

Tackling a Shortest Path Having a Negative Cycle by using Johnson's Calculation

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Abstract:

In graph theory numerous calculations are utilized to discover shortest path, for example, Dijkstra's calculation and Bellman fort calculation. In this paper, we solve a negative cycle in the directed graph and find shortest path for it. Here, we use Johnson's calculation for finding shortest path between the pair of vertex. Johnson's calculation use Bellman-ford calculation to distinguish negative cycle and afterward it is utilized by Johnson's calculation to wipe out the negative edge and reweight the edges in directed graph. Presently the Johnson's calculation solves the upgraded directed graph utilizing Dijkstra's shortest path calculation to discover most brief way between all pair of vertices. The yield of the calculation is the result of the original directed graph. Consider 5 towns A, B, C, D, and E. The sales guy begins from the metropolis A and he visits all other cities B, C, D, E. Find a minimal route of for all pair of vertices. The result is received through the use of Johnson's calculation.

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I. INTRODUCTION

The first paper in Graph Theory was written by Euler in 1736 when he settled the famous unsolved problem of his day known as the Konigsberg Bridge. A graph G is a pair represented by $G = (V, E)$ consisting of a finite set V and a set E of 2-element subsets of V . The elements of V are called vertices and elements of E are called edges. The set V is known as vertex set of G and E as the edge set of G . Suppose $\{u, v\}$ is a member of $E(G)$, then we say u and v are joined by an edge in G . If we denote the edge by e , we can then write $e = uv$ which is an edge of G . Thus two vertices u and v of G are said to be adjacent if they have a common vertex. The length of the edges is called weight.

II. DIRECTED GRAPH

A directed graph is a graph which is which is comprised of a lot of vertices associated by edges where the edges have a course connected with them.

III. SHORTEST PATH DEFINITION

The shortest path problem is about finding the path between two vertices or nodes such that the total sum of the weight in corresponding edges is minimal. Here nodes constitute the town or town and edges represent the routes among vertices.

IV. APPLICATIONS OF GRAPH THEORY

There are many applications of graph theory to a wide form of topics which encompass Operations Research, Physics, Chemistry, Economics, Genetics, Engineering, Computer Science, Sociology, Linguistics, etc. In our everyday existence Graph Theory is utilized in Google maps, traffic lights, social network, to clear road blockage and in Google page rank. Graph Theory is used to find out the shortest path between two vertices. By using Dijkstra's calculation and Bellman Fort calculation we can find a single source shortest path. If the directed graph contains negative cycle, we wipe out the negative edge and reweight the edges in the directed graph through the use of Johnson's calculation.

V. BASIC CONCEPTS

Dijkstra's calculation is used to find the shortest path between the two nodes. It is applicable for both directed and undirected graph. Dijkstra's calculation finds shortest path only if the weight of the edges is positive. It fails when the weight of the edges is negative. It finds the most limited way from a source vertex to all different vertices in a graph. Bellman Fort calculations find the most limited way between two vertices. It detects the negative cycle in the graph. Bellman Fort calculation unearths the most limited way from a source vertex to all different vertices in a graph.

Johnson's calculation is used for graph with negative cycle. It permits some of the negative weight edges however no negative cycle exist inside the graph. The weight of the edges in the graph can be reweighted through the use of Johnson's calculation.

VI. METHODOLOGY

The beneath problem contains negative cycle, Johnson's calculation utilizes both Bellman Fort calculation and Dijkstra's calculation to solve the problem. In Bellman Fort calculation, a source vertex S is introduced and we calculate the shortest path from the source vertex S to all other vertices. Then, reweight the edges to make all edges positive by using the formula, $w(u,v) = w(u,v) + h(u) - h(v)$. Apply Dijkstra's calculation for each vertex and calculate the shortest path for every vertex A, B, C, D, E. If A is considered as source vertex then A is assigned to zero, then by using the formula, $\delta(u,v) = \delta'(u,v) - h(u) + h(v)$.

