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Analysis of Fuzzy Tandem Terror Queues

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Abstract

Fuzziness gravels an astounding fragment to modify the state of perplexity correlated with vague data. Terror queues network looks for terror plots in preparatory work and when they discover one, the undercover agents try to detect/demolish the conspiracy. This paper depicts a tandem framework of Terror queue model performance measures like expectations where terror plots are 'customers' and group of agents/informants are 'servers'. Fuzzy tandem terror queues are discussed on the basis of Baurah's Randomness-Fuzziness consistency principle involving Zadeh's extension principle with parametric programming approach upon which the queue models represents fuzzy entry and exit rate of customers.

Keywords; *Fuzzy sets, fuzzy queues, parametric programming, terror plots, distribution function.*

I. INTRODUCTION

The spectacular extension of queues to fuzzy world have wide-ranging real life implications in decision analysis, operations research, computer technologies, science and abstract theories. The perception of fuzziness is one of the enhancement, not of auxiliary.

The origin of fuzzy queues throws magnificient stretchability in all research grounds. Many explorers rooted impact in fuzzy waiting line paradigms like Li R.J. and Lee E.S. [7], Negi D.S. and Lee E.S. [11] introduced α-cut and two random variable simulation approach for studying queues in uncertain environs, Chiang Kao, Chang-Chung Li, Shih-Pin Chen [6], applied parametric programming approach to fuzzy variables with its problem formulation, Nagoor Gani A. and Ritha W. [10] have discussed fuzzy tandem queues and also analyzed fuzzy queues using parametric programming. Dhruba Das, Hemanta K. Baruah [3], constructed parametric programming approach to two fuzzy waiting line conception based on the Randomness-Fuzziness consistency principle.

In 2010, Edward H. Kaplan [4], published a paper on 'Terror queues' and in 2013 investigated on different staffing models for counter terrorism deeply-rooted on queuing theory. He has researched a tandem version of the terror queue model. The terror plots in preparatory work are treated as 'customers' in a network of two queues. One queue includes all undetected terror plots, after detection they move to the other queue of detected terror plots until they are blocked. Kaplan presumes that the completion, detection and interdiction times of terror plots are exponentially distributed. The consequence of all terror plots either lead to attacks, or are detected and interdicted.

This paper intends to find the membership functionality portraying the performance mechanism concurrent to the designated principle associated to the entry and service level of the consumers.

II. PRELIMINARIES

Notations used are:

$X(t)$: Number of undetected terror plots at time t.

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$Y(t)$: Number of spotted, but not prohibited plots.
f : Total number of trained workers/service agencies.
λ : Arrival time between two subsequent terror plots.
δ : Detection intensity.
μ : Time until a plot is executed.
δf : Detection time.
$\mu + \delta f$: Service time of a terror plot in queue X.
ρ : Interdiction of a terror plot.
$\frac{\mu M}{\mu+\delta f}$: Expected amount of terror attacks per unit time.
$\frac{\lambda\delta f}{\mu+\delta f}$: Departure process for detected terror plots.

III. THE RANDOMNESS-FUZZINESS CONSISTENCY PRINCIPLE

A normal fuzzy number N_{FN} = [a, b, c] with its membership functionality is expressed by

$$\mu_{FN}(x) = \begin{cases} \omega_1(x), & \text{if } a \le x \le b \\ \omega_2(x), & \text{if } b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

with $\omega_1(a) = \omega_1(c) = 0$ and $\omega_1(b) = \omega_2(b) = 1$ with $\omega_1(x)$ being a increasing function in $a \le x \le b$ and $\omega_2(x)$ is a decreasing function in $b \le x \le c$ along with a stationary reference function 0 in real line.

The field of uncertainty is framed as

$$\{x, \mu_{FN}(x), 0 : x \in R\}$$

A fuzzy quantity with its membership functionality $\mu_{FN}(x)$, in the Dubois-Prade nomenclature, $\omega_1(x)$ and $\omega_2(x)$ are designated as left and right reference functions.

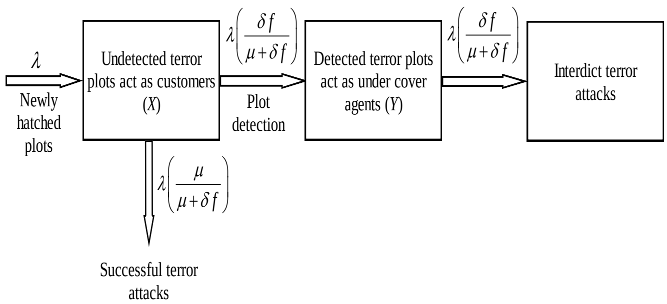
IV. TANDEM TERROR QUEUE MODEL

The tandem terror queue model is an interconnected structure of two M/M/∞ queues in tandem.

