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A Theoretical Approach to the Study of Optimal Public Goods and Olson Conjecture

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Abstract

The free-riders' problem associated with the Olson conjecture increases with the size of the group, is based on non-zero conjectural variation explained through the public goods model involving Non-Nash behavior pattern. A hybrid solution between Kantian behavior and Nashian behavior is emerged when we look at the elasticity of the conjectured response with respect to the relative importance of the individual's contribution. When the number of contributors in the community for the provision of public goods increases infinitely, the path that emerges through hybridization converges to that of the Nash type. This convergence holds for all elasticity of conjectured response, which is greater than or equal to one. The larger is the elasticity of conjectured response, the faster will be the rate of convergence and thus free-riding.

Keywords:*Olson conjecture, Nash behaviour, Non-Nash behaviour, Hybrid solution, Elasticity of conjectured response, Pareto Points.**JEL Classification Code: H41, D82***

I. INTRODUCTION

A public good is not public just because its supply is made as a part of the public policy process or just because the government of the day finances it for public uses. What defines a public good is not the source i.e. public or private from which it has been financed, but how many people it provides the benefit with. Public goods are associated with collective benefits and involvement of large numbers of users. Due to its very character of non-rivalness and non-excludability, public goods exhibit joint consumption. The nature of the public good gives scope to individuals not to reveal their true preferences for it and free-ride. When the individuals are identical in tastes and in terms of their endowments, the work of Olson (1965) about collective action reveals that the deficiency of the equilibrium level of provision of public goods from its optimal level is influenced monotonically with respect to the group-size. This particular study is based on the objective of how to capture the 'Olson conjecture' concerning free-riding increasing with group size through a theoretical analysis.

The plan of the proceeding of this paper is as follows. Section II introduces Nash Equilibrium for individual contributions while the third section carries an analysis of Non-Nash behaviour as regards the public goods. Conclusion follows in section IV.

II. NASH EQUILIBRIUM:

Let individual i has income to allocate between personal private goods consumption and a contribution to a public good from which n other people benefit. Given the public good's relative price, the budget constraint of the individual is

$$Y_i = x_i + P_G g_i \quad \text{……………………..(i)}$$

The availability of total supply of public good is-

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$$Q = g_i + G_i \quad \text{.............................(ii)}$$

where G_i is the amount provided by others.

The individual's utility function is

$$U^i = U^i(x_i, Q) \quad \text{........................ (iii)}$$

$$=> U^i = U^i[y_i - P_G g_i, G_i + g_i]$$

$$= V^i[g_i, G_i, P_G, y_i] \quad \text{..................(iv)}$$

$dV^i = 0,$ on an indifference curve,

$$=> dV^i = \tfrac{dV^i}{dg_i}, dg_i + \tfrac{dV^i}{dG_i} dG_i = 0 \quad \text{....(v)}$$

Therefore, the indifference curve's slope in Figure 1 to be

$$\frac{dG_i}{dg_i} = -\left(\frac{dV^i}{dg_i}\right) \Big/ \left(\frac{dV^i}{dG_i}\right) \quad \text{.............(vi)}$$

The own-contribution g_i is personally financed at a marginal cost MC and so

$$\frac{\partial V^i}{\partial g_i} = MB_i - MC \quad \text{............(vii)}$$

Other people's contribution G_i, however, provide marginal benefit at no personal cost, and

$$\frac{\partial V^i}{\partial G_i} = MB_i \quad \text{...........................(viii)}$$

Substituting equation (vii) and (viii) into equation (vi), the expression for the slope of the indifference curve is

$$\frac{dG_i}{dg_i} = -\frac{MB_i - MC}{MB_i} \quad \text{...............(ix)}$$

At the point H in Figure 1, the individual chooses a personally best or utility maximizing own contribution g_i when others are providing G_i.

$$\text{At H,} \quad \frac{dV^i}{dg_i} = 0 = MB_i - MC \quad \text{............(x)}$$

The choice at H is a Nash response. Given contribution of others G_i, the individual has chosen a personal utility-maximizing contribution of self to the public good that satisfied MB_i =MC. Let individual i has income to allocate between personal private goods consumption and a contribution to a public good from which n other people benefit. Given the public good's relative price, the budget constraint of the individual is

$$\frac{dg_i}{dG_i} = -1 + P_G \frac{dg_i}{dI_i} \quad \text{................(xi)}$$

