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A Measure of Modified Second Order Slope Rotatable Designs using Pair Wise Balanced Designs

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Abstract

In this paper, a new measure of modified second order slope rotatable designs which enables us to assess the degree of modified slope rotatability for a given response surface design using pair wise balanced designs is suggested for $6 \le v \le 15$ (v-stands for number of factors).

Some keywords and phrases*: Slope rotatable designs, modified slope rotatable designs, measure of modified slope rotatable designs.*

1. Introduction

The property of rotatability was proposed by Box and Hunter [1] for series of response surface designs. Das and Narasimham [2] developed rotatable designs using balanced incomplete block designs (BIBD). Second and third order rotatable designs using pairwise balanced designs (PBD) and doubly balanced designs was altered by Tyagi [6]. Das et al. [3] suggested modified response surface designs. Victorbabu et al. [18] recommended modified response surface designs, rotatable designs using pairwise balanced designs. Victorbabu et al. [19] developed modified rotatable central composite designs. A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design centre. The study of rotatable designs is mainly emphasized on the estimation of differences of yields and its precision. Estimation of differences in responses at two different points in the factor space will often be of great importance. If differences in responses at two points close together are of interest then estimation of local slope (rate of change) of the response is required. Hader and Park [4] presented slope rotatability for central composite designs. Victorbabu and Narasimham [11] further examined in detail the conditions to be satisfied by a common second order slope rotatable designs (SOSRD) and constructed SOSRD using BIBD. Victorbabu and Narasimham [12] constructed SOSRD using PBD. Victorbabu [7] examined Modified slope rotatable central composite designs (SRCCD). Later several methods on modified SOSRD using PBD and BIBD were suggested by Victorbabu [8, 9]. Measure of slope rotatable designs was developed by Park and Kim [5]. Victorbabu and Surekha [15], [16], [17]studied several methods for measure of second order slope rotatable designs using central composite designs (CCD), BIBD and PBD respectively. Victorbabu and Jyostna [13], [14] gave measure of modified SOSRD using CCD and BIBD respectively. In this paper, measure of modified SOSRD using PBD is studied. These measures are useful to enable us to assess the degree of modified slope rotatability for a given second order response surface designs.

2. Conditions for Second Order Slope Rotatable Designs

In this section a second order response surface design $D=((x_{iu}))$ to fitting,

$$Y_u = \beta_0 + \sum_{i=1}^{v} \beta_i x_{iu} + \sum_{i=1}^{v} \beta_{ii} x_{iu}^2 + \sum_{i=1}^{v}\sum_{j=1}^{v} \beta_{ij} x_{iu} x_{ju} + e_u \quad (2.1)$$

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where x_{iu} denotes the level of the i^{th} factor $(i=1,2,\ldots,v)$ in the u^{th} run $(u=1,2,\ldots,N)$ of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2. D is said to be SOSRD if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance $(d^2=\sum_{i=1}^{v} x_i^2)$ of the point $(x_1,x_2,\ldots,x_v)$ from the origin of the design.

The general conditions for second order slope rotatability can be obtained as follows. [cf. Box and Hunter [1], Hader and Park [4] and Victorbabu and Narasimham [11]]

1. $\sum_{u=1}^{N}\prod_{i=1}^{v} x_{iu}^{\gamma_i} = 0$ if any γ_i is odd, for $\sum \gamma_i \le 4$

2. (i) $\sum x_{iu}^2 = \text{constant} = N\mu_2$;

   (ii) $\sum x_{iu}^4 = \text{constant} = cN\mu_4$; for all i

3. $\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\mu_4$; for $i \ne j$ (2.2)

where c, μ_2 and μ_4 are constants.

