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Study of the Impact of Differential Air Speed on the Character of Thermal Convection

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Abstract:

Issues of wet convection in an analytical formulation are all the most open in some aspects. Summarizing the above, we can separately single out some currently unresolved problems in the analytical description of atmospheric convection. Such problems include, in particular, the study of the influence of the horizontal temperature gradient and, in general, inhomogeneities of the temperature field on the development of convection, the development of a moist adiabatic model of convection taking into account nonlinear terms in two- and three-dimensional formulation, the study of the influence of humidity and phase transitions of heat on the development of convection, the study of the influence of the divergence of the velocity field on the nature of thermal convection, the study of the influence of orography on the development of convection.

Keywords: *Equations of free convection , Differential air speed , Thermal convection*

I. Introduction

The study of convective stability goes back to Benard's experiments, in which the formation of cellstrue convection in thin layers of a viscous fluid. Later, Rayleigh analyzed the stability problem analytically .equilibrium layer with free boundaries. Thus, the classical theory of convection in the horizontal liquid layer describing the instability threshold and the beginning of the convection process, which is still time is used in the description of convective phenomena in geophysic[1,2]. Currently, on the theory of convection, There is an extensive literature describing convective movements in various geophysical problems. So, review the state of the problem of stability of convective motion for an incompressible fluid is presented in the monograph[3]. Here the main attention is paid to the analysis of the theory of linear stability of convective motion[4]. Rayleigh's theory is developed taking into account the influence of a magnetic field on the stability of convective motion. Theory classical (Benar) convection is used to study many applied problems, including to describe convective processes in the atmosphere. Despite the importance of research on the linear theory of stability of convective they do not reflect and do not describe the very nature of convective motion. So with a linear analysis of the, they are shaking nonlinear terms that are included in the equation of motion, and it is these terms that determine the essence convective movement. Concerning atmospheric convection, an overview of this is given in the monograph[5]. The monograph is devoted both to experimental studies of thermal convection in the atmosphere, and theoretical studies in the framework of one dimensional model of convection. Despite the simplicity of the one-dimensional model, they take into account precisely the non-linear term of the equation of motion, namely its vertical component. The profiles of the distribution of the vertical speed of the ascending flows were obtained, which qualitatively correctly reflect the observed distribution of the speed of the ascending flows in the atmosphere. However, the application of the mentioned theories of convection to display real geophysical processes encounters certain difficulties.

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Liquids used in laboratory modeling of convection are usually characterized by fairly large values of viscosity and thermal diffusivity, while in a real atmosphere, the values of dissipative air coefficients are substantially less. Moreover, convection in the thickness of the atmosphere usually takes place in space, vertical and horizontal dimensions which are not fixed. Therefore, an attempt to calculate the parameters of convective instability based on the theory of convection, where the atmosphere as a whole is chosen as a layer, leads to not always plausible results. The tasks of geophysical convection can be divided into two classes. In problems of the first class, which are characteristic of the ocean and the mantle of the Earth, we can assume that the density weakly depends on the pressure as compared with temperature dependence (the so-called Boussinesq approximation). The tasks of the second class are connected with the solution of the equations of deep convection when the Boussinesq approximation cannot be used. These problems arise in the study of atmospheric circulation. Despite what has been said, there are a large number of works in which atmospheric convection is considered in the approximation of the Business as well as in the approximation of a thin layer[6]. An attempt to resolve this situation led to the formulation of the so-called quasi-elastic approximation (anelastic approximation) in analytical models of the atmosphere. Various approaches of this kind are being actively developed[7,8]. Consideration of the issues of atmospheric convection in the presence of phase transitions of moisture is of paramount importance. A review of the current state of research in this area is presented in[9-13]. It should be noted that the analytical description of dry air convection still has many unresolved and unresolved aspects.

So for example, there is no analytical solution to the two-dimensional model of convection, etc. Although dry convection is also of practical implementation and essential in studying the dynamics of the planets' atmosphere, as shown, for example,[14].

