TEST Engineering & Management

*November-December 2019*
*ISSN: 0193-4120 Page No. 3848 - 3851*

# Rough Algebraic Structures in Approximation Space

K. Anitha

Department of Mathematics, SRM Institute of Science and Technology, India

*anithamaths2019@gmail.com*

***Article Info***
***Volume 81***
***Page Number: 3848 - 3851***
***Publication Issue:***
***November-December 2019***

***Article History***

***Article Received:*** *5 March 2019*
***Revised:*** *18 May 2019*
***Accepted:*** *24 September 2019*
***Publication:*** *19 December 2019*

***Abstract:***
Rough groups are framed by approximation spaces of Rough set. This Paper generalises the properties of upper and lower approximation in algebraic platform.

***Keywords:*** *Rough Approximation, Normal Subgroup, Upper, Lower Subgroup*

***MSC*:** 03E99, 20A05, 22A99

## I. INTRODUCTION

The theory of Rough set was proposed by Pawlak in 1982. For Mathematical Reason he considered equivalence relation to define this Set. It is the extension of crisp set in which subset of universal set is described by pair of crisp sets called upper and lower approximation. Yao used constructive and algebraic methods to generalise Pawlak Rough set model. Biswas and Nanda introduced Rough Subgroups. Nurettin Bağırmaz et al introduced the concepts of Topological Rough Group. In their paper they combined topological space and Rough Groups. Paul Isaac and Ursala Paul introduced the notion of G-Module along with Rough Homomorphism in G-Module. Cheng and Iwinski defined the properties of lower and upper approximation spaces. Zhaohao Wang and Lan Shu gave some equivalent conditions for Rough Subgroups and gave the Rough version of Lagrange's Theorem. This paper analyses the properties of Rough approximations in group theory. The concept of Rough set starts with Indiscernibility Relation, which is an equivalence relation defined by

**Indiscernibility**: For any $\mathfrak{R} \subseteq A$ there is an equivalence relation $IND(\mathfrak{R})$ which is defined as

$$IND(\mathfrak{R}) = \{ (x,y) \in U^2 / \forall\, a \in P , a(x) = a(y)\} \qquad (1.1)$$

## II. PRELIMINARIES

**Definition 2.1 Approximation Space**

A pair $(U, \mathfrak{R})$, where $U$ is a non-empty set and $\mathfrak{R}$ is an equivalence relation defined on $U$ is called an Approximation Space.

**Proposition (2.1):** The Set $IND(\mathfrak{R}) = \{ (x,y) \in U / \forall\, a \in \mathfrak{R} , a(x) = a(y)\}$ forms Partition of $U$.

Proof: The Partition of $U$, determined by $IND(\mathfrak{R})$ is denoted by $U/IND(\mathfrak{R})$, which is the set of equivalence classes generated by $IND(\mathfrak{R})$.The Partition characterizes aapproximation space $(U, \mathfrak{R})$,

November-December 2019
ISSN: 0193-4120 Page No. 3848 - 3851

**Definition 2.2 Upper and Lower Approximation**

From the Approximation Space $(U, \mathfrak{R})$ let $X \in U$. The lower and approximation spaces are defined by the following terms

$$\underline{X} = \{x \in U \colon [X]_{\mathfrak{R}} \subseteq X\} - - - - - - - - -(Lower\ Approximation\ of\ X\ in\ (U, \mathfrak{R})$$

$$\overline{X} = \{x \in U \colon [X]_{\mathfrak{R}} \cap X \neq \emptyset\} - - - -(Upper\ Approximation\ of\ X\ in\ (U, \mathfrak{R})$$

$$BN_{\mathfrak{R}}(X) = \overline{X} - \underline{X} \qquad - - - - - - - - -(Boundary\ Region\ \ of\ X\ in\ (U, \mathfrak{R})$$

If $BN_{\mathfrak{R}}(X) = \emptyset$ the set $X$ is Crisp, otherwise it is Rough.