The variances and covariances of the estimated parameters become

$$V(\hat{\beta}_0) = \frac{\mu_4(c+v-1)\sigma^2}{N\left[\mu_4(c+v-1)-v\mu_2^2\right]},$$

$$V(\hat{\beta}_i) = \frac{\sigma^2}{N\mu_2},$$

$$V(\hat{\beta}_{ij}) = \frac{\sigma^2}{N\mu_4},$$

$$V(\hat{\beta}_{ii}) = \frac{\sigma^2}{(c-1)N\mu_4}\left[\frac{\mu_4(c+v-2)-(v-1)\mu_2^2}{\mu_4(c+v-1)-v\mu_2^2}\right],$$

$$Cov(\hat{\beta}_0,\hat{\beta}_{ii}) = \frac{-\mu_2\sigma^2}{N[\mu_4(c+v-1)-v\mu_2^2]},$$

$$Cov(\hat{\beta}_{ii},\hat{\beta}_{jj}) = \frac{(\mu_2^2-\mu_4)\sigma^2}{(c-1)N\mu_4[\mu_4(c+v-1)-v\mu_2^2]} \quad (2.3)$$

and other covariances are zero.

An inspection of the variance of $\hat{\beta}_0$ shows that a necessary condition for the existence of a non-singular second order design is

4. $\frac{\mu_4}{\mu_2^2} > \frac{v}{(c+v-1)}$ (2.4)

The condition (2.4) exists then only the design exists.

For the second order model

$$\frac{\partial \hat{Y}}{\partial x_i} = \hat{\beta}_i + 2\hat{\beta}_{ii}x_{iu} + \sum_{j\ne i}\hat{\beta}_{ij}x_{ju} \quad (2.5)$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right) = V(\hat{\beta}_i) + 4x_{iu}^2 V(\hat{\beta}_{ii}) + \sum_{j\ne i} x_{ju}^2 V(\hat{\beta}_{ij}) \quad (2.6)$$

The condition for RHS of the equation (2.6) to be a function of $d^2=\sum_{i=1}^{v} x_i^2$ alone (for slope rotatability) is

$$4V(\hat{\beta}_{ii}) = V(\hat{\beta}_{ij}) \quad (2.7)$$

On simplification of (2.7), using (2.3) we get,

5. $[v(5-c)-(c-3)^2]\mu_4 + [v(c-5)+4]\mu_2^2 = 0$ (2.8)

Therefore 1, 2 and 3 of (2.2), (2.4) and (2.8) give a set of conditions for slope rotatability in any general second order response surface design (cf. Hader and Park [4], Victorbabu and Narasimham [11]).

3. Conditions for Modified Second Order Slope Rotatable Designs

Following [4], [11], equations 1, 2 and 3 of (2.2), (2.3), (2.4) and (2.8) give the necessary and sufficient conditions for modified second order slope rotatable designs (cf. Victorbabu, [7], [9]).

The usual method of construction of symmetrical designs is to take some combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at +1 and -1levels to make the level codes equidistant. All such combinations form a design. Generally SOSRD need at least five levels (suitably coded) at $0, \pm 1, \pm a$ for all factors $((0,0,\ldots,0)$ – chosen center of the design, unknown level 'a' to be chosen suitably to satisfy slope rotatability). Generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknowns. Fixing the unknowns arbitrarily also gives a design without associating the design with any property. Alternatively by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives some of the unknowns and the rest. In SOSRD the restriction used is $V(\beta_{ij})=4V(\beta_{ii})$ viz. equation (2.8). Other restrictions are also possible though, it seems, not exploited well. We shall investigate the restriction

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$$\left(\sum x_{iu}^2\right)^2 = N\sum x_{iu}^2 x_{ju}^2$$

i.e., $\left(N\mu_2\right)^2 = N\left(N\mu_4\right)$ i.e., $\mu_2^2=\mu_4$ to get modified SOSRD. By applying the new restriction in equation (2.8), we get $c=1$ or $c=5$. The non-singularity condition (2.4) leads to $c=5$. It may be noted $\mu_2^2=\mu_4$ and $c=5$ are equivalent conditions. The variances and covariances of the estimated parameters are,

$$V(\hat{\beta}_0)=\frac{(v+4)\sigma^2}{4N},$$

$$V(\hat{\beta}_i)=\frac{\sigma^2}{N\sqrt{\mu_4}},$$

$$V(\hat{\beta}_{ij})=\frac{\sigma^2}{N\mu_4},$$

$$V(\hat{\beta}_{ii})=\frac{\sigma^2}{4N\mu_4}$$

$$Cov(\hat{\beta}_0,\hat{\beta}_{ii})=\frac{-\sigma^2}{4N\sqrt{\mu_4}} \text{ and other covariances are zero.} \tag{3.1}$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right)=\left[\frac{\sqrt{\mu_4}+d^2}{N\mu_4}\right]\sigma^2.$$