II. Equations of free convection

Consider the momentum balance equation for an ideal fluid in the Euler form in the inertial reference system, without taking into account the rotation of the Earth:

$$\frac{\partial \rho_i v}{\partial t} + (v, \nabla) = -\nabla p + \rho_i \mathrm{g} \qquad (1)$$

Consider the continuity equation

$$\frac{\partial \rho_i}{\partial t} + div(\rho_i v) = 0 \qquad (2)$$

Taking into account the continuity equation, the equation of motion will be written as

$$\frac{\partial v}{\partial t} + (v, \nabla)v - v(\nabla, v) = -\frac{1}{\rho_i}\nabla p + \mathrm{g} \qquad (3)$$

We will consider the flat case, i.e. motion in the *(x, z)* plane. We write equation (3) in projections on the coordinate axes:

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} + u\frac{\partial w}{\partial z} = -\frac{1}{\rho_i}\left(\frac{\partial p}{\partial x}\right) \qquad (4)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial u}{\partial x} = -\frac{I}{\rho_i}\left(\frac{\partial p}{\partial z}\right) - \mathrm{g} \qquad (5)$$

III. The distribution of atmospheric parameters in a state of static

In a state of equilibrium (static):

$$\frac{\partial \overline{p}}{\partial z} = 0, \frac{\partial \overline{p}}{\partial z} = -\overline{\rho}_e g \qquad (6)$$

Equation (6) is the equation of atmospheric static. Here? am the density of the air particleρ_e is the density of air surrounding the air particle of the atmosphere. We regard the parameters of the surrounding atmosphere as an unperturbed state.

In a state of the static atmosphere

$$\frac{\partial \overline{T}_e}{\partial z} = 0\,, \frac{\partial \overline{\rho}_e}{\partial z} = 0 \qquad (7)$$

Consequently, in the state of atmospheric static, the temperature changes with height according to a linear law

$$\overline{T}_e = \overline{T}_{e0} - \gamma z \qquad (8)$$

Thus, in the state of atmospheric static, the horizontal temperature gradient is zero. In other words, the presence of a horizontal temperature gradient will always cause convective movement. Next, we have

$$\bar{\rho_e} = \bar{\rho_{e0}} \left(\frac{\overline{T}_{e0} - \gamma z}{\overline{T}_{e0}} \right)^{(\gamma_A - \gamma)/\gamma} \quad (9)$$

where T_{e0} is the ambient air temperature at the earth at some point of reference; $\gamma_A = \frac{g}{R_d}$
g - gravitational acceleration; R_d is the specific gas constant of dry air. Or, if in expression (11) to represent

$$\frac{1}{\overline{T}_e} \approx \frac{1}{\overline{T}_{e0}} = \propto \quad (10)$$

where $\propto$ - coefficient of thermal expansion of air, then approximately (9) can be written as

$$\overline{\rho}_e = \overline{\rho}_{e0} e^{-\alpha(\gamma_A - \gamma)z} \quad (11)$$

The pressure in the ambient atmosphere in a static state is determined by the barometric formula
If also in (11) we use the permissible atmospheric approximation (9), then the barometric formula can be approximately represented as

$$\overline{p}_e = \overline{p}_0 \mathrm{e}^{-\frac{-g}{R_d T_{e0}}} = \overline{p}_0 e^{-\alpha \gamma_A z} \quad (12)$$

IV. Overheating function
We assume that the temperature of the surrounding atmosphere is not able to statics changes according to the law:

$$T_e(x, z) = T_{e0} - \gamma \mathrm{z} - \gamma_1 x \quad (13)$$

where γ - gradient of ambient air temperature; T_{e0} is the ambient air temperature on the ground. We will also assume that the rise of the air particle occurs adiabatically. Then the temperature of the rising air particle will vary according to the law.

$$T_i(z) = T_{i0} - \gamma_a z \quad (14)$$

where T_{i0} is the temperature of the rising air particle at the ground $\gamma_a -$ is a dry adiabatic temperature gradient. Imagine

$$T_i(z) = T_e(x, z) + \Delta T(x, z) \quad (15)$$

Where *T (x, z)* -overheating function. Taking into account formulas (14) and (15), the overheating function is written in the form:

$$\Delta T(x, z) = \Delta_0 T - \Delta\gamma . z + \gamma_1 x \quad (16)$$

where $\Delta_0 T$ is the value of the overheating function at the surface of the earth; $\Delta\gamma = \gamma_a - \gamma$

V. disturbed the state of the atmosphere
In a perturbed state, in general, for pressure, we can write

$$p(x, z) = \bar{p}(z) + \acute{p}(x, z) \quad (17)$$

Find the change in air density in a disturbed state:

$$\bar{p_e} = (\bar{\rho_e} + \acute{\rho}) R_d (\overline{T}_e + \acute{T}_e) \acute{\rho}_e \approx \bar{\rho_e} \alpha \gamma_1 x \quad (18)$$

Then

$$\frac{\partial ln\rho_e}{\partial x} \approx \alpha\gamma_1; \ \frac{\partial ln\rho_e}{\partial z} = -\alpha(\gamma_A - \gamma) \quad (19)$$