The upper and lower approximations have the following properties:

(i) $\underline{X} \subseteq \bar{X}$

(ii) $\underline{X}(\emptyset) = \overline{X(\emptyset)} = \emptyset$ , $\underline{X}(U) = \overline{X(U)} = U$

(iii) $\bar{X}(AUB) = \bar{X}(A)U\bar{X}(B)$ , $\underline{X}(A \cap B) = \underline{X}(A) \cap \underline{X}(B)$

(iv) $\underline{X}\left(\underline{X}(A)\right) = \bar{X}(\underline{X}(A) = \underline{X}A$

The following Fig.2.1 is the diagrammatic representation of Rough approximations

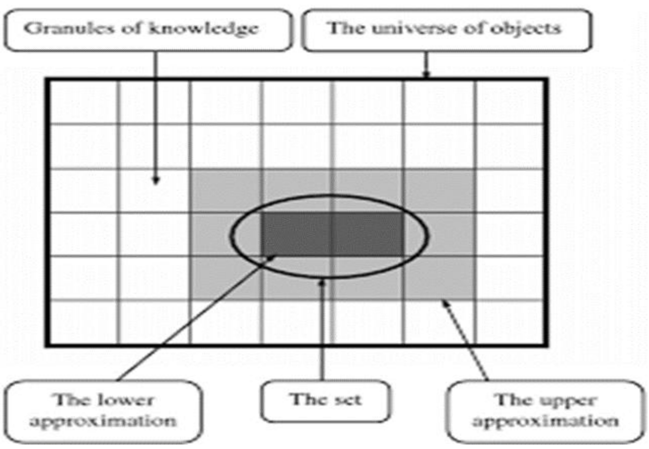


**Fig:2.1** The lower and upper Approximations to the concept in grey are marked in thick contours on a grid of equivalence classes

**Rough Topology**: Let$(U, \tau)$ be a Topological space and and let $R(\tau)$ be the equivalance relation on $P(U)$ defined as $(A, B)\ \epsilon\ R(\tau) \Leftrightarrow \{A^I = B^I\ \ and\ A^C = B^C\}$. A topology $\tau$ is Quasi-discrete if the sets in $\tau$ are Clopen ( Krieger and Sierpenski 1956). The following are the properties of Rough Topology(Raghavan and Tripathy , 2011):

(i) The members of $\tau\ are$ called $Open\ Sets$.

(ii) The complement of $Open\ Sets$ are Called $Closed\ Sets$

(iii) The Interior of X $\epsilon U$ is the largest Open Subset of X and it is denoted as $X^I$

(iv) The Closure of X $\epsilon U$ Is the smallest closed subset that includes $X$ and it is denoted as $X^C$

(v) A Topological Rough Set in $(U, \tau)$ is the Element of the Quotient set $P(U)/\mathfrak{R}(\tau)$ Let $X^C$ , $X^I$ and $X^B$ be Closure, Interior and Boundary Points respectively and they satisfy the following axioms:

(i) If $X^B = \emptyset$, then $X$ is Exact otherwise it is said to be Rough.

(ii) If $X^C = X = X^I$ , then $X$ is Totally Definable.

(iii) If $X^C \neq X$ , $X^I \neq X$ then $X$ Undefinable.

(iv) If $X^B \neq X$ , $X^C = X$ then X is Externally Definable.

(v) If $X^B = X$ , $X^C \neq X$ then X is Internally Definable.

**Proposition (2.2)**: If $A$ is Exact in $(U, \tau)$ and $\tau \subset \tau'$ then $A$ is Exact with respect to $\tau'$

Proof :If A is Exact means then $X^B = \emptyset$ in $(U, \tau)$

Since $\tau \subset \tau'$ , $X^B = \emptyset$ in $(U, \tau')$ and hence $A$ is Exact with respect to $\tau'$

November-December 2019
ISSN: 0193-4120 Page No. 3848 - 3851

**Definition 2.3 Rough Group**

Let $(U, \mathfrak{R})$ be the Approximation Space and $*$ be the binary operation defined on it. Let $G$ be a subset of $U$. If $G$satisfies the axioms of Group then it is called Rough Group.