The total supply of public good available is

$$Q^N = g_A^N + g_B^N \quad \text{..................(xii)}$$

where is the amount provided by others

The individual's utility function is

$$-\frac{MB_A - MC}{MB_A} = -\frac{MB_B}{MB_B - MC} \quad \text{.........(xiii)}$$

$$=> -1 + \frac{MC}{MB_A} = \frac{-MB_B}{MB_B - MC}$$

$$=> \frac{MC}{MB_A} = 1 - \frac{MB_B}{MB_B - MC}$$

$$=> \frac{MC}{MB_A} = \frac{-MC}{MB_B - MC}$$

$$=> \frac{MB_A}{MC} = -\frac{MB_B}{MC} + 1$$

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$$=> \frac{MB_A + MB_B}{MC} = 1$$

$$=> MB_A + MB_B = MC$$

Hence equation (xiii) is rearranged to establish

$\sum MB_i = MC$ …………………………..(xiv)

This is the efficiency condition for public good supply

Let $Q^E = g_A^E + g_B^E$ …………………………(xv)

Q^N/Q^E is used to measure the relative inefficiency of Nash provision of the public good.

With n identical individuals, each person contributes

$g_i = g = \frac{Q}{n}$ ……………(xvi)

d on an indifference curve,

$=> g_i = \frac{G_i + g_i}{n} = \frac{G + g}{n} = g$...... ….(xvii)

Or, g(n-1) =G

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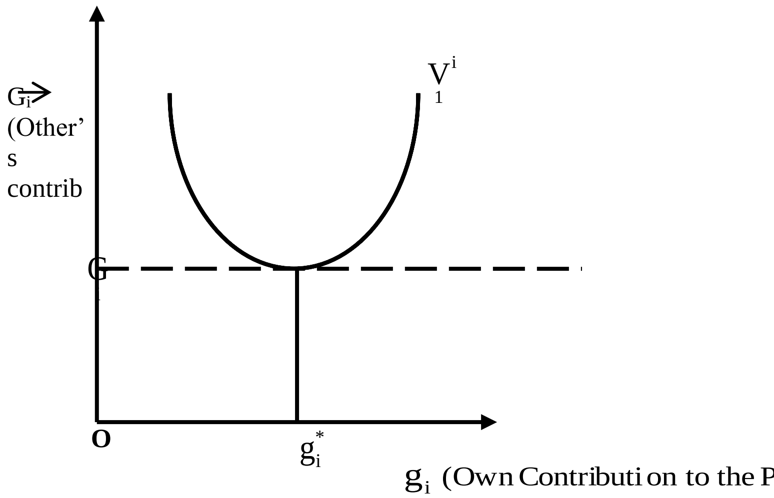

The quantity of public goods provided by others increases from G_{i1} to G_{i2} in Figure 2. The individual's response is given by

$$\frac{dg_i}{dG_i} = -1 + P_G \frac{dg_i}{dI_i}$$ ………………………………(xi)

The value -1 is a one-for-one negative substitution effect. The substitution effect is one-for one because of the perfect substitutability in consumption between the public good provided by own spending and the public good provided by the spending of others. The second term in the equation (xi) is an income effect. The income effect occurs because the increase in the public good provided by others is like receiving a gift of money. The income effect is positive if the public good in question is a normal good, which is taken as the case.

Figure 2, shows two alternatives. In case 1, the substitution effect dominates the income effect, and the own contribution declines as the contribution of others increases. In case 2, the income effect dominates, and the own contribution increases. If there were only a substitution effect (if the income effect were precisely zero), the slope of the reaction function would be precisely –1, or 45°.

