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Fixed Point Results for Pata Type Contractions in G - Metric Spaces

G Sudhaamsh Mohan Reddy
Faculty of Science and Technology, ICFAI Foundation for Higher Education, Hyderabad- 501203, INDIA

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Abstract:
In this article, the existence of fixed point for G-Pata type Zamfirescu mapping in a complete G-metric space is proved. Our result give existence of fixed point for a wider class of functions.

Keywords: *Pata, contractions, G-metric spaces*

I INTRODUCTION

Banach [1] proved the existence of fixed point on a complete metric space (X,d) in 1992. The mapping f has been considered to be a contraction and f takes points of X to itself. Later, several interpretations for the existence of fixed point with weaker conditions to contraction mapping were given. Later Kannan type [6], Chatterjea type [5] and Hardy-Rogers type mappings were introduced. Zamfirescu [21] introduced and gave the existence of fixed point for a generalized contraction mapping in 1972. In 2005, Zead Mustafa et al.[11] introduced the notion of G-metric spaces and they established new fixed point results in G-metric spaces. Later, several authors were established for fixed point results in this area.

Geno Kadwin Jacob et.[9] introduced the Pata typeZamfirescu contraction in complete metric space. In this paper, "define a G-Pata type Zamfirescu contraction and give fixed point results in complete G-matric spaces based on G-Pata type Zamfirescu contraction.

Throughout the paper, Θ denotes the class of all increasing functions. $\psi:[0,1]\to[0,\infty)$ such that Ψ is continuous at '0' with $\Psi(0)=0$.

Definition *1.1 [21]:* Let (X,d) be a metric space. A mapping $f:X\to X$ is said to be a Zamfirescu mapping if, for all $x,y\in X$ and $a,b,c\in[0,1]$, it satisfies the contraction.

$$d(f(x),f(y))\le \max\{ad(x,y),\frac{b}{2}[d(x,f(x))+d(y,f(y))],$$
$$\frac{c}{2}[d(x,f(y))+d(y,f(x))\}.$$

In a recent, Pata [7] obtained the following refinement of the classical Banach contraction principle.

Let $\Lambda\ge 0,\ \alpha\ge 1, \beta\in[0,\alpha]$ be any constants. For each $\varepsilon\in[0,1]$,

$$d(f(x),f(y))\le(1-\varepsilon)d(x,y)+\Lambda\varepsilon^{\alpha}.\Psi(\varepsilon)[1+\|x\|+\|y\|]^{\beta},$$

Where $\|x\|=d(x,x_0)$ for arbitrary $x_0\in X$ and $\Psi\in\Theta$.

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$$d(f(x),f(y)) \le (1-\varepsilon)M(x,y) + \Lambda\varepsilon^{\alpha}\Psi(\varepsilon)[1+\|x\|+\|y\|+\|f(x)\|+\|f(y)\|]^{\beta}$$

Where

$$M(x,y) = \max\{d(x,y), \frac{d(x,f(x))+d(y,f(y))}{2}, \frac{d(x,f(y))+d(y,f(x))}{2}\}$$

and

$\|x\| = d(x,x_0)$ for arbitrary $x_0 \in X$ and $\Psi \in \Theta$.

The following Lemma is used to prove our results.

Lemma 1.1:[10] If a sequence $x_n \in X$ is not G-Cauchy, then there exist $\delta > 0$ and two subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $m(k)$ is the smallest index for each $m(k) > n(k) > k$,

$G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \ge \delta$ and

$G(x_{m(k)-1}, x_{n(k)}, x_{n(k)}) < \delta$

Moreover, Suppose that $\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = 0$.

Then we have

1) $\lim_{n\to\infty} G(x_{m(k)}, x_{n(k)}, x_{n(k)}) = \delta$ 2) $\lim_{n\to\infty} G(x_{m(k)-1}, x_{n(k)-1}, x_{n(k)-1}) = \delta$

3) $\lim_{n\to\infty} G(x_{m(k)}, x_{n(k)-1}, x_{n(k)-1}) = \delta$ 4) $\lim_{n\to\infty} G(x_{m(k)-1}, x_{n(k)}, x_{n(k)}) = \delta$

II MAIN RESULTS

Existence of fixed point for G-Pata type mappings

In this section, we prove the existence of unique fixed point for G-Pata type Zamfirescu mappings. Let (X,G) be a G-metric space. In the sequal, we write $\|x\| = G(x,x,x_0)$, where x_0 is an arbitrary element in X.

