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A Fuzzy Two-Warehouse Inventory System for Stock-Dependent Demand

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Abstract:

In this paper we have formed an inventory system for a single decaying item with two distinct storage facilities and in which demand is inventory dependent. Shortage is permitted and is partially backlogged. It is considered that the holding cost of the rented warehouse is higher than that of owned warehouse. As demand, holding- cost, shortage, lost- sale, decaying- rate are uncertain in nature, we assume them as triangular fuzzy numbers and formed the model for fuzzy total cost function and is defuzzied by using Signed Distance and Centroid methods. In order to show the model, we contrast the results of crisp and fuzzy models through a numerical example and depend on the example the effect of different variable have been exactly considered by sensitivity analysis taking one variable at a time keeping the other variables unchanged.

Keywords: *Inventory; Two-Warehouse System; Triangular Fuzzy Number; Signed Distance Method; Centroid Method, Shortage.*

I. INTRODUCTION

Regularly, inventory models are mostly formed with single warehouse space. But basically, it is virtually impossible for big shops or showrooms placed in main market places of town having a bigger warehouse due to inaccessibility of space. Moreover, if a firm get it, they have to pay very high rents. When a firm vast amount of goods for the future demand, then they have a need for OW in existing market place. On the other hand, the excess units are stocked in a rented warehouse (RW) with an infinite capacity, i.e. it is as large as it may be required as per the time. Since the holding cost per unit in RW is much higher in compared with the OW.

In the last few years, two-warehouse inventory models have been generally covered in business world. This type of model was first formed by Hartley [6] in 1976. The cost of transportation from RW to OW was not assumed later by initiating the transportation costs. Sarma [27] enlarged the model of Hartley. Again, Murdeswar and Sathe [22] extracted the model in the case of finite rate. Dave [4] established the model with the both finite and infinite refilling rates. Goswami and Chaudhuri [5] provided an EOQ model for linear demand with two types of storage facility. Benkherouf [1] formed a model for deteriorating items. For knowing more research works in this field, one can see Yang [34],Huang [7], Lee and Hsu [14], Liang and Zhou [15], Yang and Chang [35], Jaggi et al. [12,11] , Bhunia et al. [2,3], Xu et al. [31], Mandal and Giri [18], Sheikh and Patel [29] , Saha, Sen and Nath [26], Singh et. al. [38, 39], Bhunia et al. [40], and Panda et al. [42] etc.

Demand engages the basic part in inventory management. There are several types of demands like price-dependent demand, time-dependent demand, inventory dependent demand, ramp-type demand etc. A plenty of research papers have been issued in inventory-dependent demand rate. Rong et al. [23] formed a model of deteriorating items with selling-price dependent demand and shortages. Jaggi and Verma [10] formulated a two-warehouse model of non-decaying items with selling-price dependent demand and under fully backlogged shortages. However, Jaggi et al. [9] developed an inventory

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model for decaying items with limited capacity and time-proportional backlogging rate. Mishra et al. [21] have established a model with suppliers joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. Mishra et al. [20] have provided a model under price and stock dependent demand for controllable decaying rate and preservation technology under shortage. In distinct conditions, it is more logical to consider the demand, shortage and holding cost as fuzzy variables. Zadeh [36] firstly gave the idea of fuzzy in stock problem. Bellman & Zadeh [37] offered fuzzy idea in decision making. A multi-goods problem related to inventory for decaying with fuzzy was given by Roy and Maiti [25]. A two-warehouse stock problem with advertisement and trapezoidal fuzzy numbers are recommended by Maiti and Maiti [16]. They used goal programming procedure to find result. Rong [23] suggested a fuzzy system with partially or fully scarcity for decaying items. They used nearest approximation procedure and global criteria technique to reduce the function of total cost. Again, Roy [24] explored a production structure for faulty and accessible goods with fuzzy variables.

Singh et al [33] established a genetic method with fuzzy parameters for decaying items with stock-based demand rate. Singh and Malik [17] suggested a two-warehouse system with rate of linear pattern and different decaying rate. Kumari et al [13] have worked on three-component demand rate with backlogging rate which is proportional to time in a fuzzy area. Anuradha and Singh [30] preferred an inventory system with increasing demand and backlogged scarcity. They concerned graded mean integration representation procedure. Shabani [28] explored a fuzzy structure with trade credit policy where demand and deterioration are considered to be fuzzy. Islam and Mandal [19] a two-warehouse model with fuzzy variables by applying graded mean integration representation procedure for Weibull decaying goods. Sharma et al [32] investigated a system for non-decaying items wit trade credit policy. The element of cost and demand are taken as triangular fuzzy numbers. Misra et al [8] proposed model with stock-based demand in fuzzy environment. Recently, Indrajitsingha [41] formulated a fuzzy model for a single item with selling price-based demand for decaying goods. They took the element of cost as triangular fuzzy numbers. They fuzzified the function of total cost by applying signed distance procedure and centroid procedure. Shee et al [43] investigated a fuzzy production system for decaying goods. Maiti [44] formulated fuzzy structure for multi goods for decaying goods.

Many of articles was formed in the region of two-warehouse system in crisp model. Although a less system was formed in uncertainty. Here we discussed many of fuzzy system related to two-warehouse in the review of literature. In current, ruthless market outline, the satisfaction of customer enacts an important part for a firm to boost the gain. The intensity of inventory should be accordingly to join with customers supposition. No firm avoids the impact of demand in his business. If the selling price of an item increases, then demand of that decreases. Here, we have assumed two-warehouse system where holding cost of rented house is greater than owned house. The demand of items is based on stock. The element of cost of two-warehouse are considered as triangular fuzzy numbers.