4. Conditions of Measure of Second Order Slope Rotatable Designs

Following [4],[11], [5], equations (2.2), (2.3), (2.4) and (2.8) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further we have,

$V\left(\beta_i\right)$ are equal for all i,

$V\left(\beta_{ii}\right)$ are equal for all i,

$V\left(\beta_{ij}\right)$ are equal for all i, j, where $i \neq j$,

$Cov\left(\beta_i,\beta_{ii}\right) = Cov\left(\beta_i,\beta_{ij}\right) = Cov\left(\beta_{ii},\beta_{ij}\right) = Cov\left(\beta_{ij},\beta_{il}\right)$ for all $i \neq j \neq l$.

Then the following measure assess the degree of slope rotatability for a design D with v independent variables (cf. Park and Kim, [5], page 398).

$$\begin{aligned}
Q_v(D)=\frac{1}{2(v-1)\sigma^4}\Bigg\{&(v+2)(v+4)\sum_{i=1}^{v}\left[\left(v(\beta_i)-\frac{1}{v}\sum_{i=1}^{v}v(\beta_i)\right)+\frac{\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)-\frac{1}{v}\sum_{i=1}^{v}\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)}{v+2}\right]^2\\
&+\frac{4}{v(v+2)}\sum_{i=1}^{v}\left(\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)-\frac{1}{v}\sum_{i=1}^{v}\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)\right)^2+2\sum_{i=1}^{v}\left[\left[4v(\beta_{ii})-\frac{\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)}{v}\right]^2+\sum_{\substack{j=1\\j\neq i}}^{v}\left[v(\beta_{ij})-\frac{\left(4v(\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}v(\beta_{ij})\right)}{v}\right]^2\right]\\
&+4(v+4)\left[4cov^2(\beta_i,\beta_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}cov^2(\beta_i,\beta_{ij})\right]+4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\j\neq i}}^{v}cov^2(\beta_{ii},\beta_{ij})+\sum_{\substack{j<l\\j,l\neq i}}^{v}\sum cov^2(\beta_{ij},\beta_{il})\right]\Bigg\}
\end{aligned} \tag{41}$$

where $Q_v\left(D\right)$ is the measure of slope-rotatability. It can be verified that $Q_v\left(D\right)$ is zero if and only if a design D is slope-rotatable. Measure of slope rotatability becomes larger as D deviates from a slope-rotatable design. Further (4.1) is simplified to

$$Q_V(D)=\frac{1}{\sigma^4}\left[4V(\beta_{ii})-V(\beta_{ij})\right]^2.$$

5. Modified SOSRD Using Pairwise Balanced Designs

Pair wise balanced designs: The arrangement of v treatments in b blocks will be called a PBD of index λ and type $(v, k_1, k_2, \ldots, k_m)$ if each block contains $k_1, k_2, \ldots, k_m$ treatments $(k_i \leq v, k_i \neq k_j)$ and every pair of distinct treatments occurs in exactly λ blocks of size $k_i (i=1,2,\ldots,m)$ then

$$b=\sum_{i=1}^{m} b_i \text{ and } \lambda v(v-1)=b=\sum_{i=1}^{m} b_i k_i (k_i-1).$$

Let $(v, b, r, k_1, k_2, \ldots, k_m, \lambda)$, be an equi-replicated PBD and $k=\max(k_1,k_2,\ldots,k_m)$. Let $2^{t(k)}$ denote a resolution V fractional factorial of 2^k with 1 or -1 levels, such that no interaction with less than five factors is confounded. $[1-(v, b, r, k_1, k_2, \ldots k_m, \lambda)]$ denote the design points generated from the transpose of incidence matrix of PBD, $[1-(v, b, r, k_1, k_2, \ldots k_m, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by multiplication (cf. Das and Narasimham, [2]).

Let $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set. Repeat this set of additional design points n_a times when $r < 5\lambda$.

