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# On Π-Generalized Α-Closed Sets and Its Applications in Neutrosophic Topological Spaces

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**Abstract**

The concept of neutrosophic set was introduced by Smarandache. Atlansov introduced the concept of neutrosophic topological spaces. The main aim of this paper is to introduce the notation πgα–closed sets in neutrosophic topological spaces. We investigate its properties and its relationships are derived. Further πgα-closed mappings are also defined and studied and its properties.

## I. INTRODUCTION

The notion of netrosophic sets was developed by FloretinSmarandache&Atanasov generalized this idea to neutrosophic sets using the notation of fuzzy sets. In addition to Coker introduced neutrosophic topological spaces using the notion of neutrosophic sets.

## II.PRELIMINARIES

Let $X \neq \varphi$. A neutrosophic set(NS) N in NTS is defined as $N=\{<x, \mu_N, \sigma_N, \gamma_N>: x \epsilon X\}$,and $\mu_N$: $X \rightarrow [0,1]$ &$\gamma_N$:$X \rightarrow [0,1]$ denotes the degree of membership $\mu_N(x)$ & the degree of non-membership function $\gamma_N(x)$ for all $x \epsilon X$ to the set N resp. $0 \leq \mu_N + \gamma_N \leq 1$ for all $x \epsilon X$. $0 = <0, 1>$: $1 = <1, 0>: x \epsilon X$ are called empty and whole neutrosophic sets on X resp.. A neutrosophic set $N=\{<x, \mu_N, \sigma_N, \gamma_N>: x \epsilon X\}$, called a subset of neutrosophic set $C = \{< x, \mu_C, \sigma_C, \gamma_C>: x \epsilon X\}$ if $\mu_N \gamma \leq_C$ and $\gamma_N \leq \gamma_C$ for each $x \epsilon X$.

The complement of a neutrosophic set $N= \{<x, \mu_N, \sigma_N, \gamma_N>: x \epsilon X\}$ is $N^c=\{<x, \mu_N, \sigma_N, \gamma_N>: x \epsilon X\}$ the intersection(resp. union) of any arbitrary family neutrosophic sets is given by

$N \cap C=\{<x, \mu N \wedge \mu C, \sigma N \wedge \sigma C, \nu N \vee \nu C>/ x \in X\}$
$N \cup C=\{<x, \mu N \vee \mu C, \sigma N \vee \sigma C, \nu N \wedge \nu C>/ x \in X\}$

A family τ of neutrosophic sets on X is said to be a neutrosophic topology on X if the neutrosophic sets 0, $1 \in \tau$, & τ is closed under arbitrary union and finite intersection. The ordered pair (X, τ) is said to be a neutrosophic topological space . Let N be a NS on X Then

(i) $int(N) = \cup\{G, G$ is a NOS in X & $G \subseteq N\}$

(ii)$cl(N) = \cap\{K, K$ is a NCS in X & $N \subseteq K\}$

**Definition2.1**

A subset of N of a space (X, τ) is called:

(i) Regular open if N = int (cl(N)).

(ii) π open if N is the union of regular open sets.

**Definition2.2**

A NS $N = <x, \mu, \nu>$ in a NTS is called a neutrosophic semi closed if

(NCS) $N \subseteq cl(int(N))$.

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**Definition2.3**

A NS N=<x,μ,ν> in a NTS is called a neutrosophic semi open set (NSOS) if $N \subseteq cl(int(N))$.

**Definition2.4**

A NS N in a NTS is a neutrosophic pre closed set (NPCS) if $cl(int(N)) \subseteq N$,its complement is (NPOS).

**Definition2.5**

Consider a NS N in NTS, then it is called:

(i)neutrosophicα-openset(NαOS) if $N \subseteq int(cl(int(N)))$,

(ii)neutrosophicα-closedset(NαCS)if$cl(int(cl(N))) \subseteq N$.