2.1 Assumptions

The assumptions which are used in this model are as follows:

i. This structure concerns only one product.
ii. The order occurs immediately limitless.
iii. The lead time is zero.
iv. The demand rate is deterministic and a function of instantaneous stock level: $D(t)=\begin{Bmatrix} P+qI(t), 0\le t\le t_1 \\ P\ , t_1\le t\le T \ \ where\ P,q>0 \end{Bmatrix}$.
v. The shortages are permitted and partially backlogged.

vi. The owned warehouse (OW) has a capacity constraint of W units.
vii. The rented warehouse (RW) has indefinite capacity.
viii. Time horizon is finite.
ix. The goods assumed here are decaying in nature.
x. The goods are placed first in OW.
xi. The goods placed in RW will be consumed first.

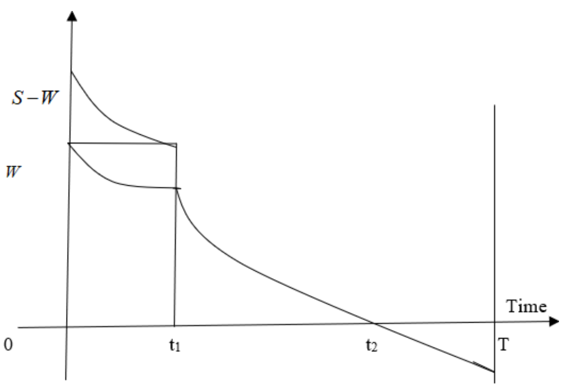
2.2 Notations

The following notations are used:

(t): the quantity of stock in RW at time t, $t \geq 0$.
(t): the quantity of stock in OW at time t, $t \geq 0$.
P : Starting demand rate.
q : Positive demand parameter.
t_1 : Time at which inventory level of RW comes to zero.
t_2 : Time at which inventory level of OW comes to zero.
W : Storage capacity of OW.
A: the ordering cost.
S : Starting stock level.
q_1 : Backorder quantity during stock out.
C: deterioration cost per unit.
T: total length of inventory time.
k: Rate of backlogging.
h_r : Holding cost in RW.
h_o : Holding cost in OW.
a: deterioration rate in RW, $0 \leq a < 1$.
b: deterioration rate in OW, $0 \leq b < 1$
s: Cost in shortage
π : Cost in lost sale.
$TAC(t_1, t_2)$: Total average cost.
P : Fuzzy Starting demand rate.
q : Fuzzy positive demand parameter.
k : Fuzzy rate of backlogging.
h_r : Fuzzy holding cost in RW.
h_o : Fuzzy holding cost in OW.
$\tilde{s}$: Fuzzy shortage.
$\tilde{b}$: Fuzzy deterioration rate in OW.
a : Fuzzy deterioration rate in RW
π : Fuzzy opportunity cost due to lost sale.
$TAC\ (t_1, t_2)$: Fuzzy total cost.
$TAC_S\ (t_1, t_2)$: Defuzzified value by using Signed Distance Method.
$TAC_C\ (t_1, t_2)$: Defuzzified value by using Centroid Method.

3. Mathematical Formulation

Let, the stock level at time $t = 0$ is S ,out of which W_1 units are stored in OW and the rest W_2 =(S − W) units are stored in RW. Since holding cost of RW is greater than the holding cost of OW, the items in RW are consumed first. During the period (0, t_1) stock level of RW reduces due to demand only, the inventory level of OW is reduced due to deterioration only. After time t = t_1, the inventory level of RW reaches zero due to demand and deterioration and demand is fulfilled during the time period [t_1 , t_2] by using stock of OW. At time t = t_2 , stock level of OW reaches to zero due to demand and deterioration and after that shortage occurs. This is shown in the Figure. 1.



4.1 Crisp Model

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Here, stock level at RW and OW are governed by the following differential equations during time $0 \le t \le T$:

$$\frac{dI_r(t)}{dt} = -(P + qI_r(t)) - aI_r(t) \ ,0 \le t \le t_1 ....eq(1)$$

With $I_r(t_1) = 0$ and

$$\frac{dI_o(t)}{dt} = -bI_o(t) \ ,0 \le t \le t_1 ....eq(2)$$

With $I_o(t) = W$

$$\frac{dI_o(t)}{dt} = -(P + qI_o(t)) - aI_o(t) \ ,t_1 \le t \le t_2 ....eq(3)$$

With $I_o(t_2) = 0$

Solving equation (1), (2) and (3), we have

$$I_r(t) = \frac{P}{(q+a)}\left[e^{(q+a)(t_1-t)} - 1\right] ,0 \le t \le t_1 ....eq(4)$$

$$I_o(t) = We^{-bt} \ ,0 \le t \le t_1 ....eq(5)$$

$$I_o(t) = \frac{P}{(q+b)}\left[e^{(q+b)(t_2-t)} - 1\right] ,t_1 \le t \le t_2 ....eq(6)$$

At $t=t_1$, from eq.(5) and eq. (6), we have

$$We^{-bt} = \frac{P}{(q+b)}\left[e^{(q+b)(t_2-t_1)} - 1\right] ......eq(7)$$

$$t_2 = t_1 + \frac{1}{(q+b)} In\left[1 + \frac{(q+b)We^{-bt_1}}{P}\right] ......eq(8)$$

Which shows that t_2 is a function of t_1.