Terror queue is a part of "Intelligence Operation Research". The major contribution for this model was done by Edward H. Kaplan which aims to reduce the proportion of successful horror attacks.

The intelligence agents are partitioned into two groups: (i) the detecting team searching for non-detected terror plots and (ii) prevention team attempts to inderdict detected terror plots.

The queues of non-detected (X) and discovered (Y) terror plots can be described as M/M/∞ queues organised in tandem. Terror plots (customers) arrive at queue X based on Poisson process with parameter λ. Undetected terror plots exit queue X as a result of completing their preparation based on Poisson process with parameter $\frac{\lambda\mu}{\mu+\delta f}$. The existing customers (terror plots) leave queue X and move to Y. The detected terror plots (Y) leave queue Y based on arrival process, in accordance to Poisson process with parameter $\frac{\lambda\delta f}{\mu+\delta f}$.



Tandem Terror Queue Model

The FM/M/∞ Queue

Consider a network of two queues, in which the terror plots in preparation enter in concert with Poisson process with a fuzzy parameter $\tilde{\lambda}$. The time until a plot is executed and detected are exponentially distributed with parameter μ and δf. The service time of a terror plot in queue X is $\mu + \delta f$. The queue X has Poisson arrivals and exponentially distributed service time which indicates queue X as an M/M/∞ queue. The departure process for detected

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terror plots follows Poisson process with parameter $\lambda\left(\dfrac{\delta f}{\mu+\delta f}\right)$. Interdiction of a terror plot takes an exponentially distributed time with parameter ρ.

The membership function is

$$\phi_{f(\tilde{\lambda},\mu+\delta f)}(\omega)=\sup\left\{\phi_{\tilde{\lambda}}(\varepsilon)/\,\omega=f(\varepsilon,\mu+\delta f)\right\}$$

where the fuzzy arrival rate $\tilde{\lambda}$ is defined as

$$\tilde{\lambda}=\left\{(\varepsilon,\phi_{\tilde{\lambda}}(\varepsilon))\,|\,\varepsilon\in R^{+}\right\}$$

where $\phi_{\tilde{\lambda}}(\varepsilon)$ is the membership functionality of the entry level of consumers and $f(\varepsilon,\mu+\delta f)$ is the production measure.

The α-cut of $\tilde{\lambda}$ is $\lambda(\alpha)=\left\{\varepsilon\in R^{+}\,|\,\phi_{\tilde{\lambda}}(\varepsilon)\geq\alpha\right\}$

where $\lambda(\alpha)$ is an ordinary set and can be described as $\left[\min\left\{\varepsilon/\phi_{\tilde{\lambda}}(\varepsilon)\geq\alpha\right\},\max\left\{\varepsilon/\phi_{\tilde{\lambda}}(\varepsilon)\geq\alpha\right\}\right]$

By convexity of a fuzzy number, the interval constraints are functions with respect to α denoted as $C^{L}_{\lambda(\alpha)}=\min\cdot\phi_{\tilde{\lambda}}^{-1}(\alpha)$ and $C_{\lambda(\alpha)}U=\max\cdot\phi_{\tilde{\lambda}}^{-1}(\alpha)$

To find the interval bounds for $f(\tilde{\lambda},\mu+\delta f)$ at possibility level α the mathematical programs are formulated as:

$$C^{L}_{f(\alpha)}=\min\cdot f(\varepsilon,\mu+\delta f)\quad \text{s.t.}\quad C^{L}_{\lambda(\alpha)}\leq\varepsilon\leq C^{U}_{\lambda(\alpha)}$$

$$C^{U}_{f(\alpha)}=\max\cdot f(\varepsilon,\mu+\delta f)\quad \text{s.t.}\quad C^{L}_{\lambda(\alpha)}\leq\varepsilon\leq C^{U}_{\lambda(\alpha)}$$

Suppose $C^{L}_{f(\alpha)}$ and $C^{U}_{f(\alpha)}$ are subjected to inversion with respect to α, then $L(\omega)=C^{L^{-1}}_{f(\alpha)}$ and $R(\omega)=C^{U^{-1}}_{f(\alpha)}$ can be determined where $L(\omega)$ and $R(\omega)$ are leftward and rightward reference functions of the membership functionality $\phi_{f(\tilde{\lambda},\mu+\delta f)}$.