**Definition2.6**

Let Z be a NS in (X,Ʈ) is a

(i)neutrosophicα-openset(NαOS)if$Z \subseteq cl(int(cl(Z)))$,

(ii)neutrosophicα-closedset(NαCS)

If $int(cl(int(Z))) \subseteq Z$.

**Definition2.7**

Let Z be a NS in (X,Ʈ) is a

(i)neutrosophicregularopenset(NROS)if$Z= int(cl(Z))$,

(ii)neutrosophic regular closed set(NRCS)

If $Z=cl(int(Z))$.

**Definition2.8**

A NS N of a NTS is a neutrosophic w-closed (NWCS) if $cl(N) \subseteq O$ whenever $N \subseteq O$ & O is a NSO . Otherwise it is called a Nw-open (NWOS) if Nc = NWCS.

**Definition2.9**

A NS N of a NTS is a neutrosophic generalized closed set (NGCS) if $cl(N) \subseteq O$ whenever $N \subseteq O$ and O is a NOS. The class of all NαCSs (NαOSs) of a NTS is denoted by NαC(X).

**Definition2.10**

Consider a NS N. Then the alpha interior & alpha closure of N are defined by

$\alpha int(N)=\cup\{L ,L$ is a NαOS in (X,Ʈ) & $L \subseteq N\}$

$\alpha cl(N)=\cap\{M ,M$ is a NαCS in (X,Ʈ) & $N \subseteq M \}$

**Remark2.1**

Let N be a NS (X,Ʈ), then

(i) $\alpha\, cl(N)= N\cup int(cl(int(N)))$,

(ii) $\alpha int(N)= N\cap cl(int(cl(N)))$

**Definition2.11**

Consider N be a NS of a NTS. Then the semi closure of N (scl(N)) is given by $scl(N)= \cap\{M/M$ is a NSCS in NTS & $N \subseteq M$ .

**Definition2.12**

A NS N of a NTS is a neutrosophic generalized semi closed set if $scl(N) \subseteq O$ whenever $N \subseteq O$ & O is a NTS.

**Definition2.13**

A NS N of a NTS is a neutrosophic semi pre closed set (NSPCS) if there exists a NPCS B with $int(B) \subseteq N \subseteq B$.

**Definition2.14**

A NS N of a NTS is a neutrosophic generalized alpha closed set (NGαCS) if $\alpha cl(N) \subseteq O$ whenever $N \subseteq O$ and O is a NOS.

**Remark2.2**

Each NOS is NSOS in NTS.

**Remark2.3**

Union of two NROS is NOS in NTS.

**Remark2.4**

Each NπOS is NOS in NTS..



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## III. NEUTROSOPHIC Π-GENERALIZED ALPHA CLOSED SETS

In this chapter, the notion of NπGαCS is defined and few of their characteristics are investigated.

**Definition3.1**

A NS N is said to be a neutrosophic πg alpha closed sets(NπGαCS)in NTS, Nαcl(A)⊆O wherein A⊆O & O is a NπOS.

**Example3.1**

Consider X = {z1, z2} & Ʈ= {0, NG, 1} is a NTS, Where NG=<z, (0.6, 0.5, 0.5), (0.4, 0.5, 0.3)>Then NG=<z, (0.4, 0.1, 0.5), (0.3, 0.1, 0.6)> is a NπgαCS.

**Theorem3.1**

Each NCS is a NπGαCS, not the converse.

Proof: Suppose N be a NCS and let N⊆O and O is a NπOS. By given αcl(N)⊆scl(N)⊆cl(N) and N is a NCS , αcl(A)⊆cl(A)=A⊆NU. Then it is NπGαCS.

**Example3.2**

Consider X= {z1, z2} & Ʈ= {0, NG, 1}, NG=<z, (0.6,0.1,0.4), (0.6,0.1,0.3)>. Then the NS N=<z, (0.4,0.1, 0.5), (0.3, 0.1, 0.6)>, then N is Nπgα-closed. But not NSC-set.

**Theorem3.2**

Each NαCS is a NπGαCS ,not the converse.