Consider the design points, $[1-(v,b,r,k_1,k_2,...k_m,\lambda)]2^{t(k)} \cup n_a(a,0,0,...,0)2^1 \cup (n_0)$ will give a v dimensional modified SOSRD in $N=\frac{(r2^{t(k)}+2n_a a^2)^2}{\lambda 2^{t(k)}}$ design points if,

$$a^4=\frac{(5\lambda\text{-}r)2^{t(k)}}{2n_a}, \qquad (5.1)$$

$$n_0=\frac{(r2^{t(k)}+2n_a a^2)^2}{\lambda 2^{t(k)}} - b2^{t(k)} - 2n_a v \qquad (5.2)$$

and n_0 turns out to be an integer.

6. Measure of Second Order Slope Rotatable Designs Using Pbd

The PBD design points, $[1-(v,b,r,k_1,k_2,...k_m,\lambda)]2^{t(k)} \cup n_a(a,0,0,...,0)2^1 \cup ($ will give a v dimensional measure of second order slope rotatable designs using PBD in $N{=}b2^{t(k)}+2vn_a+n_0$ design points, as follows:

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e\text{-}V(\beta_{ij})\right]^2$$

$$\text{where } e=V(\beta_{ii})=\frac{(v\text{-}1)\left[2^{t(k)}\lambda n_0+2^{t(k)+1}\lambda v n_a-2^{t(k)+2}r n_a a^2-r^2 2^{2t(k)}+b\lambda 2^{2t(k)}\right]+\left[2^{t(k)+1}bn_a+2n_a n_0+4n_a^2\right]a^4+(r-\lambda)\left[2^{t(k)+1}vn_a+2^{t(k)}n_0+b2^{2t(k)}\right]}{\left[2^{t(k)}(r\text{-}\lambda)+2n_a a^4\right]\left[v2^{t(k)}\left(\lambda n_0-4rn_a a^2\right)+2^{t(k)+1}n_a\left(v^2\lambda+ba^4\right)+2n_a n_0 a^4+(r-\lambda)\left(b2^{2t(k)}+2^{t(k)+1}vn_a+2^{t(k)}n_0\right)+2^{2t(k)}v\left(b\lambda-r^2\right)\right]}$$

It can be verified that $Q_v(D)$ is zero, if and only if, a design 'D' is slope-rotatable. Measure of slope rotatability becomes larger as 'D' deviates from a Slope Rotatable Design (cf. Park and Kim [5], Surekha and Victorbabu [17]).

7. Measure of Modified Second Order Slope Rotatable Designs Using Pair wise Balanced Designs

A new measure of modified SOSRD using PBD is given here. The parameters of the PBD are denoted by $(v,b,r,k_1,k_2,...,k_m,\lambda)$. Then the design points, $[1-(v,b,r,k_1,k_2,...,k_m,\lambda)]2^{t(k)} \cup n_a(a,0,0,...,0)2^1 \cup (n_0)$ generated from PBD in $N=\frac{(r2^{t(k)}+2n_a a^2)^2}{\lambda 2^{t(k)}}$ design points, will give a measure of modified second order slope rotatable designs using PBD with

$$a^4=\frac{(5\lambda\text{-}r)2^{t(k)}}{2n_a}, \qquad (7.1)$$

$$n_0=\frac{(r2^{t(k)}+2n_a a^2)^2}{\lambda 2^{t(k)}} - b2^{t(k)} - 2n_a v \qquad (7.2)$$

and n_0 turns out to be an integer.

(Alternatively N directly obtained as $N{=}b2^{t(k)}+2vn_a+n_0$ design points)

For the design points the conditions in equations of 1, 2, 3 of (2.2) are satisfied. The conditions in equation 2 and 3 of (2.2) are verified as follows:

$$2.\text{(i)} \sum x_{iu}^2=r2^{t(k)}+2n_a a^2=N\mu_2 \qquad (7.3)$$

$$\text{(ii)} \sum x_{iu}^4=r2^{t(k)}+2n_a a^4=5N\mu_4 \qquad (7.4)$$

$$3. \sum x_{iu}^2 x_{ju}^2=\lambda 2^{t(k)}=N\mu_4 \qquad (7.5)$$

From equations (7.3), (7.4) and (7.5), and on simplification, we get

$$a^4=\frac{(5\lambda\text{-}r)2^{t(k)\text{-}1}}{n_a},$$

$$\mu_2=\frac{r2^{t(k)}+2n_a a^2}{N} \text{ and } \mu_4=\frac{\lambda 2^{t(k)}}{N}.$$