The TAC per cycle consists of the following elements:

1. Ordering cost =A ……..eq (9)
2. Holding cost in RW

   $HC_{RW} = h_r \int_0^{t_1} I_r(t)dt$ [from eq (4)]

   $$= \frac{h_r P}{(q+a)^2}\left[e^{(q+a)t_1} - (q+a)t_1 - 1\right] ......eq(10)$$
3. Holding cost in OW

   $$HC_{OW} = h_o\left[\int_0^{t_1} I_o(t)dt + \int_{t_1}^{t_2} I_o(t)dt\right]$$

   $$= h_o\left[\int_0^{t_1}(We^{-bt})dt + \int_{t_1}^{t_2}\frac{P}{(q+b)}(e^{(q+b)(t_2-t)} - 1)dt\right]$$

   $$= h_o\left[\frac{W}{b}(1-e^{-bt_1}) + \frac{P}{(q+b)^2}(e^{(q+b)(t_2-t_1)} - (q+b)(t_2-t_1) - 1)\right] .....eq(11)$$
4. Shortage cost

   $$SC = s\int_{t_2}^{T} -k\, I_o(t) ....eq(12)$$

   $$= s\int_{t_2}^{T} -k\, P\, dt = -skP(T - t_2) ......eq(12)$$
5. Lost sale cost

   $$LS = \pi\int_{t_2}^{T}(1-k)(P)dt$$

   $$= \pi P(1-k)(T - t_2) .....eq(13)$$
6. Deterioration cost

   The no. of deteriorated goods in RW in [0, t_1] is

   $$D_r = I_r(0) - \int_0^{t_1} D(t)dt$$

   $$= I_r(0) - \int_0^{t_1}\left[P + q(t)\right]dt = \frac{Pa}{(q+a)^2}\left[e^{(q+a)t_1} - 1 - (q+a)t_1\right]$$

   and the no. of deteriorated goods in OW in [0, t_2] is

   $$D_o = I_o(0) - \int_{t_1}^{t_2} D(t)dt$$

   $$= I_o(0) - \int_{t_1}^{t_2}(P + q(t))dt$$

   $$= W - P(t_2 - t_1) - \frac{Pq}{(q+b)^2}\left[e^{(q+b)(t_2-t_1)} - 1 - (q+b)(t_2 - t_1)\right]$$

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DC = $C(D_r + D_o)$

=

$$C\left[\begin{array}{l}\left\{\dfrac{Pa}{(q+a)^2}(e^{(q+a)t_1}-1-(q+a)t_1)\right\}\\ +W-P(t_2-t_1)\\ -\dfrac{Pq}{(q+b)^2}\left\{e^{(q+b)(t_2-t_1)}-1-(q+b)(t_2-t_1)\right\}\end{array}\right].....eq(14)$$

Total average cost per unit time is

TAC(t_1,t_2)= $\dfrac{1}{T}\left[OC+HC+DC+SC+LC\right]$

$$\frac{1}{T}\left[\begin{array}{l}A+\dfrac{h_rP}{(q+a)^2}\left[e^{(q+a)t_1}-(q+a)t_1-1\right]\\ +h_o\left[\begin{array}{l}\dfrac{W}{b}(1-e^{-bt_1})+\dfrac{P}{(q+b)^2}(e^{(q+b)(t_2-t_1)}\\ -(q+b)(t_2-t_1)-1)\end{array}\right]\\ -skP(T-t_2)+\pi P(1-k)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\dfrac{Pa}{(q+a)^2}(e^{(q+a)t_1}-1-(q+a)t_1)\right\}\\ +W-P(t_2-t_1)\\ -\dfrac{Pq}{(q+b)^2}\left\{\begin{array}{l}e^{(q+b)(t_2-t_1)}-1\\ -(q+b)(t_2-t_1)\end{array}\right\}\end{array}\right]\end{array}\right].....eq(15)$$

To minimize TAC, t_1, t_2 can be obtained by solving equations

$$\frac{dTAC(t_1,t_2)}{dt_1}=0 \ \ and \ \ \frac{dTAC(t_1,t_2)}{dt_2}=0........eq(16)$$

Equation (16) is equal to

$$\left[\frac{1}{T}\left\{\begin{array}{l}\left\{\dfrac{h_rP}{(q+a)}(e^{(q+a)t_1}-1)\right\}+\\ h_o\left\{\begin{array}{l}We^{-bt_1}\\ +\dfrac{P}{q+b}(1-e^{(q+b)(t_2-t_1)})\end{array}\right\}\\ +C\left\{\begin{array}{l}P+\dfrac{Pa}{(q+a)}(e^{(q+a)t_1}-1)\\ -\dfrac{Pq}{q+b}(1-e^{(q+b)(t_2-t_1)})\end{array}\right\}\end{array}\right\}\right]=0$$

and

The TAC function can be minimized if they satisfied the equations

$$\frac{\partial^2 TAC(t_1,t_2)}{\partial t_1^2}>0 \ and \ \frac{\partial^2 TAC(t_1,t_2)}{\partial t_2^2}>0$$

$$\&\left(\frac{\partial^2 TAC(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2 TAC(t_1,t_2)}{\partial t_2^2}\right)-\left(\frac{\partial^2 TAC(t_1,t_2)}{\partial t_1^2\partial t_2^2}\right)^2>0.$$

....$eq(17)$

4.2 Fuzzy Model

We consider the parameters as fuzzy parameters. These parameters may be considered as triangular fuzzy numbers.

