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# The Evaluation of Robust Outlier Detection Procedure in Bilinear (1,0,1,1) Model

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***Abstract:***

The existence of outliers in bilinear time series model will causedistortion in parameter estimation, thus the existence must be detected before the next step is taken. In the outlier detection process, bootstrap method is commonly used to calculate the mean and variance magnitude of outlier effect. To improve the efficiency of detection process, this study proposes three robust estimators,namely, MOM to calculate the mean magnitude of outlier effect, while MADn and Tn to calculate the variance magnitude of outlier effect. Next, the effectiveness of the detection procedures was evaluated based on the probability of outlier detection obtained from simulation studies, focusing on two types of outlier which are often found in bilinear data i.e. additional outlier (AO) and innovational outlier (IO). The findings revealed that MOMTn with bootstrap procedure performs the best, followed by MOMMADn with bootstrap.

***Keywords:*** *Outlier Detection Procedure, Robust Procedure, Bilinear model, Additional Outlier, Innovational Outlier*

## 1. Introduction

Parameter estimation is a very important process in bilinear time series. However, the existence of outliers in time series datawill affect the estimated value, which consequentlywill affect the validity of the bilinear model. Thus, it is crucial to improve the outlier detection procedure to get the best possible parameter estimation results.Generally there are four types of outliers (OT) namely additional outlier (AO), innovational outlier (IO), level change (LC) and temporary change (TC). AO and IO are the common types of outliersin bilinear time series data [7]. AO is known through a single strange significant observation in time series data. It is the type of outlier that affects a single observation at a single time point and may occurs as a result of a mistake made by a person during observation or recording, while IO is characterized by a single strange observation at a time point, and at the same time affects subsequent observations with the effect gradually dying out [1].

In outlier detection process for bilinear models, bootstrap method is commonly employedto obtain the mean and variancemagnitude of outlier effects. This method is carried out through the process of drawing random samples with replacement.To improve the efficiency of outlier detection process, this study proposes the substitution of mean and variance magnitude of outlier effect with highly robust estimators. A robust trimmed location estimatorknown as modified one-step *M*-estimator(*MOM*)will substitute the meanmagnitude of outlier effect, while median absolute deviation (*MADn*) and alternative median based deviation called *Tn*will be used to substitute the variance magnitude of the outlier effect.Trimming is the method of eliminating outliers from each distribution tail. This method is useful in terms of achieving good efficiency and high power [3]. Since the estimators chosen possess highest breaking point, which are least affected by extreme values, the method which employed these estimators is deemed robust [4].

Wilcox and Keselman[6] introduced *MOM* as a measure of central tendency when testing the effects of treatment. This estimator which possesses highest breakdown point is calculated based on the remaining data of empirically determined trimming. Meanwhile *MADn* and *Tn* are among the best two robust scale estimators suggested by Rousseeuw and Croux[4]. *MADn* is a popular robust scale estimator withhighest breakdown point and least affected by extreme values. The other scale estimator is *Tn,* which also has the

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highest breakdown point of 50% and its efficiency is about 52%, making it more effective than *MADn* and furthermore, it has a continuous influence function.

In this study, the focus is on the process of detecting AO and IO in the bilinear (1,0,1,1) model. In section 2, we will introduce two robust detection procedures namely *MOMMADn* and *MOMTn* with bootstrap method to evaluate AO and IO detection performance in bilinear (1,0,1,1) model. The simulation process will be discussed in section 3, where the performance of robust detection procedures and standard bootstrap detection procedure will be compared and measured in terms of probability of detection before arriving to the conclusion in the last section.

## 2. Outlier Detection Procedure

In improving the outlier detection process, this study proposed two robust detection procedures namely *MOMMADn* and *MOMTn* with bootstrap method to detectAO and IO in bilinear (1,0,1,1) model. The full standard bootstrap detection procedure and robust detection procedures without bootstrap method can be found inIsmail, Ali and Syed Yahaya (2019)[2].The following phases described the robust detection procedure with bootstrap method.