To get measure of modified second order slope rotatable designs using PBD we investigate the modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e., $(N\mu_2)^2{=}N(N\mu_4)$ i.e., $\mu_2^2{=}\mu_4$ (cf. Victorbabu, [7], [9]) and on simplification we get

$$n_0=\frac{(r2^{t(k)}+2n_a a^2)^2}{\lambda 2^{t(k)}}\text{-}b2^{t(k)}\text{-}2n_a v.$$

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e\text{-}v(\hat{\beta}_{ij})\right]^2$$

$$\text{where } e=V(\hat{\beta}_{ii})=\frac{b2^{t(k)}+2vn_a+n_0}{4[r^2 2^{2t(k)}+4r2^{t(k)}n_a a^2+4n_a^2 a^4]} \quad \left(\text{since } \mu_2^2=\mu_4\right) \qquad (7.6)$$

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Measure of modified second order slope rotatable designs using PBD

$$Q_v(D) = \left[\frac{r2^{t(k)}+2n_a a^2}{N}\right]^4 \left[4e-\frac{1}{\lambda 2^{t(k)}}\right]^2 \qquad (7.7)$$

Example: We illustrate the measure of modified SOSRD for v=9 factors with the help of a PBD. The design points,

$$[1\text{-}(v=9,\ b=15,\ r=6,\ k_1=4,\ k_2=3,\ \lambda=2)]2^4 \cup n_a(a,0,\ldots,0)2^1 \cup n_0$$

will give a measure of modified SOSRD in $N=392$ design points. We have from equations (7.3), (7.4) and (7.5) are

$$\sum x_{iu}^2 = 96+2n_a a^2 = N\mu_2 \qquad (7.8)$$

$$\sum x_{iu}^4 = 96+2n_a a^4 = 5N\mu_4 \qquad (7.9)$$

$$\sum x_{iu}^2 x_{ju}^2 = 32 = N\mu_4 \qquad (7.10)$$

Equations (7.9) and (7.10) leads to $n_a a^4 = 32$, which implies $a^2=4$ for $n_a=2$. From equations (7.8), (7.10) using the modified condition $(\mu_2^2=\mu_4)$ with $a^2=4$ and $n_a=2$, we get $N=392, n_0=116$. At $a=2$, we get $e=7.8125\times10^{-3}$ then measure of modified slope rotatability is zero. Then the design is modified slope rotatable. Variance of the estimated response for measure of modified second order slope rotatable designs using PBD is

$$V\left(\frac{\partial \hat{y}}{\partial x_i}\right) = 0.0089\sigma^2+0.03125d^2\sigma^2$$

Suppose if we take $a=3.5$ instead of taking $a=2$ for 9 factors we get $e=4.6611\times10^{-3}$ then $Q_v(D)=2.9747\times10^{-6}$. Here measure of modified slope rotatability becomes larger it deviates from modified slope rotatability. Variance of the estimated response for measure of modified second order slope rotatable designs using PBD is

$$V\left(\frac{\partial \hat{y}}{\partial x_i}\right) = 0.0069\sigma^2+0.0186d^2\sigma^2 .$$

Table gives the values of measure of modified second order slope rotatable designs using PBD, at different values of 'a' for $6 \le v \le 15$. It can be verified that measure of modified slope rotatability is zero, if and only if a design 'D' is modified slope rotatable. Measure of modified slope rotatability becomes larger as 'D' deviates from a modified slope rotatable design. Variance of the estimated responses for measure of modified second order slope rotatable designs using PBDfor different values of 'a' is also included in the table.