Based on the designated uncertainty principle, the leftward reference function $L(\omega)$ is a distribution function and the rightward reference function $R(\omega)$ is a complementary dispersion.

$$\phi_{f(\tilde{\lambda},\mu+\delta f)}(\omega)=\begin{cases}L(\omega), & \omega_1\leq\omega\leq\omega_2\\ R(\omega), & \omega_2\leq\omega\leq\omega_3\\ 0, & \text{otherwise}\end{cases}$$

such that $L(\omega_1)=R(\omega_3)=0$

$$L(\omega_2)=R(\omega_2)=1$$

with stationary reference function zero, $L(\omega)$ is the distribution function in $\left[\omega_1,\omega_2\right]$ and $R(\omega)$ is the complementary distribution function in $\left[\omega_2,\omega_3\right]$

The predicted number of non-detected and discovered terror plots is

$$E\left[X,Y\right]=\left(\frac{\tilde{\lambda}}{\mu+\delta f},\frac{\tilde{\lambda}\delta f}{\rho(\mu+\delta f)}\right)$$

The membership functionality for the predicted number of non-detected and detected terror plots FM/M/∞ terror queuing model is

$$\phi_{E(X,Y)}(\omega)=\sup\left\{\phi_{\tilde{\lambda}}(\varepsilon)/\,\omega=\left(\frac{\tilde{\lambda}}{\mu+\delta f},\frac{\tilde{\lambda}\delta f}{\rho(\mu+\delta f)}\right)\right\}$$

where $\phi_{\tilde{\lambda}}(\varepsilon)$ is the membership function of $\tilde{\lambda}$.

NUMERICAL EXAMPLE

Consider a terror queue with fuzzy arrival rate $\tilde{\lambda}=\left[99,100,101\right]$, $\mu=1$, $\delta=0.1$, $\rho=4$, $f=30$. The confidence interval is $\left[\alpha+99,101-\alpha\right]$. The parametric constraints to construct the membership functionality for $\tilde{E}(X,Y)$

$$C^{L}_{E(\alpha)}=\min\cdot\left(\frac{\tilde{\lambda}}{\mu+\delta f},\frac{\tilde{\lambda}\delta f}{\rho(\mu+\delta f)}\right)$$

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Subject to: $\alpha+99\leq\tilde{\lambda}\leq 101-\alpha$

$$C^U_{E(\alpha)}=\max\left(\frac{\tilde{\lambda}}{\mu+\delta f},\frac{\tilde{\lambda}\delta f}{\rho(\mu+\delta f)}\right)$$

Subject to: $\alpha+99\leq\tilde{\lambda}\leq 101-\alpha$

when $\tilde{\lambda}$ procures its lower bound, $\tilde{E}(X,Y)$ acquires its minimum.

Hence the optimum solution is

$$C^L_{E(\alpha)}=\left[\frac{\alpha+99}{4},\frac{3\alpha+297}{16}\right]$$

when $\tilde{\lambda}$ reaches its upper bound, $E(X,Y)$ attains its maximum with its optimal solution as

$$C^U_{E(\alpha)}=\left[\frac{101-\alpha}{4},\frac{303-3\alpha}{16}\right]$$

Then there exists invertible functions $C^L_{E(\alpha)}$ and $C^U_{E(\alpha)}$ generating the membership functionality $\mu_{\tilde{E}(X,Y)}(\omega)$

$$\mu_{\tilde{E}(X,Y)}(\omega)=\begin{cases}4\omega_1-99,\frac{16\omega_2-297}{3}, & 24.5\leq\omega_1\leq 25 \text{ and } 18.56\leq\omega_2\leq 18.75\\ 101-4\omega_1,\frac{303-16\omega_2}{3}, & 25\leq\omega_1\leq 25.25 \text{ and } 18.75\leq\omega_2\leq 18.9\\ 0, & \text{otherwise}\end{cases}$$

The M/FM/∞ Queue

Consider a M/FM/∞ server queueing system where the service rate with fuzzy parameter $\tilde{\mu}$ follows Poisson process and there exists a possibility distribution with the fuzzy parameter $\tilde{\mu}$ and arrival rate follow Poisson process with crisp parameter λ. The fuzzy queue M/FM/∞ is a normal queue modelled as an M/M/∞ queue with its membership function as

$$\phi_{f(\lambda,\tilde{\mu}+\delta f)}(\omega)=\sup_{\varepsilon\in R^+}\left\{\phi_{\tilde{\mu}+\delta f}(\varepsilon)/\omega=f(\lambda,\varepsilon)\right\}$$

The fuzzy service rate is defined as

$$\tilde{\mu}+\delta f=\left\{(\varepsilon,\phi_{\tilde{\mu}+\delta f}(\varepsilon)/\varepsilon\in R^+\right\}$$

where $\phi_{\tilde{\mu}+\delta f}(\varepsilon)$ represents the membership functionality of the servicing level and $f(\lambda,\varepsilon)$ is the performance measure.

The *alpha* cut is $\mu(\alpha)=\left\{\varepsilon\in R^+/\phi_\mu(\varepsilon)\geq\alpha\right\}$ where $\mu(\alpha)$ an ordinary set.