We can calculate the shortest distance from the source vertex A to all other vertices. Consider B, C, D, E as source vertex and calculate the shortest path.

VII. PROBLEM DESCRIPTION

Let A, B, C, D, E be Five Cities having a negative edges. Using Johnson's algorithm the negative edges are reweighted and the shortest path is found for All- pair of Vertices.

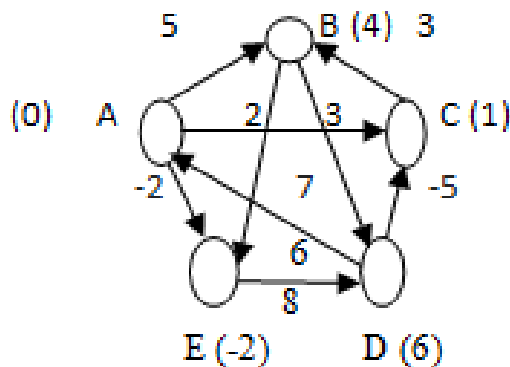


Fig (1)

STEP1: Now introduce a new source vertex

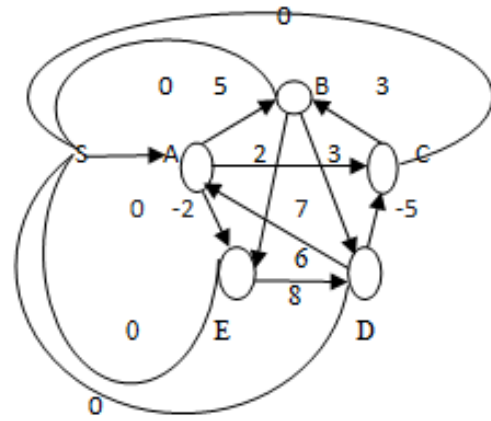


Fig (2)

STEP 2: Here we introduce Bell-man ford calculation for calculating the shortest distance from the source vertex S to all different vertices.

$$h(a) = \delta(s, a) = S \rightarrow A = 0$$

$$h(b) = \delta(s, b) = S \rightarrow D \rightarrow C \rightarrow B = 0 - 5 + 3 = -2$$

$$h(c) = \delta(s, c) = S \rightarrow D \rightarrow C = 0 - 5 = 5$$

$$h(d) = \delta(s, d) = S \rightarrow D = 0$$

$$h(e) = \delta(s, e) = S \rightarrow A \rightarrow E = 0 - 2 = -2$$

STEP 3: Reweighting the edges,

$$w'(u, v) = w(u, v) + h(u) - h(v)$$

$$w'(a, b) = w(a, b) + h(a) - h(b) = 5 + 0 - (-2) = 7$$

Similarly,

$$w'(a, c) = 2, w'(a, e) = 0, w'(b, d) = 1,$$

$$w'(c, b) = 0, w'(b, e) = 2, w'(d, a) = 6,$$

$$w'(e, d) = 6, w'(s, a) = 0, w'(s, b) = 2, w'(s, c) = 5,$$

$$w'(s, d) = 0, w'(s, e) = 2.$$

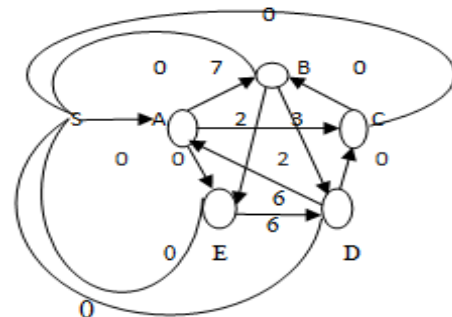


Fig (3)

STEP 4: Now remove the source vertex and we applying aDijkstra's calculation on each source vertex.