Suppose,

$P=(P_1,P_2,P_3), q=(q_1,q_2,q_3), h_r=(r_1,r_2,r_3),$
$h_o=(o_1,o_2,o_3), b_o=(b_1,b_2,b_3), a_r=(a_1,a_2,a_3),$
$\pi=(l_1,l_2,l_3), \tilde{s}=(s_1,s_2,s_3), k=(k_1,k_2,k_3)$

Then TAC is

$$\left[\frac{1}{T}\left\{\begin{array}{l}h_o\left\{\dfrac{P}{q+b}(e^{(q+b)(t_2-t_1)}-1)\right\}\\ +skP-\pi(1-k)P-\\ C\left\{\dfrac{Pq}{q+b}(e^{(q+b)(t_2-t_1)}-1)+P\right\}\end{array}\right\}\right]=0$$

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$$TAC(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l} A+\frac{h_r P}{(q+a)^2}\left[\begin{array}{l} e^{(q+a)t_1}-(q+a)t_1\\ -1\end{array}\right]\\ +h_o\left[\begin{array}{l}\frac{W}{\tilde{b}}(1-e^{-\tilde{b}t_1})\\ +\frac{P}{(q+\tilde{b})^2}(e^{(q+\tilde{b})(t_2-t_1)}\\ -(q+\tilde{b})(t_2-t_1)-1)\end{array}\right]\\ -\tilde{s}kP(T-t_2)+\pi P(1-k)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\begin{array}{l}\frac{Pa}{(q+a)^2}(e^{(q+a)t_1}-1\\ -(q+a)t_1)\end{array}\right\}\\ +W-P(t_2-t_1)\\ -\frac{\widetilde{Pq}}{(\widetilde{q+b})^2}\\ \left\{e^{(\widetilde{q+b})(t_2-t_1)}-1-(\widetilde{q+b})(t_2-t_1)\right\}\end{array}\right]\end{array}\right].....eq(18)$$

We fuzzified the fuzzy TAC by Signed Distance Method as

$$TAC_S(t_1,t_2)=\frac{1}{4}\left[\begin{array}{l} TAC_{S1}(t_1,t_2)\\ ,TAC_{S2}(t_1,t_2)\\ ,TAC_{S3}(t_1,t_2)\end{array}\right]$$

Where

$$TAC_{S1}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l} A+\frac{r_1 p_1}{(q_1+a_1)^2}\left[e^{(q_1+a_1)t_1}-(q_1+a_1)t_1-1\right]\\ +o_1\left[\begin{array}{l}\frac{W}{b_1}(1-e^{-b_1t_1})+\\ \frac{p_1}{(q_1+b_1)^2}(e^{(q_1+b_1)(t_2-t_1)}-\\ (q_1+b_1)(t_2-t_1)-1)\end{array}\right]\\ -s_1k_1p_1(T-t_2)+l_1p_1(1-k_1)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\begin{array}{l}\frac{p_1a_1}{(q_1+a_1)^2}(e^{(q_1+a_1)t_1}-1\\ -(q_1+a_1)t_1)\end{array}\right\}\\ +W-p_1(t_2-t_1)-\frac{p_1q_1}{(q_1+b_1)^2}\\ \left\{e^{(q_1+b_1)(t_2-t_1)}-1-(q_1+b_1)(t_2-t_1)\right\}\end{array}\right]\end{array}\right]$$

$$TAC_{S2}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l} A+\frac{r_2 p_2}{(q_2+a_2)^2}\left[\begin{array}{l}e^{(q_2+a_2)t_1}\\ -(q_2+a_2)t_1-1\end{array}\right]\\ +o_2\left[\begin{array}{l}\frac{W}{b_2}(1-e^{-b_2t_1})+\frac{p_1}{(q_2+b_2)^2}\\ (e^{(q_2+b_2)(t_2-t_1)}-(q_2+b_2)(t_2-t_1)-1)\end{array}\right]\\ -s_2k_2p_2(T-t_2)+l_2p_2(1-k_2)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\begin{array}{l}\frac{p_2a_2}{(q_2+a_2)^2}(e^{(q_2+a_2)t_1}-1\\ -(q_2+a_2)t_1)\end{array}\right\}\\ +W-p_2(t_2-t_1)-\frac{p_2q_2}{(q_2+b_2)^2}\\ \left\{e^{(q_2+b_2)(t_2-t_1)}-1-(q_2+b_2)(t_2-t_1)\right\}\end{array}\right]\end{array}\right]$$

$$TAC_{S3}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l} A+\frac{r_3 p_3}{(q_3+a_3)^2}\left[e^{(q_3+a_3)t_1}-(q_3+a_3)t_1-1\right]\\ +o_2\left[\begin{array}{l}\frac{W}{b_3}(1-e^{-b_3t_1})+\frac{p_3}{(q_3+b_3)^2}\\ (e^{(q_3+b_3)(t_2-t_1)}-(q_3+b_3)(t_2-t_1)-1)\end{array}\right]\\ -s_3k_3p_3(T-t_2)+l_3p_3(1-k_3)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\frac{p_3a_3}{(q_3+a_3)^2}(e^{(q_3+a_3)t_1}-1-(q_3+a_3)t_1)\right\}\\ +W-p_3(t_2-t_1)-\frac{p_3q_3}{(q_3+b_3)^2}\\ \left\{e^{(q_3+b_3)(t_2-t_1)}-1-(q_3+b_3)(t_2-t_1)\right\}\end{array}\right]\end{array}\right]$$

$$TAC_S(t_1,t_2)=\frac{1}{4}\left[\begin{array}{l} TAC_{S1}(t_1,t_2)+2TAC_{S2}(t_1,t_2)\\ +TAC_{S3}(t_1,t_2)\end{array}\right].....eq(19)$$