Proof: Let N be a NαCS and N⊆O where O is a NπOS in NTS.By given, cl(int(cl(N)))⊆N. Then int(cl(N))⊆N . we have int(N)⊆N, cl(int(N)⊆cl(N),hence int(cl(int(N)))⊆int(cl(N))⊆N, it leads to αcl(N)⊆N⊆ O . Therefore it is NπGαCS.

**Example3.3**

Let X= {z1, z2} & NG=<z, (0.7, 0.1, 0.2), (0.6, 0.1, 0.3)> and Ʈ= {0, NG, 1} be a NT. Then the NS N= NG, is a Nπgα-closed set.

**Theorem3.3**

Each NαCS is a NπGαCS, not the converse.

Proof: Let N be a NαCS. By given, αcl(N)⊆ N wherein N ⊆O & O∈ NπOS. Then αcl(N)⊆ O wherein N ⊆O & O∈ NOS. Hence N is a NπGαCS.

**Example3.4**

Let X= {z1, z2}& Ʈ= {0, NG, 1} is a NTS,whenever NG=<z, (0.5, 0.2, 0.5), (0.5, 0.4, 0.5)> Then NS N=<z, (0.4, 0, 0.6), (0.5, 0, 0.5)> is an NPCS-set. But not Nπgα-closed.

**Theorem3.4**

Each NRCS is a NπGαCS, not the converse..

Proof: Let N be a NRCS. By Def. N= cl(int(N)). Then cl(N)=cl(int(N)). Thus cl(N)=N. Hence N is a NCS. By Thm. 3.1, N is a NπGαCS.

**Example3.5**

Consider X= {z1, z2}& Ʈ= {0, NG, 1} is a NTS, NG=<z, (0.6, 0.3, 0.5), (0.5, 0.6, 0.5)> then NS N=<z,(0.2, 0.1, 0.7), (0.3, 0.1, 0.6)> then N is NπGα-closed but not NPCS.

**Theorem3.5**

Each NWCS is a NπGαCS, not the converse.

Proof: Suppose N be a NWCS and N⊆O&O is a NπOS∈ NTS. By assumption cl(N)⊆O wherein N⊆O ,because αcl(N)⊆ cl(N) & N is NWCS , αcl(N)⊆cl(N)⊆ O, wherein N ⊆ O & O∈ NSO. Thus N is a NπGαCS.

**Example3.6**

Consider X= {z1, z2} and Ʈ= {0, NG, 1} is a NTS, NG=<z, (0.6, 0.1, 0.3), (0.7, 0.1, 0.2)> then the NS N=<z, (0.2, 0.1, 0.7), (0.3, 0.1, 0.6)> then NG is NπgαCS, not NWCSs.

**Theorem3.6**

Each NGSCS is a NπGαCS, not the converse.

Proof: Let N be a NGSCS. By given, scl(N)⊆O, wherein N⊆O& O∈NπOS. By Remark αcl(N)⊆O , wherein N⊆O & O ∈NOS. Hence it is NπGαCS.



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**Theorem3.7**

Each NGαCS is an NπGαCS, not the converse.

Proof: Let N be a NGαCS. By given, αcl(N)⊆O, wherein N⊆O & O∈NπOS. By assumption and (Remark2.3)αcl(N)⊆scl(N)⊆O,wherein N ⊆O & O ∈ NOS. Then N is NπGαCS.

**Theorem3.8**

Each NπGSCS is a NπGαCS not the converse.

Proof: Consider N is a NπGSCS ∈NTS.Given that, scl(N)⊆O, wherein N⊆O&O∈NπOS. αcl(N)⊆scl(N)⊆O ,wherein N⊆O & O ∈ NπOS. Hence N is NπGαCS.

**Remark3.1**

Let N and M be in NπGαCS. Then N∩M is not a NπGαCS.

**Theorem3.9**

Consider X is a NTS. Then for each P ∈ NπGαC(X) & every all Q∈NS(X), P⊆Q ⊆αcl(P) => Q ∈NπGαC(X).

