March-April 2020
ISSN: 0193-4120 Page No. 5418 - 5421

Bycentric Subdivision of Graphs and their Magic Labeling

Rajpal[1] - Department of Mathematics, Indira Gandhi University, Meerpur, Rewari, India

Mamta Kamra[2] - Department of Mathematics, Indira Gandhi University, Meerpur, Rewari, India

Article Info
Volume 83
Page Number: 5418 - 5421
Publication Issue:
March - April 2020

Article History
***Article Received**: 24 July 2019*
***Revised**: 12 September 2019*
***Accepted**: 15 February 2020*
***Publication**: 28 March 2020*

Abstract:
Proper goal of this paper is to extend the concept of magic labeling to some bycentric subdivided special graphs like, Shell graphs (Sn), Cycle graphs (Cn), Wheel graphs (Wn) and some more cyclic graphs. We also obtained some results on acyclic graph and bycentric subdivision of product graphs. In bycentric subdivision we insert a vertex on each edge so that every edge of the graph becomes a path of length two. In this process the resultant graphs will look like a graph embedded their image inside them, as for example a cycle inscribed in a cycle. We obtained some theorems and algorithms to generalize the concept to similar types of graphs.

2010 AMS Classification: 05C76, 05C78.
Keywords: *Shell graphs, graph labeling, Magic labeling.*
Article type: *Research article*

INTRODUCTION

We considered only the finite, simple, connected and undirected graphs throughout in this paper. Basic conventional notions and denotions are taken from the book of graph theory written by D. West [9]. We also prefer Gallian [3] for other remarkable problems and results on graph labeling. Graph labeling associates with the process of logical assignment of integers to the vertices of any graph or its edges or to both follows a certain pattern corresponding to certain condition(s). If we label only the vertex (edge) set of a graph then such assignment is named as vertex labeling (an edge labeling) of a graph. Graph labeling have a significant role in the field of optimization of network problems. It plays magnificent role in designing an algorithm and for analysis of complexity of any algorithm.

Any graph can be labeled by many different ways, various method of labeling of vertices of any graph is present in the literature. One of these method of labeling of vertices is with numbers. An interesting approach of vertex labeling with numbers is vertex-magic labeling. The vertex-magic graphs are obtained by labeling of graphs with numbers in such a way such that the assignment of numbers to every vertex and to all its incident edges add up to the _x number. This _x number is known as magic number. One popular graph that hold this interesting vertex-magic properties is the cycle graph. Here in this paper, we introduce magic labeling to the bycentric subdivision of cycle graphs. In bycentric subdivision of a graph we subdivided every edge of a graph in such a way so that each edge converts into the path of length two. In other words in barycentric subdivision of a graph we insert a vertex with degree two to every edge of the given graph. We use the notation S(G) for the barycentric subdivision of any graph G.

2 Main Results

Suppose G (V, E) is a simple graph where V is the vertex-set of G and E is the edge-set of the graph G. If μ is a one-to-one mapping from E ∪ V onto the set of integers {1, 2, …… e + v}, we define the magic sum of the vertex x by $wt(x) = \mu(x) + \sum \mu(xy)$, where the sum is over all vertices y

March-April 2020
ISSN: 0193-4120 Page No. 5418 - 5421

adjacent to x. We say μ is a total vertex-magic labeling if there exist a constant h such that wt(x) = h for every vertex x of graph G. A graph in which this type of labeling is possible is called vertex-magic graph.

Lemma 1 If any connected graph G having n vertices and m number of edges admits total magic labeling then there must exist an positive number l such that

$$\sum_{1}^{n} \mu(n) + \sum_{1}^{m} \mu(m) = nl$$

Proof: Let $V_{sum} = \sum_1^n \mu(n)$be the sum of the labels of all vertices and $E_{sum} = \sum_1^n \mu(n)$ be the sum of the labels of all edge of graph G. Since each edge is incident to two vertices thus, each edge label is counted twice in the order of counting magic number at each vertex. Therefore, in order to add magic number at each vertex we get $V_{sum} + 2E_{sum} = nl$.