To diminish $TAC_S(t_1,t_2)$, t_1 and t_2 can be derived by solving equations

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$$\frac{\delta TAC_S(t_1,t_2)}{\delta t_1}=0\ \&\ \frac{\delta TAC_S(t_1,t_2)}{\delta t_2}=0......eq(20)$$

$$\left[\begin{array}{l}\frac{1}{4T}\left\{\begin{array}{l}\left\{\frac{r_1P_1}{(q_1+a_1)}(e^{(q_1+a_1)t_1}-1)\right\}+o_1\left\{We^{-b_1t_1}+\frac{P_1}{q_1+a_1}(1-e^{(q_1+a_1)(t_2-t_1)})\right\}+\\ C\left\{P_1+\frac{P_1a_1}{(q_1+a_1)}(e^{(q_1+a_1)t_1}-1)-\frac{P_1q_1}{q_1+b_1}(1-e^{(q_1+b_1)(t_2-t_1)})\right\}\end{array}\right\}\\ +\frac{2}{T}\left\{\begin{array}{l}\left\{\frac{r_2P_2}{(q_2+a_2)}(e^{(q_2+a_2)t_1}-1)\right\}+o_2\left\{We^{-b_2t_1}+\frac{P_2}{q_2+a_2}(1-e^{(q_2+a_2)(t_2-t_1)})\right\}+\\ C\left\{P_2+\frac{P_2a_2}{(q_2+a_2)}(e^{(q_2+a_2)t_1}-1)-\frac{P_2q_2}{q_2+b_2}(1-e^{(q_2+b_2)(t_2-t_1)})\right\}\end{array}\right\}\\ +\frac{1}{T}\left\{\begin{array}{l}\left\{\frac{r_3P_3}{(q_3+a_3)}(e^{(q_3+a_3)t_1}-1)\right\}+o_3\left\{W_3e^{-\theta_3t_3}+\frac{P_3}{q_3+a_3}(1-e^{(q_3+a_3)(t_2-t_1)})\right\}+\\ C\left\{P_3+\frac{P_3a_3}{(q_3+a_3)}(e^{(q_3+a_3)t_1}-1)-\frac{P_3q_3}{q_3+b_3}(1-e^{(q_3+b_3)(t_2-t_1)})\right\}\end{array}\right\}\end{array}\right]=0$$

$$\left[\begin{array}{l}\frac{1}{4T}\left\{\begin{array}{l}o_1\left\{\frac{P_1}{q_1+b_1}(e^{(q_1+b_1)(t_2-t_1)}-1)\right\}+s_1P_1+\\ l_1(1-k)P_1-C\left\{\frac{P_1q_1}{q_1+b_1}(e^{(q_1+b_1)(t_2-t_1)}-1)+P_1\right\}\end{array}\right\}\\ +\frac{2}{T}\left\{\begin{array}{l}o_2\left\{\frac{P_2}{q_2+b_2}(e^{(q_2+b_2)(t_2-t_1)}-1)\right\}+s_2P_2+\\ l_2(1-k)P_2-C\left\{\frac{P_2q_2}{q_2+b_2}(e^{(q_2+b_2)(t_2-t_1)}-1)+P_2\right\}\end{array}\right\}\\ +\frac{1}{T}\left\{\begin{array}{l}o_3\left\{\frac{P_3}{q_3+b_3}(e^{(q_3+b_3)(t_2-t_1)}-1)\right\}+s_3P_3+\\ l_3(1-k)P_3-C\left\{\frac{P_3q_3}{q_3+b_3}(e^{(q_3+b_3)(t_2-t_1)}-1)+P_3\right\}\end{array}\right\}\end{array}\right]=0$$

Here, t_1 and t_2 can be derived by solving above equations will reduce fuzzy TAC if they satisfy the equations

$$\frac{\delta^2 TAC_S(t_1,t_2)}{\delta t_1^2}>0\ \&\ \frac{\delta^2 TAC_S(t_1,t_2)}{\delta t_2^2}>0\ \&$$

$$\left(\frac{\delta^2 TAC_S(t_1,t_2)}{\delta t_1^2}\right)\left(\frac{\delta^2 TAC_S(t_1,t_2)}{\delta t_2^2}\right)-\left(\frac{\partial^2 TAC_S(t_1,t_2)}{\partial t_1^2\partial t_2^2}\right)>0.$$

$....eq(21)$

We fuzzified the fuzzy TAC by Centroid Method as

$$TAC_C(t_1,t_2)=\frac{1}{3}\left[\begin{array}{l}TAC_{C1}(t_1,t_2),TAC_{C2}(t_1,t_2),\\ TAC_{C3}(t_1,t_2)\end{array}\right],$$