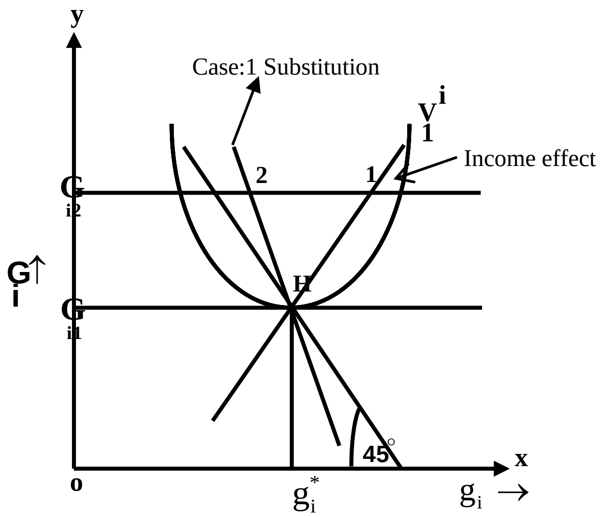

Figure 2. The Change in the contribution by others

Now consider two individuals A and B. Figure 3 shows the derivation of the reaction function $R_B R_B$ of individual B, who chooses personal utility-maximizing public-good contributions in reaction to the changes in the contributions of individual A. $R_B R_B$ shows a dominant substitution effect (the slope of the reaction function is negative).

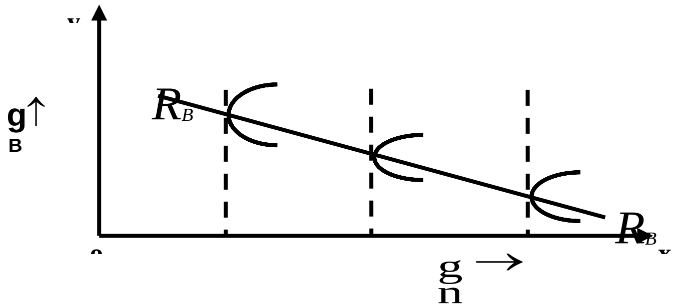

Figure 3. The reaction function of person B.

Point N in Figure 4 shows the Nash equilibrium. In this figure the reaction functions of the two persons intersect at point N and the choices of own contributions are mutually consistent given the contributions each is making to the public good. The voluntarily supplied total quantity of the public good that is at N is

$$Q^N = g_A^N + g_B^N \qquad \text{.....................................(xii)}$$

If the individuals are identical then their contributions to the public goods are also identical.

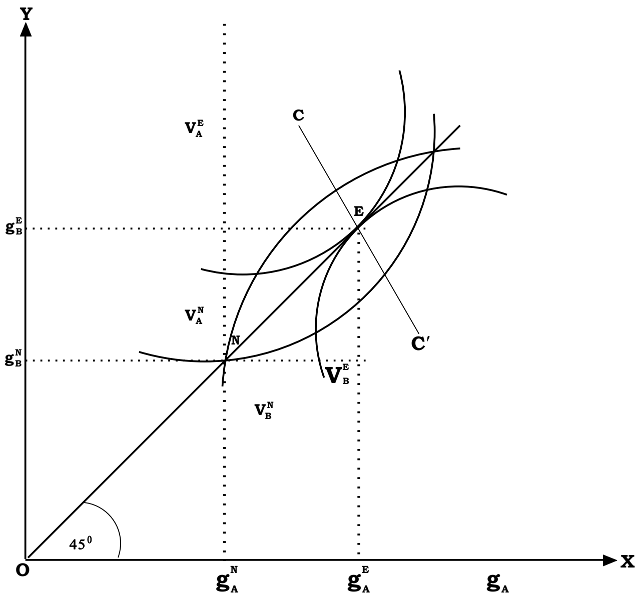

Figure: 4 The Nash Equilibrium for individuals contributions

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Point N in Figure 4 is the inefficient outcome obtained in the prisoners' dilemma. This outcome can be improved upon for both persons. Pareto-efficient combinations of the contributions of private person to the public good lie along the contract curve CC'. Since indifference curves are tangential along the contract curve, we can equate the slopes of the two individuals' indifference curves, to obtain

$$-\frac{MB_A - MC}{MB_A} = -\frac{MB_B}{MB_B - MC}$$ ……………………………………………..(Xiii)

$$=> -1 + \frac{MC}{MB_A} = \frac{-MB_B}{MB_B - MC}$$

$$=> \frac{MC}{MB_A} = 1 - \frac{MB_B}{MB_B - MC}$$

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With n identical individuals, each person contributes

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$$=> g_i = \frac{G_i + g_i}{n} = \frac{G + g}{n} = g......$$ …………(xvii)