Proof: Let Q⊆O &O ∈NπOS. Since P⊆Q, P⊆ O and P ∈ NπGαCS, αcl(P)⊆O, wherein P⊆ O, By given ,Q⊆αcl(P),αcl(Q)⊆αcl(P)⊆O. Therefore αcl(Q)⊆O .Hence Q is a NπGαCS.

**Theorem3.10**

Let N be both NπOS & NπGαCS in NTS, then N is a NαCS in NTS.

Proof: Let N be a NπOS. Since N⊆N, By given αcl(N)⊆N but always N⊆αcl(N). We get αcl(N)=N. Hence N is NαCS.

**Theorem3.11**

Let X be a NTS. If a NS N is both NπOS & NCS of X, then the following are equivalent:

(1) N is a NGCS

(2) N is a NπGαCS.

Proof: (1) ⇒ (2): Consider N is NGCS in NTS. By theorem 3.8 N is NπGαCS in NTS. (2)⇒(1):Consider N ∈ NπGαCS. Then αcl(N)⊆O wherein N⊆ O&O∈NπOS,=> αcl(N)⊆ cl(N)⊆O , wherein N⊆O , Since N is both NπOS & NCS in NTS . Then it is NGCS.

**Definition 3.17**

Let N be a NS .Then

The Nπ-kernel=∩ { K: K is a NπOS and N⊆K}

**Remark3.2**

Let N⊆X, then N is NTS is NπgαC if αcl(N) ⊆Nπ-ker(N).

**Theorem3.12**

Let N⊆X, then N is NπGα-closed iffαcl(N) CNπ-ker(N).

Proof: Since N is NπgαC, αcl (N)⊆N for any π-open set O& N⊆O and then αcl(N)⊆Nπ-ker(N). Conversely, consider O is any Nπ open set such that N⊆O.By assumption, αcl(N)⊆Nπ-ker(N)⊆O and hence N is NπgαC.

**Theorem3.13**

Let N is Nπ-open and NπgαC. Then N is α-closed.

Proof: Since N is Nπ-open & NπgαC, αCl(N)⊂ N, but N⊂αCl(N) Hence, N is αC.

**Theorem3.14**

Consider N is Nπgα-closed in NTS. Then αCl (N)\N does not contain any nonempty Nπ-closed.

Proof: Consider V is NπC subset of αCl (N)\N. Then N⊂ X \V, wherein N is NVπgαclosed and X \V is Nπ-open. Hence αCl (N) ⊂ X \V, or V⊂ X \αCl (N). By given V⊂Cl(N), we get a contradiction.

**Corollary3.1**

Consider N be a Nπgα-closed in NTS. Then N is Nα-closed iff αCl (N)|N is NπCs.



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Proof: Necessary: Let N is Nπgα-closed. By assumption αCl(N)=N & αCl(N)\N= φ which is NπCs.

Sufficient: Let αCl(N)\N ∈ NπC. Then by Thm. 3.9, αCl(N)\N= π. i.e, αCl(N)=N. Hence, N is αC.

## IV. NEUTROSOPHIC ΠGAC MAPPINGS

In this chapter we define neutrosophic πgαC mappings and discuss few of its characteristics.

**Definition 4.1**

A map f:(X,τ)→(Y,σ) is said to be neutrosophic πgαC mapping if f(N) is a NπGαCS ∈ (Y,σ) for all NCS N in (X,τ).

**Example4.1**

Consider X= {p, q} & Z= {r, s}. Now Ʈ1= <x, (0.4, 0.1, 0.5), (0.4, 0.1, 0.5) & Ʈ2= <x, (0.4, 0.1, 0.5), (0.3, 0.1) 0.6)> are NTs on (X,τ) and (Z,σ). Consider a mapping f:Ʈ1→Ʈ2 by f(p)=r & f(q)=s. Hence f is a NπGαC.

**Definition4.2**

A map f:(X,τ)→(Y,σ) is called neutrosophic πgαO map (NπGα open map)if f(M) is a NπGαOS ∈ (Y,σ) for each NOS M ∈ (X,τ).