**Phase 1:** Constructing the null and alternative hypothesis

For a robust detection procedure, the hypotheses are, $H_0 : \omega = 0$ and $H_1 : \omega \neq 0$ in bilinear (1,0,1,1)model with outlier at a time point *t*. The statistical test for the hypotheses is:

$$\hat{\tau}_{OT,MADn,t} = \frac{\left(\hat{\omega}_{OT,t} - \bar{\tilde{\omega}}_{OT,MOM,t}\right)}{\tilde{\sigma}_{OT,MADn,t}} \quad (1)$$

$$\hat{\tau}_{OT,Tn,t} = \frac{\left(\hat{\omega}_{OT,t} - \bar{\tilde{\omega}}_{OT,MOM,t}\right)}{\tilde{\sigma}_{OT,Tn,t}} \quad (2)$$

where $OT$ represents the outlier typeAO or IO, $\hat{\omega}$ is estimate magnitude of outlier effects obtain in phase 2, $\bar{\tilde{\omega}}$ is bootstrap mean, $\tilde{\sigma}$ is bootstrap standard deviation and $t = 1,\ldots,n$.

**Phase 2:** Obtaining magnitude of outliereffects

The statistics to measure the magnitude of outlier effects for AO and IO can be obtained using the least squares method. Consider the following equation:

$$S = \sum_{t=1}^{n} e_t^2 = \sum_{t=1}^{d-1} e_t^2 + \sum_{k=0}^{n-d}\left(e_{d+k}^{*} - \{-1\}^k f_{d+k}(\omega)\right) \quad (3)$$

Equation (3) is then minimized with respect to $\omega$, yielding the following measures of outlier effects:

$$\hat{\omega}_{OT} = \frac{\sum_{k=0}^{n-d}\left[\{-1\}^{l} e_{d+k}^{*} A_{k,OT}\right]}{\sum_{k=0}^{n-d} A_{k,OT}^{2}} \quad (4)$$

where

$$A_{k,AO} = \begin{cases} 1 & for \quad k = 0 \\ -(a_k + b_{k1}e_{d+k-1}) - \sum_{j=1}^{k} b_{1j} Y_{d+k-j,AO}^{*} & for \quad k \geq 1 \end{cases}$$

and

$$A_{k,IO} = \begin{cases} 1 & for \quad k = 0 \\ -\sum_{m=1}^{k} b_{1m} Y_{d+k-1,IO}^{*} A_{k-m,IO} & for \quad k \geq 1 \end{cases}$$

**Phase 3:** Obtaining robust variance of magnitude of outlier effects

Calculate $\tilde{\sigma}_{OT,MADn,t}$ using *MADn* and $\tilde{\sigma}_{OT,Tn,t}$ using *Tn*, while $\bar{\tilde{\omega}}_{OT,MOM,t}$ is obtained *MOM* (refer [2]).

**Phase 4:** Detecting the existence of AO or IO

The complete steps to detect the existence of AO or IO are described below.

(1) Compute statistical test value, $\hat{\tau}_{OT,F,t}$ based on equation (1) and equation (2) for each $t$, where $t = 1,2,\ldots,n$. *F* refers to *MADn* or *Tn*.

(2) The maximum value of $\hat{\tau}_{TP,F,t}$ is determined, which is represented by $\eta_{F,t} = \max_{t=1,2,\ldots n}\left\{\left|\hat{\tau}_{OT,F,t}\right|\right\}$.

(3) For any *t*, where $t = 1,2,\ldots,n$, if $\eta_{F,t} > C.V$ ($C.V$ is a critical value), then $H_0$ is rejected.

Finally the existence of AO or IO in $Y_t$ is detectable.

## 3. Simulation and Results

A simulation study has been carried out to investigate the performance of theproposed detection procedures in detecting AO and IO in bilinear (1,0,1,1) model. The performance of the robust outlier detection procedures using *MOMMADn* and *MOMTn* with bootstrap method will be compared with *MOMMADn* and *MOMTn* without bootstrap methodas well as the standard bootstrap method procedure. The procedures using *MOMMADn* and *MOMTn* with bootstrap method will be discussed in this section, while the other procedures for comparison had been elaborated in Ismail et al. (2019) [2]. The effectiveness of the proposed procedure is measured by the probability of outlier detection. The data are simulated using S-Plus package. To investigate on the performance of the

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*MOMMADn* and *MOMTn* with bootstrap method procedures, the combinations of the following factors are considered as well as in Ismail et al. (2019) [2]:

(a) Two types of outliers: AO and IO.
(b) Six underlying bilinear (1,0,1,1) model with different combinations of coefficients (*a*,*b*) for both types of outlier.
(c) A single outlier will be introduced at a time point $t = 40$ in sample size (*n*) of 100.
(d) $B = 100$ for the number of sets of bootstrap samples.
(e) Two different values of magnitude of outlier effect: $\omega = 3$, 5.
(f) Five different levels of critical values (C.V): C.V $= 2.0$, 2.5, 3.0, 3.5, 4.0.