Or, g(n-1) =G

$$=> g= \frac{G}{(n-1)}$$ …………………………….(xviii)

For example in Figure 5, since n=2, we have g=G, and the Nash outcome is on the 45o line from the origin. As the size of the group increases beyond 2, the ray from the origin along which the Nash equilibrium is located moves to the left as in Figure 5. It means

$$\frac{dg_i}{dn} < 0 \text{ and } \frac{d(ng_i)}{dn} = \frac{dQ^N}{dn} > 0..$$ …….(xix)

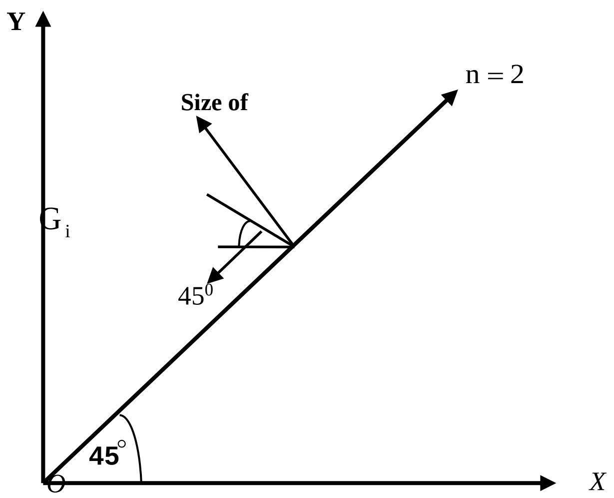

Figure 5. Changes in Nash equilibrium when group size increases

Therefore, (1) as group size increases, individuals increasingly free ride by reducing individual contribution (i.e. the average personal contribution falls); (2) at the same time, the total value of the contributions made by the population (including a newly added member) increases.

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3. Non-Nash Behaviour:

Unlike Nash or Cournotbehaviour where $\frac{dG_i}{dg_i} = 0$, known as 'zero conjectural variation' , The public goods model with Non-Nash behaviour is characterized by 'non-zero conjectural variation' i.e. $dG_i^e/dg_i \neq 0$,where G_i^e is the expected value of Gi . It implies that, the expectations of an agent is that the public good's provision of his own will have an influence, positive or negative on the provision of the community. If we talk of a symmetric equilibrium and in its neighborhood, where an identical share is being contributed by each agent, then the conjectural variation that we are talking about will depend on the size of the community, n. As it appears, the larger the values of the size 'n', the smaller becomes the share of the typical individual, and as a result the conjectural variation will become smaller, numerically.

$$\frac{dG_i^e}{dg_i} = f(\alpha, q) \quad \text{...(xx)}$$

where α is a parameter or vector of parameters. This may include n, and its influence on the anticipated responsiveness of the other members to the public good provision g_i of an agent . Speaking in a general parlance, an integration of equation (xx) can be done to find an endogenous relationship for

G_i^e : $\quad G_i^e = F(\alpha, g_i k)$ ....(xxi)

where k represents the integration constant which is dependent on the initial conditions means the actual value or the initial value for Gi)

Seeing the problem at the individual level-

Max $U^i = U^i(x_i, Q^e)$

$= U^i(x_i, Q^e)$

$= U^i[x_i, F(\alpha, g_i k) + g_i]$

Subject to $y_i = x_i + P_G g_i$

Applying Lagranges method,

Maximization: $\quad Z = U^i[x_i, F(\alpha, g_i, k) + g_i] + \lambda[y_i - (x_i + P_G g_i)]$ ....(xxii)

Here the First order condition is-

$$=> \frac{\partial z}{\partial g_i} = U_g^i(F^1 + 1) - \lambda P_G = 0 \quad \text{...............(xxiii)}$$

$$\text{And } \frac{\partial z}{\partial x_i} = U_x^i - \lambda = 0 \quad \text{................(xxiv)}$$

$$=> \frac{U_g^i(F^1 + 1)}{U_x^i} = P_G \quad \text{................................(xxv)}$$

$$=> \frac{U_g^i}{U_x^i}\left(\frac{dG_i^e}{dg_i} + 1\right) = P_G \quad \text{......................(xxvi)}$$

Given this, the condition of optimality turns out to become that of Nash when $\frac{dG_i^e}{dg_i} = 0$, otherwise the conjectural variation is an additive weightage factor applied to the MRS when we determine the optimal behaviour. If the second order condition is satisfied, then $d^2z < 0$. It implies that in the neighborhood of the point characterized by (xxvi), the expectation contours $G_i^e = F(\alpha, g_i k)$ have curvature less than the same of the indifference curves, therefore the contours touche the indifference curve from below.