Definition 2.1: Let (X,G) be a complete G-metric space. A mapping $f: X \to X$ is said to be G-Pata type Zamfirescu mapping if for all $x,y,z \in X, \Psi \in \Theta$ and for every $\varepsilon \in [0,1]$, f satisfies the inequality

$$G(f(x),f(y),f(z)) \le (1-\varepsilon)M(x,y,z) + \Lambda\varepsilon^{\alpha}.\Psi(\varepsilon).[1+\|x\|+\|y\|+\|z\|+\|f(x)\|+\|f(y)\|+\|f(z)\|]^{\beta},$$

Where

$$M(x,y,z) = \max\{G(x,y,z), \frac{G(x,f(x),f(x))+G(y,f(y),f(y))+G(z,f(z),f(z))}{3}, \frac{G(x,f(y),f(y))+G(y,f(z),f(z))+G(z,f(x),f(z))}{3}\}$$

and $\Lambda \ge 0$, $\alpha \ge 1, \beta \in [0,\alpha]$ are constants.

Theorem 2.1: Let (X,G) be a complete G-metric space and let $f: X \to X$ be a G-Pata type Zamfirescu mapping. Then, f has a unique fixed point in X..

Proof: Let x_0 be an arbitrary element in X. Define $x_{n+1} = f(x_n)$ and $c_n = G(x_n, x_n, x_0)$.

To prove that $G(x_{n+1}, x_{n+1}, x_n)$ is a non increasing sequence, take $\varepsilon = 0$. Therefore,

$$G(x_{n+1},x_{n+1},x_n) = G(f(x_n),f(x_n),f(x_{n-1}))$$

$$\le \max\{G(x_n,x_n,x_{n-1}), \frac{2G(x_n,f(x_n),f(x_n))+G(x_{n-1},f(x_{n-1}),f(x_{n-1}))}{3}, \frac{G(x_n,f(x_n),f(x_n))+G(x_n,f(x_{n-1}),f(x_{n-1}))+G(x_{n-1},f(x_n),f(x_n))}{3}\}$$

$$\le \max\{G(x_n,x_n,x_{n-1}), \frac{2G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)}{3}, \frac{G(x_n,x_{n+1},x_{n+1})+G(x_n,x_n,x_n)+G(x_{n-1},x_{n+1},x_{n+1})}{3}\}$$

$$\le \max\{G(x_n,x_n,x_{n-1}), \frac{2G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)}{3}, \frac{G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)+G(x_n,x_{n+1},x_{n+1})}{3}\}$$

$$\le \max\{G(x_n,x_n,x_{n-1}), \frac{2G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)}{3}, \frac{2G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)}{3}\}$$

$$\le \max\{G(x_n,x_n,x_{n-1}), \frac{2G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_n,x_n)}{3}\}$$

$$G(x_{n+1},x_{n+1},x_n) \le G(x_n,x_n,x_{n-1}) \le G(x_1,x_1,x_0) = c_1.$$

Claim(1): $\{c_n\}$ is bounded

$$C_n = G(x_n, x_n, x_0)$$
$$\leq G(x_n, x_n, x_{n+1}) + G(x_{n+1}, x_{n+1}, x_1) + G(x_1, x_1, x_0)$$
$$\leq (1-\varepsilon)\max.\{G(x_n, x_n, x_0), \frac{2G(x_n, x_{n+1}, x_{n+1}) + G(x_0, x_1, x_1)}{3}, \frac{G(x_n, x_n, x_1) + G(x_n, x_n, x_1) + G(x_0, x_{n+1}, x_{n+1})}{3}\}$$
$$+2C_1 + \Lambda\varepsilon^{\alpha}\psi(\varepsilon)[1 + \|x_n\| + \|x_n\| + 0 + \|x_{n+1}\| + \|x_{n+1}\| + \|x_1\|]^{\beta}$$

$$\leq (1-\varepsilon)\max.\{G(x_n, x_n, x_0), \frac{2G(x_n, x_{n+1}, x_{n+1}) + G(x_0, x_1, x_1)}{3}, \frac{2G(x_n, x_n, x_1) + G(x_0, x_{n+1}, x_{n+1})}{3}\}$$
$$+2C_1 + \Lambda\varepsilon^{\alpha}\psi(\varepsilon)[1 + 2\|x_n\| + G(x_{n+1}, x_{n+1}, x_n) + 2G(x_n, x_n, x_0) + \|x_1\|]^{\beta}$$
$$\leq (1-\varepsilon)\max.\{C_n, C_1, G(x_n, x_n, x_0) + G(x_1, x_1, x_0) + G(x_{n+1}, x_{n+1}, x_n) + G(x_n, x_n, x_0)\}$$
$$+2C_1 + \Lambda\varepsilon^{\alpha}\psi(\varepsilon)[1 + \|x_n\| + \|x_n\| + \|x_n\| + \|x_1\| + \|x_1\| + \|x_1\|]^{\beta}$$
$$\leq (1-\varepsilon).\max.\{C_n, C_1, C_n + C_1\} + 2C_1\Lambda\varepsilon^{\alpha}\psi(\varepsilon)[1 + 3C_n + 3C_1]^{\beta}$$