**Remark**

(i) Let $H \subseteq$ G and if $H$ satisfies the axioms of group then it is called Rough Subgroup.

(ii) If $G$satisfies commutative group then it is called Commutative Rough Group.

**Definition 2.4 Rough Normal Subgroup**

Let $N$be the Normal subgroup of $G$and $K \subseteq G$ . Then approximations of $K$with respect to N are defined as follows

$$\underline{N_K} = \{x \in G : xN \subseteq K\}$$

$$\overline{N_K} = \{x \in G : xN \cap K \neq \emptyset\}$$

If $\underline{N_K}$&$\overline{N_K}$ are subgroups of $G$, then $K$ is called Lower and Upper Normal Rough Sub groups of $G$.

## III. PROPERTIES OF UPPER AND LOWER APPROXIMATIONS IN GROUP

**Proposition 3.1.**

Let $N$ be the Normal subgroup of, $e \in K$ and $K \subseteq G$. Then $N \supseteq \underline{N_K}$

**Proof:** Since $e \in G \Rightarrow e \in K$ & $xN \subseteq K \Rightarrow N \supseteq \underline{N_K}$

**Proposition 3.2.**

Let $N_1$&$N_2$ are the Normal subgroups of , $e \in K$ and $K \subseteq G$. Then

$$\underline{[N_1 N_2]_K} \supseteq \underline{[N_1]_K [N_2]_K}$$

**Proof:**

Since$\underline{[N_1]_K} = KN_1$, $\underline{[N_2]_K} = KN_2 \Rightarrow \underline{[N_1 N_2]_K} = KN_1 N_2$

$xN_1 \subseteq K$, $xN_2 \subseteq K, e \in K$ & $N$ *is Normal*
$\Rightarrow \underline{[N_1 N_2]_K} \supseteq \underline{[N_1]_K [N_2]_K}$

**Proposition 3.3.**

Let $N$ be the Normal subgroup, $N_1$- Lower Rough Subgroup$N_2$ - Lower Normal subgroup then $N_1 N_2$ is the Lower Rough Normal Subgroup.

**Proof:** From the definition of Upper and Lower Rough Subgroup $N(\underline{N_1})$ is lower subgroup and $N(\underline{N_2})$ is Lower rough Normal Subgroup.

Hence $N(\underline{N_1})N(\underline{N_2}) = N(\underline{N_2})N(\underline{N_1}) \Longrightarrow N_1 N_2$is Lower Rough Normal Subgroup.

**Definition 3.1**. Let $K$ be the subgroup of $G$&$S \subseteq G$. Then $S$ is called Transversal of $K$ *in* $G$ if $K$ contains exactly one element of every right coset$Hx$, $x \in G$.

**Definition 3.2**. Let $N$ be the Normal subgroup of $G$ and $K$ be the Upper Rough Subgroup of$G$. Then $K$ is called Minimal Upper Rough Subgroup of G if for all $K_1 \subset K$ we have

$\overline{N(K)} \neq \overline{N(K_1)}$. Similarly for Minimal Lower Rough Subgroup.

**Proposition 3.4.**Let $N$ be the Normal subgroup of $G$ and $K$ be the Lower Rough Subgroup of $G$. Then $K$ is minimal lower Rough subgroup if $|K|/|G|$

**Proof:** Since $K$ is minimal lower rough subgroup there exists the transversal $S$ of $K$ in $G$ such that $S \subseteq K$.