For each given bilinear (1,0,1,1) model, 100 series of length 100 are generated using *rnorm* procedure in S-Plus. The generated series areto contain only one of the outlier types. The performance of the outlier detection procedure in bilinear (1,0,1,1) model can be observed in table 1to table 6 for AO and IO. In the tables, the values in columns 3-7 represent the probability of outlier detection of the respective procedure with correct location at a time point $t = 40$ .The results across the tables show that the performances of MOMTn and MOMMADn with bootstrap outlier detection procedure arebetter than the other outlier detection procedures for almost all the models used.

In table 1 the result shows that the *MOMTn* with bootstrap is the best regardless of the critical values for both AO and IO case. Meanwhile, *MOMMADn* with bootstrap performs the best when $\omega = 5$ and $\text{C.V} = 3$ for AO case, while for IO case, it is the best when $\omega = 5$ and $\text{C.V} = 2.5$ to 3.In regards to the standard bootstrap procedure, it is the best for $\omega = 5$ and $\text{C.V} = 2.5$ for IO case.

In table 2, *MOMTn* with bootstrap is the best procedure, followed by *MOMMADn* with bootstrap and standard bootstrap procedure. Meanwhile in table 3,it can be observed that the results of AO and IO for all $\omega$ and the critical values (C.V) used are in favor of the *MOMTn* with bootstrap outlier detection procedure, while the results in table 4 and 5 are almost similar for *MOMTn* and *MOMMADn* with bootstrap outlier detection procedure, whereby both are the best among the rest.In table 6, the pattern is the same as in table 1, where the best procedure is *MOMTn* with bootstrap, followed by *MOMMADn* with bootstrap and standard bootstrap procedure.

Overall there are three factors that influence the outlier detection level, i.e. $\omega$ values, combinations of coefficients and type of procedures used. The performance of the outlier detection procedures is better when larger $\omega$ value is used. Meanwhile, the selection of a combination of coefficients (*a*,*b*) also affects the probability of detection. When the coefficient is increased, for example the combinations of (0.5,0.2) and (-0.4,0.2) from table 5 and table 6 respectively, the probability of detection decreases for both AO and IO cases due to the larger value of coefficients used compared to table 1 to table 4. The type of procedures used is also important in controlling the probability of outlier detection, for example *MOMTn* with bootstrap is the best, followed by *MOMMADn* with bootstrap and then standard bootstrap procedure. *MOMTn* and *MOMMADn* with bootstrap procedure are found to be suitable for all levels of critical values, while standard bootstrap procedure, *MOMTn* and *MOMMADn* without bootstrap procedure are applicable forC.V $= 2.0$ to 3 only. Based on the probability of detection, the performance of *MOMTn* and *MOMMADn* with bootstrap procedure and standard bootstrap procedure are excellent, but for *MOMTn* and *MOMMADn* without bootstrap procedure, the performance isjust moderate.