By the same reason as in [8], it follows that is $\{C_n\}$ bounded. Let $\lim_{n\to\infty} G(x_n, x_n, x_{n-1}) = G$. Since $G(x_n, x_n, x_{n-1})$ is non increasing.

$$G(x_{n+1}, x_{n+1}, x_n) = G(f(x_n), f(x_n), f(x_{n-1}))$$
$$\leq (1-\varepsilon)\max.\{G(x_n, x_n, x_{n-1}), \frac{2G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_n, x_n)}{3}, \frac{G(x_n, x_{n+1}, x_{n+1}) + G(x_n, x_n, x_n) + G(x_{n-1}, x_n, x_n)}{3}\}$$
$$+\Lambda\varepsilon^{\alpha}\psi(\varepsilon)[1 + \|x_n\| + \|x_n\| + \|x_{n-1}\| + \|x_{n+1}\| + \|x_{n+1}\| + \|x_n\|]^{\beta}$$
$$\leq (1-\varepsilon)\max.\{G(x_n, x_n, x_{n-1}), \frac{2G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_n, x_n)}{3}, \frac{G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_n, x_n) +}{3}\} + k\varepsilon\Psi(\varepsilon).$$

Now, as $n \to \infty$, we get that $G \leq k\,\varepsilon\,\Psi(\varepsilon)$ and hence $G = 0$.

*Claim(2):*The sequence $\{x_n\}$ is a G-Cauchy. Suppose that $\{x_n\}$ is not a G-Cauchy sequence, then by lemma 1.1, there exist sub sequences $\{x_{n_k}\}$ and $\{x_{m_k}\}$ with $n_k > m_k > k$ such that

$$\delta \leq G(x_{m_k}, x_{m_k}, x_{n_k}) = G(f(x_{m_k} - 1), f(x_{m_k} - 1), f(x_{n_k} - 1))$$
$$\leq (1-\varepsilon)\max.\{G((x_{m_k} - 1), (x_{m_k} - 1), (x_{n_k} - 1), \frac{2G(x_{m_k} - 1, x_{m_k}, x_{m_k}) + G(x_{n_k} - 1, x_{n_k}, x_{n_k})}{3}, \frac{G(x_{m_k} - 1, x_{m_k}, x_{m_k}) + G(x_{m_k} - 1, x_{n_k}, x_{n_k}) + G(x_{n_k} - 1, x_{m_k}, x_{m_k})}{3}\} + k\varepsilon\Psi(\varepsilon)$$

Now, as $n \to \infty$, we get $\delta \leq k\,\varepsilon\,\Psi(\varepsilon)$, which is a contradiction. Therefore $\{x_n\}$ is a G-Cauchy. Since X is G–complete, there exists $x \in X$ such that $x_n \to x$. Now, for all $n \in N$ and for $\varepsilon = 0$, we obtain

$$G(f(x), f(x), x) \leq G(f(x), f(x), x_{n+1}) + G(x_{n+1}, x_{n+1}, x)$$
$$\leq \max\{G(x, x, x_n), \frac{G(x, f(x), f(x) + G(x_n, x_{n+1}, x_{n+1})}{3}\} \frac{G(x, f(x), f(x) + G(x_n, x_{n+1}, x_{n+1}) + G(x_n, f(x), f(x))}{3}\}$$
$$+G(x_{n+1}, x_{n+1}, x).$$

As $n \to \infty$, the above inequality concludes that $G(f(x), f(x), x) \leq \frac{5}{3} G(f(x), f(x), x)$.

Hence, x is a fixed point of f.For the uniqueness of fixed point, suppose that x and y are fixed points of f. Then

$$G(f(x), f(y), f(y)) \leq (1-\varepsilon)\max\{G(x, y, y), \frac{G(x, f(x), f(x)) + 2G(y + f(y), f(y))}{3}, \frac{G(x, f(y), f(y)) + G(y + f(y), f(y) + G(y, f(x), f(x))}{3} + k\varepsilon\psi(\varepsilon).$$

Therefore, we get $G(x, y, y) \leq k\,\psi(\varepsilon)$ and hence $x = y$.

Therefore f has a unique fixed point in X.

III REFERENCES

[1]. Banach, S. Sur les operations dans les ensembles abstraits et leurs applications aux equations integrals. Fund. Math. 1922, 3, 133–181.
[2]. Berinde, V., Stability of Picard iteration for contractive mappings satisfying an implicit
relation, Carpathian J. Math., 27 (2011), no. 1, 13-23.
[3]. Ciric, Lj., On contraction type mappings, Math. Balcanica 1 (1971), 52-57.