8. Conclusion

In this paper, measure of modified second order slope rotatable designs using PBD has been proposed which enables us to assess the degree of slope rotatability for a given response surface design. This measure of modified second order slope rotatable designs using PBD, measure of modified slope rotatability [$Q_v(D)$] is zero, if and only if, the design 'D' is modified slope rotatable design, and becomes larger as 'D' deviates from a modified slope rotatable design. Variances of the estimated response for measure of modified second order slope rotatable designs using PBD are also obtained.

Table Measure of modified second order slope rotatable designs [$Q_v(D)$] and variances of the estimated response $V\left(\partial\hat{y}/\partial x_i\right)$ using PBD

$(6,7,3,2,3,3,4,1), N=128, a=1.4142, n_0$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	3.3528×10^{-6}	$0.0357\sigma^2+0.1633d^2\sigma^2$
1.4142	0	$0.03125\sigma^2+0.125d^2\sigma^2$
1.5	2.4593×10^{-7}	$0.0303\sigma^2+0.1175d^2\sigma^2$
2.0	1.9312×10^{-5}	$0.025\sigma^2+0.08d^2\sigma^2$
2.5	1.1037×10^{-4}	$0.0204\sigma^2+0.0533d^2\sigma^2$
3.0	3.8625×10^{-4}	$0.0167\sigma^2+0.0356d^2\sigma^2$
3.5	1.0788×10^{-3}	$0.0137\sigma^2+0.024d^2\sigma^2$
4.0	2.6286×10^{-3}	$0.0114\sigma^2+0.0165d^2\sigma^2$

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$(8,15,6,4,3,2,2), N=392, a=2, n_0=120, n_a=2$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	2.6766×10^{-7}	$0.01\,\sigma^2+0.0392\,d^2\sigma^2$
1.5	6.8466×10^{-9}	$0.0095\,\sigma^2+0.0356\,d^2\sigma^2$
2.0	0	$0.0089\,\sigma^2+0.0313\,d^2\sigma^2$
2.5	1.8187×10^{-7}	$0.0083\,\sigma^2+0.0268\,d^2\sigma^2$
3.0	9.8491×10^{-7}	$0.0076\,\sigma^2+0.0225\,d^2\sigma^2$
3.5	2.9747×10^{-6}	$0.0069\,\sigma^2+0.0186\,d^2\sigma^2$
4.0	7.0498×10^{-6}	$0.0063\,\sigma^2+0.0153\,d^2\sigma^2$

$(9,15,6,4,3,2), N=392, a=2, n_0=116, n_a=2$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	2.6766×10^{-7}	$0.01\,\sigma^2+0.0392\,d^2\sigma^2$
1.5	6.8466×10^{-9}	$0.0095\,\sigma^2+0.0356\,d^2\sigma^2$
2.0	0	$0.0089\,\sigma^2+0.0313\,d^2\sigma^2$
2.5	1.8187×10^{-7}	$0.0083\,\sigma^2+0.0268\,d^2\sigma^2$
3.0	9.8491×10^{-7}	$0.0076\,\sigma^2+0.0225\,d^2\sigma^2$
3.5	2.9747×10^{-6}	$0.0069\,\sigma^2+0.0186\,d^2\sigma^2$
4.0	7.0498×10^{-6}	$0.0063\,\sigma^2+0.0153\,d^2\sigma^2$

$(10,11,5,5,4,2), N=450, a=1.4142, n_0=74, n_a=10$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	2.2955×10^{-7}	$0.01\,\sigma^2+0.045\,d^2\sigma^2$

1.4142	0	$0.0018\,\sigma^2 + 0.03125\,d^2\sigma^2$
1.5	6.2062×10^{-8}	$0.008\,\sigma^2 + 0.0288\,d^2\sigma^2$
2.0	2.9873×10^{-6}	$0.0063\,\sigma^2 + 0.0176\,d^2\sigma^2$
2.5	1.8174×10^{-5}	$0.0049\,\sigma^2 + 0.0107\,d^2\sigma^2$
3.0	6.7402×10^{-5}	$0.0038\,\sigma^2 + 0.0067\,d^2\sigma^2$
3.5	1.9819×10^{-4}	$0.0031\,\sigma^2 + 0.0043\,d^2\sigma^2$
4.0	5.0486×10^{-4}	$0.0025\,\sigma^2 + 0.0028\,d^2\sigma^2$