**Example4.2**

Consider X= {p, q} and Y= {x, y}.Let Ʈ1= <x, (0.6, 0.1, 0.3), (0.5, 0.1, 0.4)> and Ʈ2= <x, (0.2, 0.1, 0.7), (0.3, 0.1, 0.6)> are NTs in (X,τ) & (Y,σ) respectively. Consider a f:(X,τ)→(Y,σ) by f(p)=x and f(q)=y. Then map is a NπGαO.

**Theorem4.1**

Each NC map is a NπGαC map ,not the converse.

Proof: Consider f: (X,τ)→(Y,σ) is a NC map & N is a NCS ∈ (X,τ). But f is a NC map, f (N) is a NCS∈ (Y,σ). For each NCS ∈NπGαCS, f (N) ∈NπGαCS∈ (Y,σ). Hence map is a NπGαC.

**Example4.3**

Consider X= {p, q} and Y= {x, y}and T1= <x, (0.2, 0.1, 0.7), (0.1, 0.1, 0.8)> ,T2= <y, (0.3, 0.1, 0.6), (0.2, 0.1, 0.7)> are NTs on (X,τ)and (Y,σ) respectively. Consider f:X→ Y by f(p)=x & f(q)=y. Let NS N= <x, (0.7, 0.9, 0.2), (0.8, 0.9, 0.1)> is a NCS in X. Then f(n) is a NπGα in Y. Hence map is NπGαC map, f(n) is not a NC.

**Theorem4.2**

Each NGC map is a NπGαC ma, not the converse.

Proof: Let f is a NGC map. Consider M is a NCS in X. By assumption f(M) is a NGCS in Y. but all NGCS is NπGαCS, f(M) is a NπGαCS in Y. Then the map is a NπGαC.

**Example4.4**

Consider X= {p, q} and Y= {x, y}.let T1= <x, (0.5, 0.1, 0.4), (0.6, 0.1, 0.3)> and T2= <y, (0.6, 0.1, 0.3), (0.7, 0.1, 0.2)> are NTs on (X,τ) and (Y,σ) respectively. Consider f:(X,τ)→(Y,σ) by f(p)=x and f(q)=y. Let NS n= <x, (0.4, 0.9, 0.5), (0.3, 0.9, 0.6)> is a NCS in (X,τ).Hence map is a NπGαC map ,map is not NGC.

**Theorem4.3**

Each NαC map is a NπGαC map, not the converse.

Proof: Let f is a NαC map and M is a NCS in X. By given f(M) is a NαCS in Y. Since each NαCS is a NπGαCS, f(M) is a NπGαCS in Y. Then the map is a NπGαC.

**Example4.4**

Consider X= {p, q} and Z= {r, s}.Also T1= <x, (0.4, 0.1, 0.5), (0.5, 0.1, 0.4)> and T2= <x, (0.4, 0.1, 0.5), (0.3, 0.1, 0.6)> are NTs on (X,τ) and (Y,σ) respectively. Consider f:(X,τ)→(Y,σ) by f(p)=r & f(q)=s. Let NS n= <x, (0.5, 0.9, 0.4), (0.4, 0.9, 0.5)> is a NCS in (X,τ). Hence the map is a NπGαC map, map is not a NαC.

**Remark4.1**

A NπGαC map is independent of a NPC mapping.



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**Example4.5**

Consider X= {p, q} and Y= {x, y}.Let T1= <x, (0.1, 0.1, 0.8), (0.2, 0.1, 0.7)> and T2= <y, (0.7, 0.1, 0.2), (0.6, 0.7, 0.3)> are NTs on (X,τ)and (Y,σ) respectively. Consider f:(X,τ)→(Y,σ) by f(p)=x and f(q)=y. also NS N= <x, (0.8, 0.9, 0.1), (0.7, 0.9, 0.2)> is a NCS in X. Hence the map is a NπGαC, map is not a NPC.