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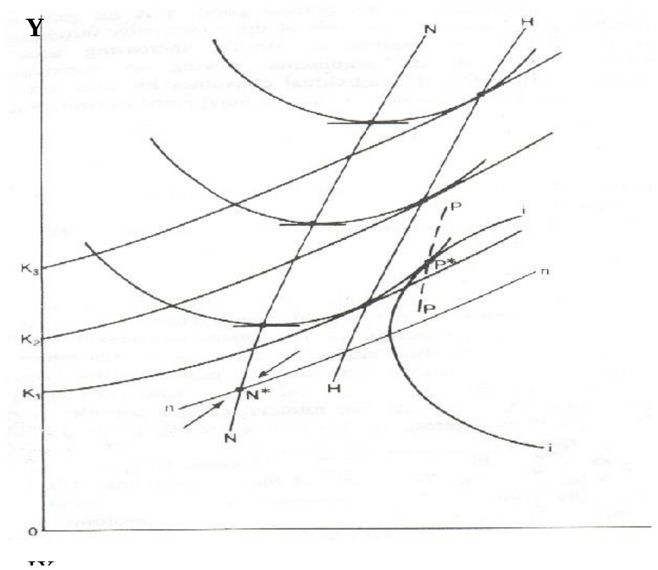

Figure 6. Nash, Non-Nash and Pareto Optimal Equilibrium

In Figure 6 curves, $\underset{1}{K}G_i^e, \underset{2}{K}G_i^e$ and $\underset{3}{K}G_i^e$ represent expectations paths for G_i^e, each differing by a constant. Their slopes $\frac{dG_i}{dg_i} > 0$, being the conjectural variation. The locus of points of tangencies between the indifference curves and the expectations paths give birth to the hybrid reaction path, HH. The reaction path are found to be located to the right side of the Nash path, and at the same time will lie closer to the locus of points indicating Pareto-optimal allocations. Basing on this if we make a comparison with the Nash equilibrium, then what emerges out of this is that individuals will naturally feel encouraged to supply more of the public good if they expect same type of matching behaviour from other members of the community.

With the help of a specific presentation of the conjectural variation we can depict an interesting and realistic type of behavior as follows:

$$dG_i^e \big/ dg_i = \left(\frac{g_i}{G_i}\right)^{\alpha} \quad \text{………………………….(xxvii)}$$

The parameter α has an interpretation which is simple and precise too

$$\therefore \mathrm{Ln}\left(\frac{dG_i^e}{dg_i}\right) = \alpha\, \mathrm{Ln}\left(\frac{g_i}{G_i}\right)$$

$$=> \frac{d\mathrm{Ln}\left(\frac{dG_i^e}{dg_i}\right)}{d\mathrm{Ln}\left(\frac{g_i}{G_i}\right)} = \alpha$$

It implies that the α represents the elasticity of the conjectured response in terms of the relative importance of the contribution at the individual level.

Seeing in terms of a symmetrical standpoint, the conjectural variation turns out to be

$$\frac{dG_i}{dg_i} = \left(\frac{1}{n-1}\right)^{\alpha} = (n-1)^{-\alpha} \quad \text{………………(xxviii)}$$