**Table 1. The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient $(a_1 = 0.1, b = 0.1)$**

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.62 | 0.80 | 0.74 | 0.39 | 0.47 |
| | | 2.5 | 0.62 | 0.72 | 0.67 | 0.16 | 0.23 |
| | | 3.0 | 0.51 | 0.66 | 0.63 | 0.05 | 0.10 |
| | | 3.5 | 0.26 | 0.60 | 0.39 | 0.00 | 0.05 |
| | | 4.0 | 0.15 | 0.54 | 0.22 | 0.00 | 0.02 |
| | 5 | 2.0 | 0.96 | 1.00 | 0.98 | 0.95 | 0.94 |
| | | 2.5 | 0.96 | 1.00 | 0.98 | 0.81 | 0.81 |
| | | 3.0 | 0.93 | 0.98 | 0.98 | 0.55 | 0.64 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 3.5 | 0.85 | 0.98 | 0.88 | 0.27 | 0.42 |
| | | 4.0 | 0.71 | 0.96 | 0.79 | 0.12 | 0.21 |
| IO | 3 | 2.0 | 0.60 | 0.79 | 0.68 | 0.39 | 0.47 |
| | | 2.5 | 0.58 | 0.68 | 0.57 | 0.16 | 0.23 |
| | | 3.0 | 0.36 | 0.64 | 0.51 | 0.05 | 0.10 |
| | | 3.5 | 0.18 | 0.58 | 0.33 | 0.00 | 0.05 |
| | | 4.0 | 0.01 | 0.55 | 0.29 | 0.00 | 0.02 |
| | 5 | 2.0 | 0.98 | 0.99 | 0.98 | 0.95 | 0.94 |
| | | 2.5 | 0.98 | 0.98 | 0.98 | 0.81 | 0.81 |
| | | 3.0 | 0.96 | 0.98 | 0.98 | 0.55 | 0.64 |
| | | 3.5 | 0.89 | 0.98 | 0.90 | 0.27 | 0.42 |
| | | 4.0 | 0.69 | 0.89 | 0.85 | 0.12 | 0.21 |

**Table 2. The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient**
$(a_1 = 0.2, b = -0.2)$

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.74 | 0.72 | 0.77 | 0.53 | 0.60 |
| | | 2.5 | 0.68 | 0.69 | 0.73 | 0.19 | 0.42 |
| | | 3.0 | 0.53 | 0.67 | 0.55 | 0.05 | 0.29 |
| | | 3.5 | 0.31 | 0.63 | 0.27 | 0.01 | 0.15 |
| | | 4.0 | 0.17 | 0.58 | 0.15 | 0.00 | 0.10 |
| | 5 | 2.0 | 0.99 | 1.00 | 1.00 | 0.97 | 0.98 |
| | | 2.5 | 0.99 | 1.00 | 1.00 | 0.76 | 0.86 |
| | | 3.0 | 0.98 | 1.00 | 1.00 | 0.51 | 0.73 |
| | | 3.5 | 0.87 | 1.00 | 0.98 | 0.25 | 0.61 |
| | | 4.0 | 0.66 | 0.98 | 0.88 | 0.08 | 0.42 |
| IO | 3 | 2.0 | 0.72 | 0.65 | 0.47 | 0.30 | 0.38 |
| | | 2.5 | 0.68 | 0.60 | 0.30 | 0.12 | 0.17 |
| | | 3.0 | 0.44 | 0.60 | 0.15 | 0.04 | 0.06 |
| | | 3.5 | 0.19 | 0.45 | 0.15 | 0.00 | 0.03 |
| | | 4.0 | 0.09 | 0.30 | 0.12 | 0.00 | 0.01 |
| | 5 | 2.0 | 0.99 | 0.98 | 0.97 | 0.96 | 0.92 |
| | | 2.5 | 0.97 | 0.95 | 0.95 | 0.77 | 0.71 |
| | | 3.0 | 0.95 | 0.95 | 0.90 | 0.46 | 0.52 |
| | | 3.5 | 0.88 | 0.95 | 0.80 | 0.13 | 0.29 |
| | | 4.0 | 0.71 | 0.92 | 0.77 | 0.03 | 0.11 |

**Table 3.The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient**
$(a_1 = 0.3, b = 0.3)$

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.68 | 0.75 | 0.70 | 0.44 | 0.57 |
| | | 2.5 | 0.68 | 0.72 | 0.64 | 0.12 | 0.43 |
| | | 3.0 | 0.55 | 0.69 | 0.54 | 0.03 | 0.25 |
| | | 3.5 | 0.33 | 0.62 | 0.25 | 0.00 | 0.13 |
| | | 4.0 | 0.15 | 0.54 | 0.20 | 0.00 | 0.05 |
| | 5 | 2.0 | 0.95 | 0.97 | 0.92 | 0.64 | 0.65 |
| | | 2.5 | 0.95 | 0.97 | 0.89 | 0.57 | 0.64 |
| | | 3.0 | 0.89 | 0.97 | 0.81 | 0.40 | 0.55 |