$(12,16,6,6,5,4,3,2)$, $N=676$, $a=2.82843$, $n_0=148$, $n_a=1$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	3.7031×10^{-8}	$0.0052\,\sigma^2 + 0.018\,d^2\sigma^2$
1.5	2.5298×10^{-8}	$0.0051\,\sigma^2 + 0.0175\,d^2\sigma^2$
2.0	1.2455×10^{-8}	$0.005\,\sigma^2 + 0.0169\,d^2\sigma^2$
2.5	2.4369×10^{-9}	$0.0049\,\sigma^2 + 0.0162\,d^2\sigma^2$
2.82843	0	$0.0048\,\sigma^2 + 0.0156\,d^2\sigma^2$
3.0	8.1708×10^{-10}	$0.0047\,\sigma^2 + 0.0153\,d^2\sigma^2$
3.5	1.5221×10^{-8}	$0.0046\,\sigma^2 + 0.0144\,d^2\sigma^2$
4.0	5.5855×10^{-8}	$0.0044\,\sigma^2 + 0.0135\,d^2\sigma^2$

$(13,16,6,6,5,4,3,2)$, $N=676$, $a=2.82843$, $n_0=138$, $n_a=1$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	3.7031×10^{-8}	$0.0052\,\sigma^2 + 0.018\,d^2\sigma^2$
1.5	2.5298×10^{-8}	$0.0051\,\sigma^2 + 0.0175\,d^2\sigma^2$
2.0	1.2455×10^{-8}	$0.005\,\sigma^2 + 0.0169\,d^2\sigma^2$
2.5	2.4369×10^{-9}	$0.0049\,\sigma^2 + 0.0162\,d^2\sigma^2$
2.82843	0	$0.0048\,\sigma^2 + 0.0156\,d^2\sigma^2$
3.0	8.1708×10^{-10}	$0.0047\,\sigma^2 + 0.0153\,d^2\sigma^2$
3.5	1.5221×10^{-8}	$0.0046\,\sigma^2 + 0.0144\,d^2\sigma^2$

4.0	5.5855×10^{-8}	$0.0044\,\sigma^2+0.0135\,d^2\sigma^2$

$(14,16,6,6,5,4,2)$, N=676, a=2.82843, $n_0=136$, $n_a=1$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	3.7031×10^{-8}	$0.0052\,\sigma^2+0.018\,d^2\sigma^2$
1.5	2.5298×10^{-8}	$0.0051\,\sigma^2+0.0175\,d^2\sigma^2$
2.0	1.2455×10^{-8}	$0.005\,\sigma^2+0.0169\,d^2\sigma^2$
2.5	2.4369×10^{-9}	$0.0049\,\sigma^2+0.0162\,d^2\sigma^2$
2.82843	0	$0.0048\,\sigma^2+0.0156\,d^2\sigma^2$
3.0	8.1708×10^{-10}	$0.0047\,\sigma^2+0.0153\,d^2\sigma^2$
3.5	1.5221×10^{-8}	$0.0046\,\sigma^2+0.0144\,d^2\sigma^2$
4.0	5.5855×10^{-8}	$0.0044\,\sigma^2+0.0135\,d^2\sigma^2$

$(15,16,6,6,5,2)$, N=676, a=2, $n_0=360$, $n_a=2$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial x_i\right)$
1.0	2.7478×10^{-8}	$0.0051\,\sigma^2+0.0176\,d^2\sigma^2$
1.5	1.0174×10^{-8}	$0.005\,\sigma^2+0.0168\,d^2\sigma^2$
2.0	0	$0.0048\,\sigma^2+0.0156\,d^2\sigma^2$
2.5	1.7105×10^{-8}	$0.0046\,\sigma^2+0.0144\,d^2\sigma^2$
3.0	8.8897×10^{-8}	$0.0044\,\sigma^2+0.013\,d^2\sigma^2$
3.5	2.5667×10^{-7}	$0.0041\,\sigma^2+0.0116\,d^2\sigma^2$
4.0	5.7992×10^{-7}	$0.0039\,\sigma^2+0.0103\,d^2\sigma^2$

***=** exact modified slope rotatability value using PBD.

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