Where

$$TAC_{C1}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l}A+\frac{r_1p_1}{(q_1+a_1)^2}\left[e^{(q_1+a_1)t_1}-(q_1+a_1)t_1-1\right]\\ +o_1\left[\begin{array}{l}\frac{W}{b_1}(1-e^{-b_1t_1})+\frac{p_1}{(q_1+b_1)^2}\\ (e^{(q_1+b_1)(t_2-t_1)}-(q_1+b_1)(t_2-t_1)-1)\end{array}\right]\\ -s_1k_1p_1(T-t_2)+l_1p_1(1-k_1)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\frac{p_1a_1}{(q_1+a_1)^2}(e^{(q_1+a_1)t_1}-1-(q_1+a_1)t_1)\right\}\\ +W-p_1(t_2-t_1)-\frac{p_1q_1}{(q_1+b_1)^2}\\ \left\{e^{(q_1+b_1)(t_2-t_1)}-1-(q_1+b_1)(t_2-t_1)\right\}\end{array}\right]\end{array}\right],$$

$$TAC_{C2}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l}A+\frac{r_2p_2}{(q_2+a_2)^2}\left[e^{(q_2+a_2)t_1}-(q_2+a_2)t_1-1\right]\\ +o_2\left[\begin{array}{l}\frac{W}{b_2}(1-e^{-b_2t_1})+\frac{p_1}{(q_2+b_2)^2}\\ (e^{(q_2+b_2)(t_2-t_1)}-(q_2+b_2)(t_2-t_1)-1)\end{array}\right]\\ -s_2k_2p_2(T-t_2)+l_2p_2(1-k_2)(T-t_2)\\ +C\left[\begin{array}{l}\left\{\frac{p_2a_2}{(q_2+a_2)^2}(e^{(q_2+a_2)t_1}-1-(q_2+a_2)t_1)\right\}\\ +W-p_2(t_2-t_1)-\frac{p_2q_2}{(q_2+b_2)^2}\\ \left\{e^{(q_2+b_2)(t_2-t_1)}-1-(q_2+b_2)(t_2-t_1)\right\}\end{array}\right]\end{array}\right]$$

and

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$$TAC_{C3}(t_1,t_2)=\frac{1}{T}\left[\begin{array}{l} A+\dfrac{r_3p_3}{(q_3+a_3)^2}\left[e^{(q_3+a_3)t_1}-(q_3+a_3)t_1-1\right] \\ +o_2\left[\begin{array}{l}\dfrac{W}{b_3}(1-e^{-b_3t_1})+\dfrac{p_3}{(q_3+b_3)^2} \\ (e^{(q_3+b_3)(t_2-t_1)}-(q_3+b_3)(t_2-t_1)-1)\end{array}\right] \\ -s_3k_3p_3(T-t_2)+l_3p_3(1-k_3)(T-t_2) \\ +C\left[\begin{array}{l}\left\{\dfrac{p_3a_3}{(q_3+a_3)^2}(e^{(q_3+a_3)t_1}-1-(q_3+a_3)t_1)\right\} \\ +W-p_3(t_2-t_1)-\dfrac{p_3q_3}{(q_3+b_3)^2} \\ \left\{e^{(q_3+b_3)(t_2-t_1)}-1-(q_3+b_3)(t_2-t_1)\right\}\end{array}\right]\end{array}\right]$$

Then

$$TAC_C(t_1,t_2)=\frac{1}{3}\left[TAC_{C1}(t_1,t_2)+TAC_{C2}(t_1,t_2)+TAC_{C3}(t_1,t_2)\right].$$

$....eq(22)$

To diminish $TAC_C(t_1,t_2)$, the value of t_1 and t_2 can be derived by solving equations

$$\frac{\delta TAC_C(t_1,t_2)}{\delta t_1}=0\ \&\ \frac{\delta TAC_C(t_1,t_2)}{\delta t_2}=0......eq(23)$$

Equation (23) are equal to

$$\left[\begin{array}{l}\dfrac{1}{3T}\left\{\begin{array}{l}\left\{\dfrac{r_1P_1}{(q_1+a_1)}(e^{(q_1+a_1)t_1}-1)\right\}+ \\ o_1\left\{We^{-b_1t_1}+\dfrac{P_1}{q_1+b_1}(1-e^{(q_1+b_1)(t_2-t_1)})\right\}+ \\ C\left\{\begin{array}{l}P_1+\dfrac{P_1a_1}{(q_1+a_1)}(e^{(q_1+a_1)t_1}-1) \\ -\dfrac{P_1q_1}{q_1+b_1}(1-e^{(q_1+b_1)(t_2-t_1)})\end{array}\right\}\end{array}\right\} \\ +\dfrac{1}{T}\left\{\begin{array}{l}\left\{\dfrac{r_2P_2}{(q_2+a_2)}(e^{(q_2+a_2)t_1}-1)\right\}+ \\ o_2\left\{We^{-b_2t_1}+\dfrac{P_2}{q_2+b_2}(1-e^{(q_2+b_2)(t_2-t_1)})\right\}+ \\ C\left\{\begin{array}{l}P_2+\dfrac{P_2a_2}{(q_2+a_2)}(e^{(q_2+a_2)t_1}-1) \\ -\dfrac{P_2q_2}{q_2+b_2}(1-e^{(q_2+b_2)(t_2-t_1)})\end{array}\right\}\end{array}\right\} \\ +\dfrac{1}{T}\left\{\begin{array}{l}\left\{\dfrac{r_3P_3}{(q_3+a_3)}(e^{(q_3+a_3)t_1}-1)\right\}+ \\ o_3\left\{We^{-b_3t_3}+\dfrac{P_3}{q_3+b_3}(1-e^{(q_3+b_3)(t_2-t_1)})\right\}+ \\ C\left\{\begin{array}{l}P_3+\dfrac{P_3a_3}{(q_3+a_3)}(e^{(q_3+a_3)t_1}-1)- \\ \dfrac{P_3q_3}{q_3+b_3}(1-e^{(q_3+b_3)(t_2-t_1)})\end{array}\right\}\end{array}\right\}\end{array}\right]=0$$