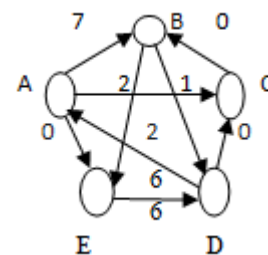


Fig (4)

Taking A as a source vertex $V = A$, we obtain a shortest path

$$\delta'(A, A) = A \rightarrow A = 0$$

$$\delta'(A, B) = A \rightarrow C \rightarrow B = 2$$

$$\delta'(A, C) = A \rightarrow C = 2$$

$$\delta'(A, D) = A \rightarrow C \rightarrow B \rightarrow D = 2 + 0 + 1 = 3$$

$$\delta'(A, E) = A \rightarrow E = 0$$

Again,

$$\delta(u, v) = \delta'(u, v) - h(u) + h(v)$$

$$\delta(A, A) = \delta'(A, A) - h(A) + h(A) = 0 - 0 + 0 = 0$$

Similarly,

$$\delta(A, B) = 0, \delta(A, C) = -3, \delta(A, D) = 3,$$

$$\delta(A, E) = -2$$

In loop we insert the δ'/δ values,

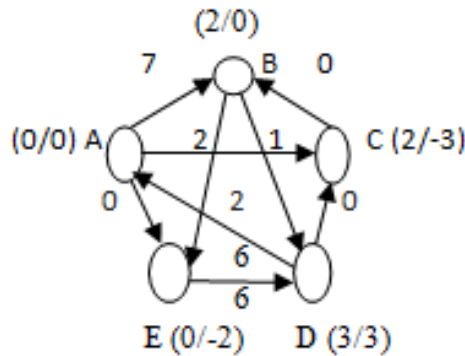


Fig (5)

Therefore the shortest path is $A \rightarrow E \rightarrow D \rightarrow C \rightarrow B$

Taking B as a source vertex $V = B$, the shortest path is

$$\delta'(B, B) = B \rightarrow B = 0$$

$$\delta'(B, A) = B \rightarrow D \rightarrow A = 7$$

$$\delta'(B, C) = B \rightarrow D \rightarrow C = 1$$

$$\delta'(B, D) = B \rightarrow D = 1$$

$$\delta'(B, E) = B \rightarrow E = 2$$

Again,

$$\delta(u, v) = \delta'(u, v) - h(u) + h(v)$$

$$\delta(B, B) = \delta'(B, B) - h(B) + h(B) = 0 - 0 + 0 = 0$$

Similarly,

$$\delta(B, A) = 9, \delta(B, C) = -2, \delta(B, D) = 3, \delta(B, E) = 2$$

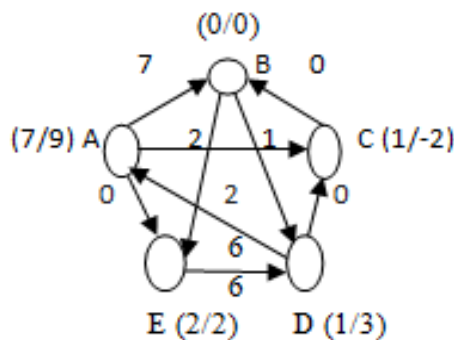


Fig (6)