Lemma 2 For any connected graph $|V| + |E| + 1$ is the lower bound for total magic labeling.

Proof: The proof is obvious because if we label all the vertices and edges by the integers from 1 to n : n = $|V| + |E|$ then at least one edge or vertex receive label n. Since we considered the graph is connected so this n must be added to at least one number from 1 to n−1. Thus, the magic number is always greater than $|V| + |E| + 1$.

Theorem 1 Every odd cycle graph C_m satisfy vertex magic total labeling if m ≥ 3.

Proof: We need to prove all the odd cycles C_m, m ≥ 3 is vertex magic total labeling. **Case (i):** Consider the cycle with odd number of vertices. Define v = 2m + 1, the vertex labeling of the cycles be (1, m + 1, 2m + 1, m,…..m + 2) here the labeling is concluded from the foregoing one by adding mmod (2m + 1). The solid match of vertices has summation m + 2, 3m + 2, 3m + 1, ……m + 3 which are all disparate. Providing magic number k = 5m + 4, then the edge labels.

are 4m + 2, 2m + 2, 2m + 3, 4m + 1 as necessaries K = 5m + 4 = $\frac{1}{2}$(5v + 3). Hence the proof. When we bycentric subdivide any cycle graph C_m it expands to cycle graph with 2m number of vertices. Hence we observed that the magic labeling of bicentric subdivision of cycle graph is similar to the magic labeling of cycle graph with even number of vertices.

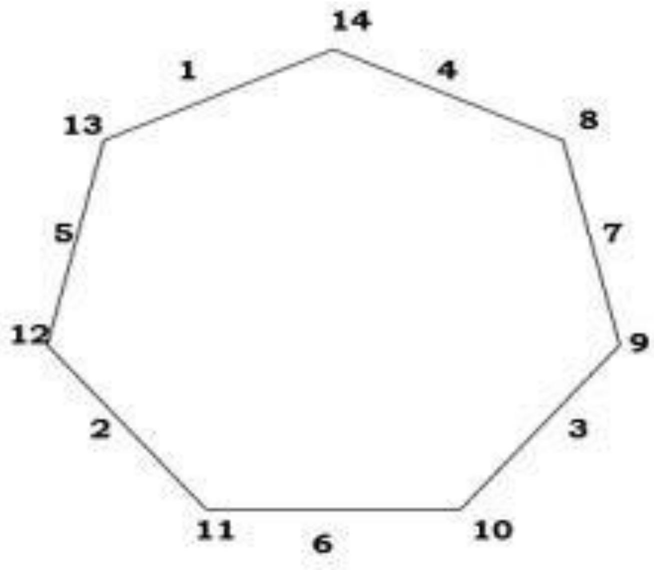

Figure 1: Magic labeling of C_7

Theorem 2 Bycetric subdivision of any cycle graph C_m always admits magic labeling.

Proof: Let C_m is a cycle graph with m number of vertices. Let B_n is the bycentric subdivision of C_m where B_n is a cycle with 2m number of vertices, i.e., n = 2m. Now we define magic labeling of B_n as follows:

Case-I: If m is odd then we define magic labeling f (x_j) as:

$$f(x_j)=\begin{cases}\frac{j+1}{2}; & j=1,3,...,m\\ 3m; & j=2\\ \frac{2m+j+2}{2}; & j=4,6,...,m-1\\ \frac{m+3}{2}; & j=m+1\\ \frac{j+3}{2}; & j=m+2,m+4,...,2m-1\\ \frac{2m+j}{2}; & j=m+3,m+5,...,2m-2\\ m+2; & j=2m\end{cases}$$