and

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$$\begin{bmatrix} \frac{1}{3T}\begin{Bmatrix} o_1\left\{\frac{P_1}{q_1+b_1}(e^{(q_1+b_1)(t_2-t_1)}-1)\right\}+s_1k_1P_1 \\ -l_1(1-k)P_1-C\left\{\frac{P_1q_1}{q_1+b_1}(e^{(q_1+b_1)(t_2-t_1)}-1)+P_1\right\} \end{Bmatrix} \\ +\frac{2}{T}\begin{Bmatrix} o_2\left\{\frac{P_2}{q_2+b_2}(e^{(q_2+b_2)(t_2-t_1)}-1)\right\}+s_2k_2P_2 \\ -l_2(1-k)P_2-C\left\{\frac{P_2q_2}{q_2+b_2}(e^{(q_2+b_2)(t_2-t_1)}-1)+P_2\right\} \end{Bmatrix} \\ +\frac{1}{T}\begin{Bmatrix} o_3\left\{\frac{P_3}{q_3+b_3}(e^{(q_3+b_3)(t_2-t_1)}-1)\right\}+s_3k_3P_3 \\ -l_3(1-k)P_3-C\left\{\frac{P_3q_3}{q_3+b_3}(e^{(q_3+b_3)(t_2-t_1)}-1)+P_3\right\} \end{Bmatrix} \end{bmatrix}=0$$

The value of t_1 and t_2 can be derived by solving above equations will reduce fuzzy TAC if they satisfy the equations

$$\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_1^2}>0\ \&\ \frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_2^2}>0\ \&$$
$$\left(\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_1^2}\right)\left(\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_2^2}\right)-\left(\frac{\partial^2 TAC_C(t_1,t_2)}{\partial t_1^2 \partial t_2^2}\right)>0.....eq(24)$$

$$\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_1^2}>0\ \&\ \frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_2^2}>0\ \&$$
$$\left(\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_1^2}\right)\left(\frac{\delta^2 TAC_C(t_1,t_2)}{\delta t_2^2}\right)-\left(\frac{\partial^2 TAC_C(t_1,t_2)}{\partial t_1^2 \partial t_2^2}\right)>0.....eq(24)$$

5. Numerical Example

To explain the outcomes of offered structure, assess an algorithmic exemplar of inventory structure with the presented values:

5.1 Crisp Structure

Let

$P=200, q=10.8, A=100, W=100, C=50, s=15,$

$a=0.07, b=0.05, k=0.08, h_o=8, h_r=10, \pi=8$

Then, t_1 =1.112, t_2=1.189, TAC=10713.285

Let

$P=(100,200,300), q=(9.8,10.8,11.8),$

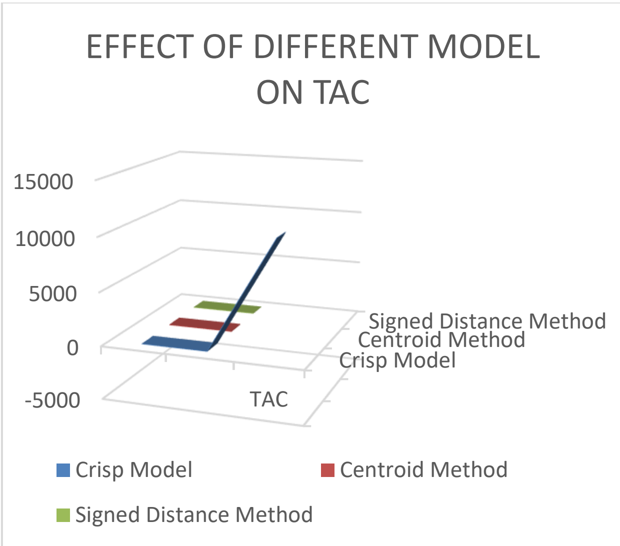
$\tilde{s}=(14,15,16), \pi=(7,8,9),$

$a_r=(0.06,0.07,0.08), h_o=(7,8,9),$

$b_0=(0.04,0.05,0.06), h_r=(9,10,11)$

$k=(0.07,0.08,0.09),$

	t_1	t_2	TAC
Crisp Model	1.112	1.189	10713.285
Centroid Method	-1.93	2.003	-3.145×10^{21}
Signed Distance Method	36.51	36.44	1.8832×10^{20}



6. Sensitivity Analysis

From Table 1 to 3, we looked over the structure variables with distinct values in fuzzy, keeping other variables in its exemplar values. Here SDM (Signed distance method) & CM (Centroid Method)

5.2 Fuzzy Structure

Table- 1 (Sensitiveness of holding cost variable h_r)