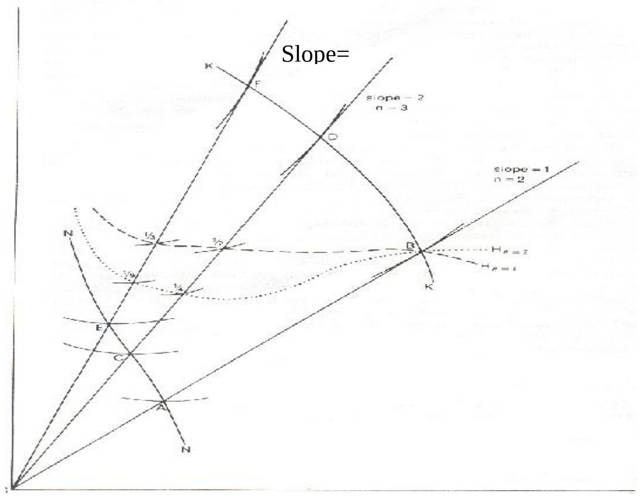

Figure 7. Non-Nash solution between Kantian and Nashianbehaviour

If the Figure7 is anything to be assumed, then if everyone is identical, the Pareto path corresponds to something called the Kantian behavior, as the 'categorical imperative', says that each agent acts as they want others to act, is satisfied. When α=-1 in (xxviii)-: there is the generation of the Kantian path. Given this, other values of α leads to hybrid types or Non-Nash types solutions, which come between the Kantian type and the Nashian type behaviour. Seeing in terms of an example, if α=1, the dotted

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line in Figure 7 shows the Non-Nash reaction curve as the group size changes. Since $\frac{dG_i}{dg_i} = (n-1)^{-1}$, equilibrium for n=3, for instance corresponds to the highest indifference curve as shown with slope of half on the ray with slope two. On the other hand, if the size of the group is 4, the position of equilibrium corresponds to the position on the ray with slope three with the indifference curve has its slope as one-third.

As the 'n' approaches to infinity, we find that the hybrid path, as shown converges to that of the Nash. This convergence holds for all $\alpha \geqq 1$. Further, it implies that the larger is the value of α, the faster becomes the rate of convergence as we can observe by seeing the dashed reaction path in Figure 7, which corresponds to a value of α=2.

CONCLUSION:

As can be inferred from the Oligopoly literature [e.g. Bresnahan (1981), Perry (1982)], a conjectural variation is consistent only when it is identical to the optimal response of the other agent(s) at the point of equilibrium which is based on the conjecture itself. In case there are two agents, a conjecture becomes consistent only when the slope of each agent's reaction path equals the corresponding conjectural variation held by the other agent at the point of equilibrium.

$$\therefore \frac{dg_h}{dg_i} = \frac{dg_h}{dG} \cdot \frac{dG}{dg_i} \left(dG_i = d\underset{h}{G} = dG \right)$$

$$= \frac{dG}{dg_i} \cdot \frac{1}{\frac{dG}{dg_h}} = (n-1)^{-\alpha} \cdot \frac{1}{\frac{1}{(n-1)^{-\alpha}}}$$

$$= (n-1)^{-2\alpha} \quad \text{………………………………….(xxix)}$$

Table-1

	$\alpha \to \infty$	α=1	α= -1
$\frac{dG_i}{dg_i}$	0	$(n-1)^{-1}$	$(n-1)$
$\frac{dg_h}{dg_i}$	0	$(n-1)^{-2}$	1
Remarks	Converges to Nash very quickly	Converges to Nash as n $\to \infty$,Kantian behaviour when n=2	Kantian behaviour for all n.

From Table 1 it is observed that

(i) when α=-1, Kantian behaviour prevails irrespective of the involvement of the number of persons in the group that is contributing.

(ii) When α=1, there will be Kantian behaviour, provided the number of contributors are two in the group.

(iii) When α=1, for n>2 and n$\to \infty$, there is the convergence of Non-Nash equilibrium to the Nash equilibrium.

(iv) When $\alpha \to \infty$, Non-Nash equilibrium converges to Nash very quickly.

As can be seen in the Figure 7, if we compute the ratio of distances from the origin between the Nash and the Pareto points along a given ray, we get an index of easy riding derived (e.g. OA/OB, OC/OD and OE/OF). More easy riding is indicated if the ratios are smaller; there is no easy riding in case of a ratio of 1. But in case the ratio is zero, it is a situation of free ride completely.

Further, the hybrid path converges to that of the Nash as 'n' approaches infinity. The fact of the matter then is, the 'Olson conjecture' associated

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with free riding increases with the size of the group which can be inferred from the very formulation itself. Further in Section-I, it is proved that as group size increases, individuals increasingly free ride by reducing individual contribution which lead to the reduction of the average personal contribution but the aggregate contributions made by the population increases on the Nash equilibrium path. This might cause the convergence between NN and KK, in Figure 7 demonstrating the possibility that easy riding may decrease with community size.

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