It can be expressed as

$\left[\min\left\{\varepsilon/\phi_\mu(\varepsilon)\geq\alpha\right\},\max\left\{\varepsilon/\phi_\mu(\varepsilon)\geq\alpha\right\}\right]$ which lies at possibility level α.

The interval of bounds, by convexity of a fuzzy number is

$$C^L_{\mu(\alpha)}=\min\phi^{-1}_\mu(\alpha) \text{ and } C^U_{\mu(\alpha)}=\max\phi^{-1}_{\mu(\alpha)}$$

The mathematical programs parametered by possibility level α is

$$C^L_{f(\alpha)}=\min f(\lambda,\varepsilon) \quad \text{s.t.} \quad C^L_{\mu(\alpha)}\leq\varepsilon\leq C^U_{\lambda\mu(\alpha)}$$

$$C^U_{f(\alpha)}=\max f(\lambda,\varepsilon) \quad \text{s.t.} \quad C^L_{\mu(\alpha)}\leq\varepsilon\leq C^U_{\mu(\alpha)}$$

where $C^L_{f(\alpha)}$ and $C^U_{f(\alpha)}$ are invertible corresponding to α. By fuzziness randomness consistency principle,

$$\phi_{f(\lambda,\tilde{\mu})}(\omega)=\begin{cases}L(\omega), & \omega_1\leq\omega\leq\omega_2\\ R(\omega), & \omega_2\leq\omega\leq\omega_3\\ 0, & \text{otherwise}\end{cases}$$

where $L(\omega_1)=R(\omega_3)=0$ and $L(\omega_2)=R(\omega_2)=1$ with constant reference function 0.

Numerical Example

Consider an M/FM/∞ queue where the arrival time is exponentially distributed with mean $\lambda=100$ and fuzzy service rate $\tilde{\mu}=[3,4,5]$, $\delta=0.1$, $f=30$ and $\rho=4$. The confidence interval is $[\alpha+3,5-\alpha]$. The

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parametric constraints to construct the membership functionality for $\tilde{E}(X,Y)$

$$C^{L}_{E(\alpha)} = \min\left[\frac{\lambda}{\tilde{\mu}+\delta f}, \frac{\lambda\delta f}{\rho(\tilde{\mu}+\delta f)}\right] \quad ; \quad \text{s.t:}$$

$\alpha + 3 \le \tilde{\mu} \le 5 - \alpha$

$$C^{U}_{E(\alpha)} = \max\left[\frac{\lambda}{\tilde{\mu}+\delta f}, \frac{\lambda\delta f}{\rho(\tilde{\mu}+\delta f)}\right] \quad ; \quad \text{s.t:}$$

$\alpha + 3 \le \tilde{\mu} \le 5 - \alpha$

when $\tilde{\mu}$ reaches its lower bound, $\tilde{E}(X,Y)$ attains its minimum.

The solution is

$$C^{L}_{E(\alpha)} = \left[\frac{100}{\alpha+6}, \frac{300}{4\alpha+24}\right]$$

when $\tilde{\mu}$ attains its upper bound, $\tilde{E}(X,Y)$ produces its maximum.

The resultant solution is

$$C^{U}_{E(\alpha)} = \left[\frac{100}{8-\alpha}, \frac{300}{32-4\alpha}\right]$$

Then there exist inverse functions $C^{L}_{E(\alpha)}$ and $C^{U}_{E(\alpha)}$ with its membership function

$$\mu_{\tilde{E}(X,Y)}(\omega) = \begin{cases} \frac{100-6\psi_1}{\psi_1}, \frac{300-24\psi_2}{4\psi_2} & \text{where } 14.3 \le \psi_1 \le 16.67 \text{ and } 10.71 \le \psi_2 \le 12.5 \\ \frac{8\psi_1-100}{\psi_1}, \frac{32\psi_2-300}{4\psi_2} & \text{where } 12.5 \le \psi_1 \le 14.23 \text{ and } 10.71 \le \psi_2 \le 14.285 \\ 0, & \text{otherwise} \end{cases}$$

V. CONCLUSION

In this paper, we have studied Kaplan's terror queue model which resulted in a tandem network of M/FM/∞ and FM/M/∞ queues and obtained the membership function for its expectations based on Randomness-Fuzziness consistency principle which examines the left and right reference functions with their appropriate ranges. It has the benefit of being easy to explore applying tools from queueing theory.

Research can be extended on incorporating different kinds of terror plots. This can be implemented by insertion of different queueing models with distinct parameters.

Fuzzy terror queue permits reasonable solution for each depository with different levels of potentiality. It helps the decision maker to estimate the detected and undetected terror plots in extensive ranges of possibility level. It is more applicable to solve real-world problems in recent years.

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