	SDM			CM		
h_r	**t_1**	**t_2**	**TAC**	**t_1**	**t_2**	**TAC**

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(9,10,11)	-0.928	-1.005	-1520.662	0.0601	-0.017	1247.597
(10,11,12)	-1.04	-1.117	-1519.146	-1.125	-1.001	932.887
(11,12,13)	-1.667	-1.542	-1489.009	-1.595	-1.469	1212.313

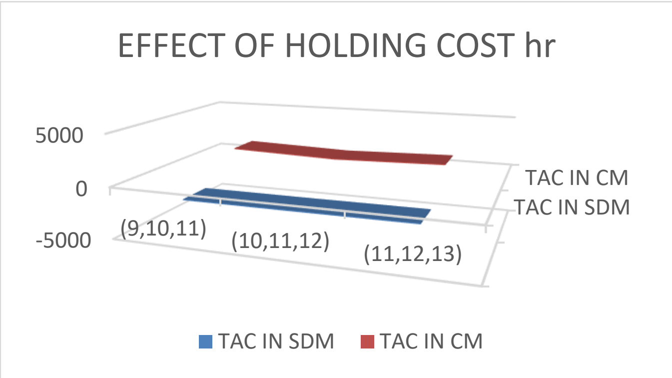

Table- 2 (Sensitiveness of holding cost variable h_o)

	SDM			**CM**		
h_o	**t_1**	**t_2**	**TAC**	**t_1**	**t_2**	**TAC**
(7,8,9)	-1.111	-1.188	1243.415	-1.94	-2.023	1246.640
(8,9,10)	-0.132	-0.028	1269.380	-1.104	-1.025	1244.147
(9,10,11)	-0.082	-0.0206	12585.980	-1.174	-1.250	1249.245

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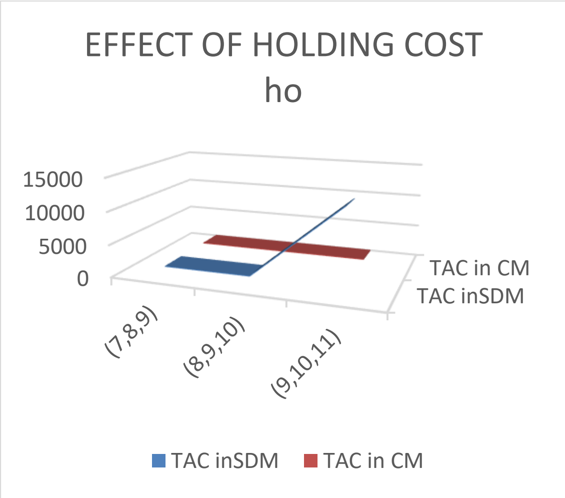
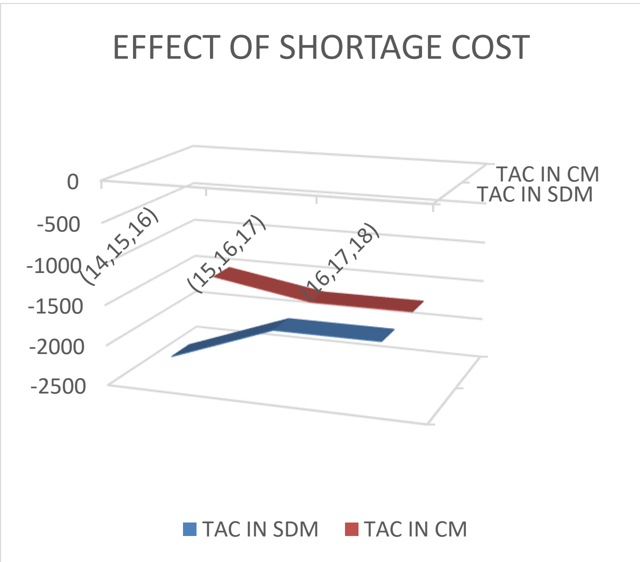


Table- 3(Sensitiveness of shortage cost variable s)

	SDM			CM		
S	**t_1**	**t_2**	**TAC**	**t_1**	**t_2**	**TAC**
(14,15, 16)	-0.811	-0.710	-2190.422	0.462	0.350	-1510.757
(15,16, 17)	-0.807	-0.884	-1721.749	-1.104	-1.182	-1724.939
(16,17, 18)	-0.804	-0.881	-1721.475	-1.099	-1.177	-1721.920



Observation: - 1. When we take holding cost variable in fuzzy and other variables in crisp, if h_r rise then in both SDM & CM method t_1, t_2 rise $\&TAC<0$.

2. When we take holding cost variable in fuzzy and other variables in crisp, if h_o rise then in both SDM & CM method t_1 & t_2 rise $\&TAC$ rises.

3. When we take shortage cost variable in fuzzy and other variables in crisp, if S rise then in both SDM & CM method t_1 rise, t_2 rises $\&TAC<0$.

CONCLUSION

Here, the inventory structure for deciding the excellent cost. The rate is based on stock and time. Shortage is permitted and partly backlogged. The rate of backloading is changeable. An accessible method is put forwarded to acquire the best cost and cycle of ordering that enlarge the entire cost. Numerical works are provided to demonstrate the procedure. The developed structure is explored for both crisp and fuzzy parameters. The cost components have been taken as triangular fuzzy numbers. The main motive is to find the best result to enlarge entire cost. We noticed that the entire cost reduces in fuzzy when compared to crisp. So, the crisp structure gives the more gain than the fuzzy. This structure can be expanded for the following features such as constant decaying rate, permissible delay in payments and multi items.

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