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ISSN: 0193-4120 Page No. 3317 - 3320

[4]. Chakraborty, M.; Samanta, S.K. On a fixed point theorem for a cyclical Kannan-type mapping, pre-print (2012). Facta Univ. Ser. Math. Inform. 2013, 28, 179–188.
[5]. Chatterjea, S.K. Fixed-point theorems. C. R. Acad. Bulg. Sci. 1972, 25, 727–730.
[6]. Kannan, V. Some results on fixed points. Bull. Calcutta Math. Soc. 1968, 60, 71–76.
[7]. Pata, V. A fixed point theorem in metric spaces. J. Fixed Point Theory Appl. 2011, 10, 299–305.
[8]. Kadelburg, Z.; Radenovic, S. Fixed point theorems under Pata-type conditions in metric spaces. J. Egypt. Math. Soc. 2016, 24, 77–82.
[9]. Geno Kadwin Jacob, M. S. Khan, Choonkil Park 3, and Sungsik Yun., On Generalized PataType Contractions, Mathematics 2018, 6, 25.
[10]. Mona Khandaqji, Sharifa Al-Sharif and Mohammad Al-Khaleel, Property P and some fixed point results on (ψ,ϕ)-weakly contractive maps in G-metric spaces, International Journalof Mathematics and Mathematical Sciences, 2012.
[11]. Mustafa.Z and B. Sims, "A new approach to generalized metric spaces," *Journal of Nonlinear and Convex Analysis*, vol. 7, no. 2, pp. 289–297, 2006.
[12]. Rhoades, B.E. A comparision of various definitions of contractive mappings. Trans. Am. Math. Soc. 1977, 226, 257–290.
[13]. Samet, B., Vetro, C., and Vetro,P., Fixed point theorems for $\alpha-\varphi$ contractivetype mappings, Nonlinear Anal. 75 (2012), 2154-2165.
[14]. G. Sudhaamsh Mohan Reddy, Generalization of Contraction Principle on G-Metric Spaces, Global Journal of Pure and Applied Mathematics, 14(9) (2018), 1177-1283.
[15]. G. Sudhaamsh Mohan Reddy, Fixed point theorems of contractions of G-metric Spaces and property 'P' in G-Metric spaces, Global Journal of Pure and Applied Mathematics, 14(6)(2018), 885-896.
[16]. G. Sudhaamsh Mohan Reddy, A Common Fixed Point theorem on complete G-metric spaces, International Journal of Pure and Applied Mathematics, 118(2)(2018), 195-202.
[17]. G. Sudhaamsh Mohan Reddy, Fixed Point Theorems for (ε, λ)-Uniformly Locally Generalized Contractions, Global Journal of Pure and Applied Mathematics, 14(9)(2018), 1177-1183.
[18]. G. Sudhaamsh Mohan Reddy, New proof for generalization of contraction principle on G-Metric spaces, Jour of Adv Research in Dynamical & Control Systems, Vol. 11, Special Issue-08, (2019), 2708-2713.
[19]. V Srinivas Chary, G Sudhaamsh Mohan Reddy,Fixed Point Results for Almost Z_G-contraction via Simulation Functions in G-metric spaces, International Journal of Control and Automation, Vol. 12, No. 6, (2019), pp. 608-615.
[20]. G. Sudhaamsh Mohan Reddy, Fixed point theorems of Rus-Reich- Ciri´c type contraction and Hardy-Rogers type contraction on G-metric spaces International Journal of Advanced Science and Technology, Vol. 29, No.02, (2020), pp. 2782-2787.
[21]. Zamfirescu, T. Fixed point theorems in metric spaces. Arch. Math. 1972, 23, 292–298.
[22]. Shafti, S.S. and Ahmadie, M., 2018. Improvement of Psychiatric Symptoms by Cardiac Rehabilitation in Coronary Heart Disease Vol 22 (2) 80, 89.
[23]. Bonsaksen, T., Opseth, T.M., Misund, A.R., Geirdal, A.Ø., Fekete, O.R. and Nordli, H., 2019. The de Jong Gierveld Loneliness Scale used with Norwegian clubhouse members: Psychometric properties and associated factors Vol 22 (2) 88, 100.
[24]. Ritter, V.C., Nordli, H., Fekete, O.R. and Bonsaksen, T., 2017. User satisfaction and its associated factors among members of a Norwegian clubhouse for persons with mental illness. International Journal of Psychosocial Rehabilitation. Vol 22 (1) 5, 14.
[25]. Ferrazzi, P., 2018. From the Discipline of Law, a Frontier for Psychiatric Rehabilitation. International Journal of Psychosocial Rehabilitation, Vol 22(1) 16, 28.
[26]. Bornmann, B.A. and Jagatic, G., 2018. Transforming Group Treatment in Acute Psychiatry: The CPA Model. International Journal of Psychosocial Rehabilitation, Vol 22(1) 29, 45.