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|---|---|---|---|---|---|---|---|
| | | 3.5 | 0.81 | 0.92 | 0.81 | 0.14 | 0.45 |
| | | 4.0 | 0.71 | 0.86 | 0.75 | 0.06 | 0.35 |
| IO | 3 | 2.0 | 0.51 | 0.79 | 0.63 | 0.33 | 0.31 |
| | | 2.5 | 0.51 | 0.75 | 0.55 | 0.12 | 0.15 |
| | | 3.0 | 0.41 | 0.70 | 0.55 | 0.03 | 0.05 |
| | | 3.5 | 0.23 | 0.70 | 0.45 | 0.00 | 0.02 |
| | | 4.0 | 0.13 | 0.59 | 0.29 | 0.00 | 0.01 |
| | 5 | 2.0 | 0.91 | 0.98 | 0.91 | 0.61 | 0.59 |
| | | 2.5 | 0.91 | 0.98 | 0.91 | 0.45 | 0.51 |
| | | 3.0 | 0.90 | 0.98 | 0.91 | 0.30 | 0.36 |
| | | 3.5 | 0.87 | 0.98 | 0.91 | 0.12 | 0.24 |
| | | 4.0 | 0.84 | 0.90 | 0.88 | 0.03 | 0.12 |

**Table 4.The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient** $(a_1 = -0.3, b = 0.1)$

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.70 | 0.72 | 0.73 | 0.52 | 0.52 |
| | | 2.5 | 0.67 | 0.68 | 0.69 | 0.17 | 0.33 |
| | | 3.0 | 0.48 | 0.64 | 0.58 | 0.06 | 0.19 |
| | | 3.5 | 0.24 | 0.64 | 0.35 | 0.02 | 0.08 |
| | | 4.0 | 0.12 | 0.59 | 0.27 | 0.01 | 0.03 |
| | 5 | 2.0 | 0.98 | 1.00 | 1.00 | 0.98 | 0.94 |
| | | 2.5 | 0.98 | 1.00 | 1.00 | 0.80 | 0.82 |
| | | 3.0 | 0.98 | 1.00 | 1.00 | 0.54 | 0.66 |
| | | 3.5 | 0.89 | 1.00 | 0.98 | 0.25 | 0.47 |
| | | 4.0 | 0.73 | 1.00 | 0.91 | 0.10 | 0.23 |
| IO | 3 | 2.0 | 0.52 | 0.63 | 0.57 | 0.52 | 0.52 |
| | | 2.5 | 0.45 | 0.55 | 0.45 | 0.17 | 0.33 |
| | | 3.0 | 0.40 | 0.55 | 0.35 | 0.06 | 0.19 |
| | | 3.5 | 0.25 | 0.45 | 0.30 | 0.02 | 0.08 |
| | | 4.0 | 0.17 | 0.40 | 0.27 | 0.01 | 0.03 |
| | 5 | 2.0 | 0.95 | 1.00 | 0.97 | 0.98 | 0.94 |
| | | 2.5 | 0.95 | 1.00 | 0.95 | 0.80 | 0.82 |
| | | 3.0 | 0.92 | 1.00 | 0.95 | 0.54 | 0.66 |
| | | 3.5 | 0.86 | 1.00 | 0.90 | 0.25 | 0.47 |
| | | 4.0 | 0.63 | 0.97 | 0.87 | 0.10 | 0.23 |

**Table 5.The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient** $(a_1 = 0.5, b = 0.2)$

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.41 | 0.42 | 0.39 | 0.32 | 0.39 |
| | | 2.5 | 0.38 | 0.39 | 0.31 | 0.15 | 0.26 |
| | | 3.0 | 0.34 | 0.36 | 0.25 | 0.07 | 0.13 |
| | | 3.5 | 0.21 | 0.36 | 0.19 | 0.03 | 0.04 |
| | | 4.0 | 0.12 | 0.33 | 0.15 | 0.01 | 0.01 |
| | 5 | 2.0 | 0.60 | 0.85 | 0.81 | 0.71 | 0.71 |
| | | 2.5 | 0.60 | 0.78 | 0.74 | 0.60 | 0.65 |
| | | 3.0 | 0.60 | 0.78 | 0.72 | 0.29 | 0.56 |