Similarly, we find:

$$p_e + \acute{p}_i = (\rho_e + \acute{\rho}_i) R_d (T_e + T'_i), \acute{\rho}_i \approx \bar{\rho_e} \frac{\acute{p}}{p_o} - \bar{\rho}_e \alpha T'_i \quad (20)$$

$$\rho_i \approx \bar{\rho_e} \left(1 - \alpha \overline{\Delta T} + \frac{\acute{p}}{p_o} \right) \quad (21)$$

$$\overline{\Delta T} = \Delta_0 T - (\gamma_a - \gamma) z \quad (22)$$

VI. The system of equations describing thermal convection

Taking into account formula (21), equations (4) and (5) are written in the form:

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - u \frac{\partial w}{\partial z} = - \frac{1}{\bar{\rho_e}} \frac{\partial \acute{p}}{\partial x} \quad (23)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} - w \frac{\partial u}{\partial x} = - \frac{1}{\bar{\rho_e}} \frac{\partial \acute{p}}{\partial z} + g \left(\alpha \overline{\Delta T} \frac{\acute{p}}{p_o} \right) \quad (24)$$

VII. Continuity equation

We write the equation of continuity in the Cartesian coordinate system

$$\frac{\partial\rho_i}{\partial t}+u\frac{\partial\rho_i}{\partial x}+w\frac{\partial\rho_i}{\partial z}+\rho_i\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0$$

In the stationary case, the continuity equation is written as

$$u\frac{\partial\rho_i}{\partial x}+w\frac{\partial\rho_i}{\partial z}+\rho_i\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=0 \qquad (25)$$

from here

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=\left(-\frac{1}{p_o}\frac{\partial \acute{p}}{\partial x}-\alpha\gamma_1\right)u+\left(-\frac{1}{p_o}\frac{\partial \acute{p}}{\partial z}+\alpha(\gamma_A-\gamma_a)\right)w \qquad (26)$$

VIII. Current function

We write the continuity equation in the form

$$\frac{\partial u}{\partial x}+\left(-\frac{1}{p_o}\frac{\partial \acute{p}}{\partial x}-\alpha\gamma_1\right)u+\left(-\frac{1}{p_o}\frac{\partial \acute{p}}{\partial z}+\alpha(\gamma_A-\gamma_a)\right)w=0 \qquad (27)$$

We introduce new features

$$\tilde{u}=ue^f, \qquad \tilde{w}=we^f$$

from here

$$\tilde{u}=ue^{-\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a zz}, \tilde{w}=we^{-\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z}$$

Then equation (27) is written in the form

$$\frac{\partial\tilde{u}}{\partial x}+\frac{\partial\tilde{w}}{\partial z}=0 \qquad (28)$$

It follows that for the converted velocities, it is possible to introduce “stream functions”:

$$\tilde{u}=\frac{\partial\varphi}{\partial}, \tilde{w}=-\frac{\partial\varphi}{\partial x}$$

from here

$$u=e^{-\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z}\frac{\partial\varphi}{\partial z}, w=-e^{-\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z}\frac{\partial\varphi}{\partial x} \qquad (29)$$

IX. Getting the basic system of equations

Express in equations (23) and (24) speeds through stream functions, we get

$$e^{-\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z}\left(-\frac{1}{p_o}\frac{\partial\acute{p}}{\partial t}\frac{\partial\varphi}{\partial z}+\frac{\partial^2\varphi}{\partial t\partial z}\right)+\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial z\partial x}-\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial z^2}$$

$$=-e^{2\left[\!\left[\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z\right]\!\right]}\frac{1}{\rho_o}\frac{\partial\acute{p}}{\partial x}, e^{\left[\!\left[\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z\right]\!\right]}$$

$$\left(-\frac{1}{p_o}\frac{\partial\acute{p}}{\partial t}\frac{\partial\varphi}{\partial x}+\frac{\partial^2\varphi}{\partial t\partial x}\right)+\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x\partial z}-\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial x^2}$$

$$=e^{2\left[\!\left[\frac{\acute{p}}{p_o}\alpha\gamma_1+\alpha(\gamma_A-\gamma_a)z\right]\!\right]}\left(-\frac{1}{\rho_e}\frac{\partial\acute{p}}{\partial z}+g\left[\alpha\overline{\Delta T}\frac{\acute{p}}{p_o}\right]\right)$$

X.Conclusions

Thus, we have obtained a system of equations describing thermal convection under the condition that the divergence of air velocity is not zero, and also in the presence of a horizontal temperature gradient. Rayleigh analyzed the problem of equilibrium stability of a layer with free boundaries. A mathematical model has been developed in the atmosphere with nonzero velocity.

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