$\langle K, N \rangle = \bigcup\{sN : s \in S\} = SN$ = which is a subgroup

$$SN = \langle K, N \rangle \Longrightarrow KN = SN \Longrightarrow \underline{N(K)} = \underline{N(S)}$$

Hence $S = K \Longrightarrow K$ is the minimal lower subgroup of G$\Longrightarrow |K|/|G|$

**Proposition 3.5.** Let $N$be the Normal subgroup of G and $K$ non-empty subset of $G$. Then the following assertions are equal.

(i) $K$ is an Lower Normal Subgroup of $G$

(ii) $KN$ is the subgroup of $G$ and for all $x \in G, k \in K, xkx^{-1} \in KN$

**Proof:** (i)$\Longrightarrow (ii)$ If $N$ is the Normal subgroup $\underline{N(K)}$ = KN $\Longrightarrow KN$ *is* Normal Subgroup

By the property of Normal Subgroup for all $x \in G, k \in K, xkx^{-1} \in KN$

$(ii) \Longrightarrow (i)$ Let $x \in G, y \in KN$

Then there exists $k \in K, n \in N$ such that $y = kn$

$$xyx^{-1} = xknx^{-1} = (xkx^{-1})(xnx^{-1})$$

Since $N$ is Normal, $xnx^{-1} \in N \Longrightarrow KN$ *is* a Subgroup

$$xyx^{-1} = (xkx^{-1})(xnx^{-1}) \in KN \Longrightarrow KN \text{ is Normal Subgroup}$$

**Corollary 3.1**

(i) Let $N$ be a normal subgroup of a group $G$ and $K$ a non-empty subset of G. If$KN$ is a normal subgroup of $G$, then $\forall x \in G, KxN = KN$.

(ii) Let $N$ be a normal subgroup of a group $G$, Kbe a non-empty subset of $G$ and $x \in G$. Then $K$ is a Lower rough normal subgroup of $G$ if and only if $Kx$ is a Lower Rough Normal Subgroup of $G$.

(iii) Let $N$ be a normal subgroup of a group $G$ and $K$ a nonempty subset of $G$. If$K$ is a Upper Rough Normal Subgroup of G, then$\overline{N(K)} = \bigcup_{x \in G} \overline{N(K^x)}$

## IV. CONCLUSION

Rough Set Theory is a tremendous mathematical tool in both pure and applied mathematics. The analyzation of Rough Set properties in group theory is a meaningful research. In this paper we have exhibited the properties of Rough Group through upper and lower approximation spaces. The extended work of these studies may be proceed in the direction of analysing Rough Homomorphism and Rough Ideals.

## V. REFERENCES

[1] Anitha. K. Thangeswari. M, A Short Note On Properties Of Rough Ideals, Journal of Applied Science and Computations, JAN/2019 DOI:16.10089. JASC.2018. V6I1.453459.1500473

[2] Biswas .R, Nanda.S, Rough groups and rough subgroups, Bull. Polish Acad. Sci. Math. 42(1994)251-254

[3] Cheng.W, Mo.Z, Wang.J, Notes On The Lower And Upper Approximations In A Fuzzy Group And Rough Ideals In Semigroups, Inform.Sci.177(2007)5134-5140

[4] Iwinski. T, Algebraic Approach To Roughsets, Bull. Polish Acad.Sci.Math.35(1987) 673–683.

[5] Nurettin Bagirmaz, Ilhan Icen, Abdullah F.Ozcan, Topological Rough Groups, Topol. Algebra Appl.(2016);4:31–38

[6] Paul Isaac, Ursala Paul, Rough G-modules and their properties, Advances in Fuzzy Mathematics,(2017), 12(1),93-100

[7] PawlakZ, Rough sets, Int. J. Comput. Inform. Sci. 11(1982)341-356.

[8] YaoY.Y., Constructive and algebraic methods of the theory of rough sets, Inform. Sci. 109(1998)21-44.

[9] Zhaohao Wang, Lan Shu, The lower and upper approximations in a group, International Journal of Mathematical and Computational Sciences, Vol:6, No:8, 2012.