**Example4.6**

Consider X= {p, q} and Y= {x, y}.Let T1= <x, (0.7, 0, 0.3), (0.6, 0, 0.4)> and T2= <y, (0.4, 0, 0.6), (0.5, 0, 0.5)>are NTs on (X,τ) and (Y,σ) respectively. Consider f:(X,τ)→(Y,σ) by f(p)=x & f(q)=y. Let NS N= <x, (0.3, 0.7, 0.7), (0.4, 0, 0.6)> is a NCS .Hence the map is a NπGαC, map is not a NπGαC.

**Remark4.2**

A NπGαC map is independent of a NGSC map.

**Example4.7**

Consider X= {p, q} and Y= {x, y}.Let T1= <x, (0.45, 0.1, 0.45), (0.5, 0.1, 0.4)> and T2= <y, (0.5, 0.1, 0.4), (0.6, 0.1, 0.3)>are NTs on (X,τ) and (Y,σ) respectively. Consider f:X→Y by f(a)=u & f(b)=v. Let NS N= <x, (0.45, 0.9, 0.45), (0.4, 0.1, 0.5)> is a NCS. Hence the map is a NπGαC, map is not a NGSC.

**Example4.8**

Consider X= {p, q} and Y= {x, y}.And T1= <x, (0.8, 0.1, 0.1), (0.7, 0.1, 0.2)> and T2= <y, (0.2, 0.1, 0.7), (0.3, 0.1, 0.6)> are NTs on (X,τ) and (Y,σ) respectively. Consider f:(X,τ)→(Y,σ) by f(p)=x and f(q)=y. Let NS N= <x, (0.1, 0.9, 0.8), (0.2, 0.9, 0.7)> is a NCS in in (X,τ). Hence the map is a NGSC, map is not a NπGαC.

**Theorem4.4**

Consider f:X→Y.Suppose f(A) is a NRCS in Y for each NCS N in X. Then f is a NπgαC map.

Proof: Let N be a NCS in (X,τ). Then f(N) is a NRCS in (Y,σ). Since each NRCS is a NπGαCS, f(N) is a NπGαCS in (Y,σ). Then f is a NπGαC.

**Theorem4.5**

Consider f:X→Y is a NπGαC map. Then f is a NFC map if (Y,σ) is a NFπααT1/2 space.

Proof: Consider a NS set N in (X,τ).. By given, f(N) is a NπGαCS in Y. Because Y is a NFπααT1/2 space, f (N) is a NCS. Hence the map is a NFC.

**Theorem4.6**

Consider f:(X,τ)→(Y,σ) be a map from a NTS (X,τ) into a NTS (Y,σ). Then the followings are equivalent if Y is a NFπααT1/2 space.

(1) f is a NπGαO map,

(2)If N is a NOS in X.Provef(N) is a NπGαOS in Y

(3)f(int(N))⊆int(cl(int(f(N)))) for all NS N in X.

Proof (1)⇒(2):The proof is trivial.

(2)⇒(3):Let N be a NS in X. Then int(N) is a NOS in X. Then f(int(N)) is a NπGαOS in (Y,σ). Since (Y,σ) is an NπααT1/2 space, f(int(N)) is a NOS in (Y,σ). Hence f(int(N))=int(f(int(N))⊆int(cl(int(f(N))).

(3)⇒(1):Let N be a NOS in (X,τ). Since f (int(N))⊆int(cl(int(f(N)))).=>f(N)⊆int(cl(int(f(N)))). Hence f (N) is a NαOS. Since every NαOS is a NπGαOS, f (N) is a NπGαOS in (Y,σ). Thus f is a NπGαO map.

**Theorem4.7**

Consider f:(X, τ)→(Y,σ) is a NπGαC map. Then f is a NGC map if (Y,σ) is a NπαbT1/2 space.

Proof: Let N be a NCS in (X,τ). Then f(N) is a NπGαCS in (Y,σ). But (Y,σ) is a NFπαbT1/2 space, f(N) is a GCS in (Y,σ). Hence the map is a NGC .