Therefore the shortest path is $B \rightarrow D \rightarrow C$,

$B \rightarrow D \rightarrow A \rightarrow E$

Taking C as a source vertex $V = C$, the shortest path is

$$\delta'(C, C) = C \rightarrow C = 0$$

$$\delta'(C, A) = C \rightarrow B \rightarrow D \rightarrow A = 7$$

$$\delta'(C, B) = C \rightarrow B = 0$$

$$\delta'(C, D) = C \rightarrow B \rightarrow D = 1$$

$$\delta'(C, E) = C \rightarrow B \rightarrow E = 2$$

Again,

$$\delta(u, v) = \delta'(u, v) - h(u) + h(v)$$

$$\delta(C, C) = \delta'(C, C) - h(C) + h(C) = 0 - 0 + 0 = 0$$

Similarly,

$$\delta(C, A) = 12, \delta(C, B) = 3, \delta(C, D) = 6, \delta(C, E) = 4$$

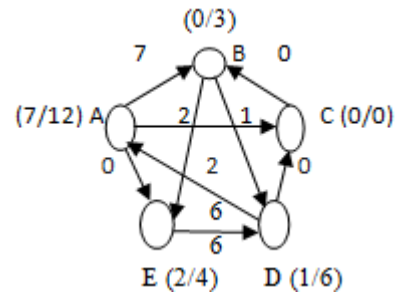


Fig (7)

Therefore the shortest path is $C \rightarrow B \rightarrow D \rightarrow A \rightarrow E$

Taking D as a source vertex $V = D$, the shortest path is

$$\delta'(D, D) = D \rightarrow D = 0$$

$$\delta'(D, A) = D \rightarrow A = 6$$

$$\delta'(D, B) = D \rightarrow C \rightarrow B = 0$$

$$\delta'(D, C) = D \rightarrow C = 0$$

$$\delta'(D, E) = D \rightarrow C \rightarrow B \rightarrow E = 2$$

Again,

$$\delta(u, v) = \delta'(u, v) - h(u) + h(v)$$

$$\delta(D, D) = \delta'(D, D) - h(D) + h(D) = 0 - 0 + 0 = 0$$

Similarly,

$$\delta(D, A) = 6, \delta(D, B) = -2, \delta(D, C) = -5, \delta(D, E) = 0$$

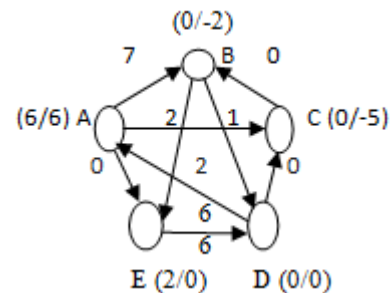


Fig (8)

Therefore the shortest path is $D \rightarrow C \rightarrow B$, $D \rightarrow A \rightarrow E$

Taking E as a source vertex $V = E$, the shortest path is

$$\delta'(E, E) = E \rightarrow E = 0$$

$$\delta'(E, A) = E \rightarrow D \rightarrow A = 12$$

$$\delta'(E, B) = E \rightarrow D \rightarrow C \rightarrow B = 2$$

$$\delta'(E, C) = E \rightarrow D \rightarrow C = 6$$

$$\delta'(E, D) = E \rightarrow D = 6$$

Again,

$$\delta(u, v) = \delta'(u, v) - h(u) + h(v)$$

$$\delta(E, E) = \delta'(E, E) - h(E) + h(E) = 0 - 0 + 0 = 0$$

Similarly,

$$\delta(E, A) = 9, \delta(E, B) = 6, \delta(E, C) = 3, \delta(E, D) = 8$$

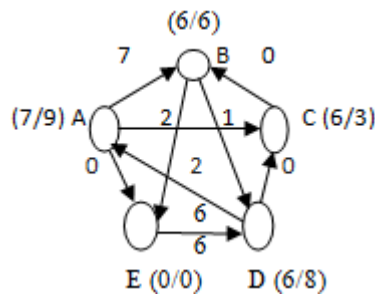


Fig (9)

Therefore the shortest path is $E \rightarrow D \rightarrow A$, $E \rightarrow D \rightarrow C \rightarrow B$

VIII. RESULT

Here. We used Johnson's algorithm and we found the shortest path for each vertex by keeping it as a source vertex. The obtained result is the result of the original graph.

IX. CONCLUSION

In this paper, we solve negative edges in directed graph and we found shortest path for each vertices. In step 1 we introduced a source vertex and in step 2 we found shortest path for each vertex. In step 3 we reweighted the edges using Bell-man ford calculation. Again, we used Dijkstra's Calculation and we found most brief way between all pair of vertices between the cities A to E.

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