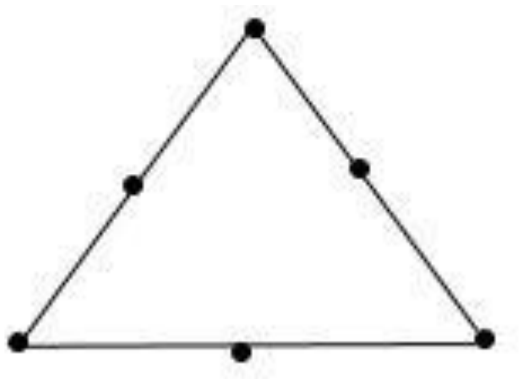

Figure 2: Bycentric subdivision of C_3

Case-II: If m is even then we define magic labeling $f(x_j)$ as:

$$f(x_j)=\begin{cases}\frac{j+1}{2}; & j=1,3,...,m\\ 3m; & j=2\\ \frac{2m+j}{2}; & j=4,6,...,m\\ \frac{j+2}{2}; & j=m+2,m+4,...,2m\\ \frac{2m+j-1}{2}; & j=m+3,m+5,...,2m-1\end{cases}$$

from these we found $f(x_j)$ and the uniform k values $f(x_jx_{j+1})$ be able to determined. Suppose we explicate a $f(x_j) = f(x_jx_{j+1})$ and $f(x_jx_{j+1}) = f(x_{j+1})$, where the subscripts are integer modulo n. After that at each respective vertex define k be a magic uniform, resigned vertex magic total labeling for cycles. Hence we obtain magic labeling of B_n. We obtained shell graph Sn by taking n−3 concurrent chords in a cycle with n vertices C_n. The vertex at which all the chords are concurrent is known as the apex vertex.

3. Conclusion

In this paper, we obtain some conditions with the help of which we become able to label any cycle graphs such that it becomes total magic labeling. Here we present some bound for magic number and an approach on how to label bycentric subdivision of any cycle graphs. Further, we want to generalize the concept to the bycentric subdivision of any graphs. Also, want to extend the concept to some other types of graphs.

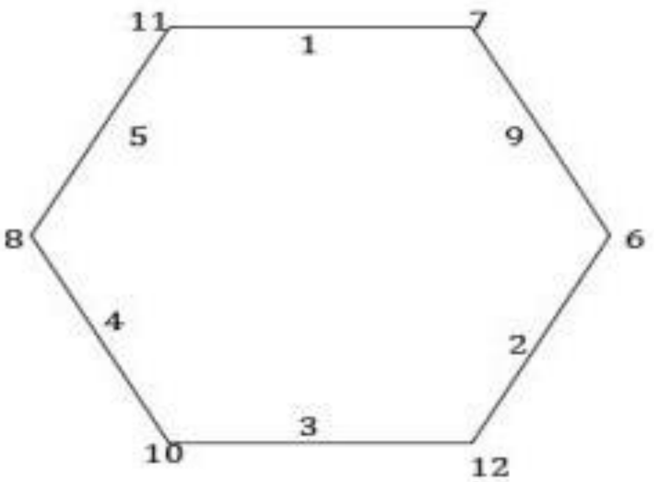

Figure 3: Magic labeling bycentric subdivision of C_3 i.e., B_6

REFERENCES

1. I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.
2. S. Cichacz, A. Gorlich, Z. Tuza, Cordial labeling of hypertrees, Discrete Mathematics, 313 (22) (2013) 2518-2524.
3. J. A. Gallian, A dynamic survey of graph labeling, Elec. J. Combin., DS6, accessed December 7, 2015, http://www.combinatorics.org/Surveys/ds6.pdf.
4. T. Grace, On sequential labelings of graphs, Journal of Graph Theory, 7(2) (1983) 195-201.
5. R.L. Graham and N. J. A. Sloane, On additive bases and harmonious graphs, SIAM J. Algebraic Discrete Methods, 1 (1980) 382-404.
6. M. Hovey, A-cordial graphs, Discrete Mathematics, 93 (1991) 183-194.
7. O. Pechenik and J. Wise, Generalized graph cordiality, Discussiones Mathematicae Graph Theory, 32 (2012) 557-567.
8. A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Sympos., Rome, 1966), New York: Gordon and Breach, (1967) 349-355.
9. D. B. West, Introduction to Graph Theory, second edition, Prentice-Hall (2001).
10. K. Driscoll, E. Krop, M. Nguyen, All Trees are Six-Cordial, Electronic J. of Graph Theory and Applications, 5(1) (2017) 1-15.