanoparticle concentration increases whereas with the increasing Nb and M the nanoparticle concentration decreases.
3. The dimensionless heat transfer coefficient enhances for increasing values of electrification parameter M whereas it reduces for increasing values of Nb and Nt .
4. The dimensionless mass transfer coefficient rises with Nb and Mbutreduces with Nt .

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