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**Theorem4.8**

Consider f:(X,τ)→(Y,σ) be a NC map and g: (Y,σ)→(Z,δ) be a NπGαC map. Then g ∘ f:(X,τ)→(Z,δ) is a NπGαC map.

Proof: Let N be a NCS in (X,τ). Then f(N) is a NCS in (Y,σ). Since g is a NπGαC map, g(f(N)) is a NπGαCS in Z. Then g∘f is a NπGαC.

**Theorem4.9**

Consider f:(X,τ) →(Y,σ) be a map from a NTS (X,τ) into a NTS (Y,σ). Then the following are equivalent if Y is a NπαaT1/2 space.

(1) f is a NπGαC map,

(2)f(int(N))⊆αint(f(N)) for each NCS N of (X,τ).
(3)int(f−1(B))⊆f−1(αint(B)) for each NS B of (Y,σ).

Proof (1)⇒(2):Let f is a NπGαC map. Let N be any NS in (X,τ). Then int(N) is a NOS in (X,τ). By given, f(int(N)) is a NπGαOS in (Y,σ). Since (Y,σ) is a NπαaT1/2 space, f(int(A)) is a NαOS in (Y,σ). Then αint(f(int(N)))=f(int(N)).Nowf(int(N))=αint(f(int(N)))⊆αint(f(N)).

(2)⇒(3):Let S is a NS in (Y,σ). Then f−1(S) is a NS in X. Since given, f(int(f−1(S)))⊆αint(f(f−1(S)))⊆αint(S). Hence int(f−1(S))⊆ f−1(αint(S)).

(3)⇒(1):Let N be a NOS in (X,τ). Then int(N)=N and f(N) is a NS in (Y,σ).Then int(f−1(f(N)))⊆f−1(αint(f(N))).Now N=int(N)⊆int(f−1(f(N)))⊆f−1(αint(f(N))).Hence f(N)⊆f(f−1(αint(f(N))))=αint(f(N))⊆f(N),then αint(f(N)) = f(N) is a NαOS in (Y,σ). We have f(N) is a NπGαOS. Then the map is a NπGαC.

**Theorem4.10**

Consider f:(X,τ)→(Y,σ) is a NπGαC map& (Y,σ) is a NπαcT1/2 space, then f is a NGSC map.

Proof: Let N is a NCS in (X,τ). By assumption, f(N) is a NπGαCS in (Y,σ). Since (Y,σ) is a NπαcT1/2 space, f(N) is a NGSCS in (Y,σ). Then the map is a NGSC.

**Theorem4.11**

Consider a map f: (X,τ) →(Y,σ) is a NπGαO if f(αint(N))⊆αint(f(N))for each N⊆X.

Proof: Let N is a NOS in (X,τ). Then int(N) = N. Nowf(N)=f(int(N))⊆f(αint(N))⊆αint(f(N)).But αintf(N)⊆f(N). Therefore αint(f(N))= f(N). i.e f (N) is a NαOS in (X,τ).This shows f (N) is a NπGαOS. Then the map is a NπGαO.

**Theorem4.12**

Consider f:(X,τ)→(Y,σ) be a map. Then the following are equivalent if Y is a NπαaT1/2 space.

(1) f is a NπGαC map,

(2) cl(int(cl(f(N))))⊆ f(cl(N)) for each NS N in (X,τ).

Proof (1) ⇒ (2) Let N be a NS in (X,τ). Then cl(N) is a NCS in (X,τ). By given, f(cl(N)) is a NπGαCS in (Y,σ). Since (Y,σ) is a NπαaT1/2 space, f(cl(N)) is a NCS in (Y,σ) Then cl(f(cl(N)))=f(cl(N)). Now cl(int(cl(f(N))))⊆cl(f(cl(N)))=f(cl(N)).Hencecl(int(cl(f(N))))⊆f(cl(N)).

(2)⇒(1)let N is a NCS. By assumption cl(int(cl(f(N))))⊆f(cl(N))=f(N). Then f(N) is a NαCS in and therefore f(N) is a NπGαCS in (Y,σ). Hence the map is a NπGαC.

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