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|---|---|---|---|---|---|---|---|
| | | 3.5 | 0.59 | 0.78 | 0.72 | 0.13 | 0.34 |
| | | 4.0 | 0.51 | 0.73 | 0.68 | 0.03 | 0.17 |
| IO | 3 | 2.0 | 0.29 | 0.45 | 0.53 | 0.32 | 0.39 |
| | | 2.5 | 0.21 | 0.33 | 0.43 | 0.15 | 0.26 |
| | | 3.0 | 0.18 | 0.33 | 0.29 | 0.07 | 0.13 |
| | | 3.5 | 0.12 | 0.27 | 0.21 | 0.03 | 0.04 |
| | | 4.0 | 0.07 | 0.25 | 0.19 | 0.01 | 0.01 |
| | 5 | 2.0 | 0.65 | 0.76 | 0.71 | 0.71 | 0.71 |
| | | 2.5 | 0.65 | 0.76 | 0.71 | 0.60 | 0.65 |
| | | 3.0 | 0.63 | 0.76 | 0.71 | 0.29 | 0.56 |
| | | 3.5 | 0.62 | 0.76 | 0.71 | 0.13 | 0.34 |
| | | 4.0 | 0.50 | 0.76 | 0.71 | 0.03 | 0.17 |

**Table 6.The performance indetecting AO and IO in bilinear (1,0,1,1) model with coefficient** $(a_1 = -0.4, b = 0.2)$

| Outlier Type | $\omega$ | C.V | Standard Bootstrap | With Bootstrap | | Without Bootstrap | |
|---|---|---|---|---|---|---|---|
| | | | | *MOMTn* | *MOMMADn* | *MOMTn* | *MOMMADn* |
| AO | 3 | 2.0 | 0.69 | 0.62 | 0.69 | 0.43 | 0.56 |
| | | 2.5 | 0.59 | 0.59 | 0.64 | 0.17 | 0.42 |
| | | 3.0 | 0.39 | 0.57 | 0.52 | 0.06 | 0.24 |
| | | 3.5 | 0.24 | 0.48 | 0.39 | 0.00 | 0.13 |
| | | 4.0 | 0.14 | 0.39 | 0.27 | 0.00 | 0.08 |
| | 5 | 2.0 | 0.96 | 1.00 | 1.00 | 0.89 | 0.94 |
| | | 2.5 | 0.95 | 1.00 | 1.00 | 0.82 | 0.86 |
| | | 3.0 | 0.95 | 1.00 | 1.00 | 0.65 | 0.75 |
| | | 3.5 | 0.86 | 1.00 | 0.98 | 0.38 | 0.55 |
| | | 4.0 | 0.81 | 1.00 | 0.95 | 0.16 | 0.41 |
| IO | 3 | 2.0 | 0.58 | 0.68 | 0.59 | 0.33 | 0.30 |
| | | 2.5 | 0.52 | 0.65 | 0.53 | 0.15 | 0.16 |
| | | 3.0 | 0.33 | 0.60 | 0.35 | 0.04 | 0.04 |
| | | 3.5 | 0.19 | 0.45 | 0.18 | 0.00 | 0.00 |
| | | 4.0 | 0.05 | 0.41 | 0.13 | 0.00 | 0.00 |
| | 5 | 2.0 | 0.87 | 0.98 | 0.95 | 0.80 | 0.77 |
| | | 2.5 | 0.86 | 0.98 | 0.95 | 0.65 | 0.61 |
| | | 3.0 | 0.83 | 0.98 | 0.95 | 0.29 | 0.40 |
| | | 3.5 | 0.90 | 0.98 | 0.90 | 0.09 | 0.20 |
| | | 4.0 | 0.87 | 0.98 | 0.88 | 0.01 | 0.13 |

## 4. Conclusion

The study demonstrates that among the investigated outlier detection procedures, *MOMTn* with bootstrap is the best procedure. Next to *MOMTn* with bootstrap is *MOMMADn* with bootstrap procedure, followed bythe standard bootstrap procedure. All the three procedures record very good detection of outliers. While, for *MOMMADn* and *MOMTn* without bootstrap procedure, both perform moderately with almost similar results.In general, the performance of outlier detection procedure for all the investigated procedures is considerably good especially for *MOMTn* and *MOMMADn* with bootstrap procedures which are found to be suitable for all levels of critical values. Meanwhile, standard bootstrap procedure, *MOMTn* and *MOMMADn* without bootstrap procedure are applicable for C.V = 2.0 to 3 only.

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