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Parallel Contextual Array Insertion P Systems and Siromoney Array Grammars

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Abstract

This paper is an extension work of James et al [4]. We show that well-known families of Siromoney Array Languages (CF:CF)AL [3] and (CS:CS)AL [3] have non-empty intersection with $\mathcal{L}$(PCAIPS) obtained by abating the deletion rules of PCAIDPS.

Keywords; *P System - Rectangular array - Contextual Array Insertion and Deletion - Siromoney Array Grammar.*

I. INTRODUCTION

The study of picture languages has gained momentum from 1970. Since then the study has encompassed problems pertaining to pattern recognition and image processing. There has been an extensive study in the literature related to picture languages that are generated by array grammars or recognized by array automata. A picture or an array over a finite set V, called an alphabet, is a rectangular arrangement of elements over V in a two-dimensional plane. In [1], a non-isometric variety of an array grammar model has been introduced to generate rectangular arrays of symbols, called as the 2-dimensional right-linear grammar. This two-dimensional grammar involves two phases, with the first horizontal phase generating strings of symbols, called intermediate symbols using regular or context-free or context-sensitive grammar rules and in the final phase, for every intermediate symbol, corresponding right-linear grammar rules are applied simultaneously, generating strings consisting of terminal symbols of same length in the vertical direction, thereby generating a rectangular array.

Due to the limitation of the two-dimensional Siromoney matrix grammars in generating certain picture languages and the complex patterns found in kolam, a traditional and popular art in India, Siromoney et al have introduced array models called Siromoney array grammars (SAG) in [3]. SAG generalised the idea of rewriting rules of 1-dimensional grammars to 2-dimensional grammars by extending the concatenation (row and column concatenations) of strings to arrays. Henceforth several types of array grammars of non-isometric and isometric varieties with various applications have been introduced in the literature.

GeorghePăun has introduced a P system (membrane system) [2], by simulating the working of a cell and its contents inside membranes, evolving in parallel. Membrane structure prescribes initial configuration of a P system with each membrane consisting of objects and evolution rules. A computation of P system begins from a start state and comes to a halt when there are no further rules to be applied. Based

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on the array insertion and deletion operations, a new P system model called parallel contextual array insertion deletion P system (PCAIDPS) was introduced in [4]. This paper is an extension of the work done by James et al in [5] and we prove that (CF:CF)AL [3] and (CS:CS)AL [3] have non-empty intersection with $\mathcal{L}$(PCAIPS) [4] obtained from PCAIDPS by abating the deletion rules.

II. PRELIMINARIES

We now recollect few notions of insertion deletion P systems with an illustration. For further details of the P system we refer to [4].

Let a finite alphabet be denoted by V and the set of all words over V encompassing the null word λ be denoted by V^*. $V^+ = V \setminus \{\lambda\}$. For $w \in V^*$ and $a \in V$, the number of occurrences of a in w is denoted by $|w|_a$. An array is an arrangement of finite number of elements from V in rows and columns and is written in short as $A = [a_{ij}]_{m\times n}, \forall\, a_{ij} \in V, i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$. The set of all arrays over V is denoted by V^{**} which also includes the empty array Λ having neither rows or columns. $V^{++} = V^{**} - \{\Lambda\}$. An array of dimension $1 \times n$ is nothing but a word of length n (and vice versa). The column catenation of two matrices $A = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nq} \end{bmatrix}$ is defined only when $m = n$, and is given by $A \oplus B = \begin{bmatrix} a_{11} & \ldots & a_{1p} & b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \ldots & a_{mp} & b_{n1} & \cdots & b_{nq} \end{bmatrix}$, which is achieved by juxtaposing A and B along their columns. In the same way, row catenation of $A$and B, is defined when $p = q$ and is given by $A \ominus B = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mp} \\ b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nq} \end{bmatrix}$. The empty array performs the role of unit element for both column and row catenation of arrays of arbitrary dimensions.

Definition 1:A Parallel contextual array insertion P system (PCAIPS) is a construct $\Pi = (V, T, \mu, C, R, (M_1, I_1), \ldots, (M_h, I_h), \varphi_r^i, \varphi_c^i, i_0)$, where,

$-\ V$is finite non-empty set called alphabet set;

$-\ T(\subset V)$ is the set of output alphabets;

$-\ \mu$is the structure of membrane with 'h' membranes or regions;

$-\ C \subset V^{**}$is a finite set of column array contexts;

$-\ R \subset V^{**}$is a finite set of row array contexts;

$-\ M_i \subset V^{**}$is a finite axiom set corresponding to the region i;

$-\ \varphi_c^i: (V^{**}, V^{**}) \to C$are mappings performing parallel contextual column insertion operations with choice;

$-\ \varphi_r^i: (V^{**}, V^{**}) \to R$are mappings performing parallel contextual row insertion operations with choice;

$-\ I_i = \emptyset$ (or) $\left\{\left(\left\{\varphi_c^i(A_i, B_i) = \begin{bmatrix} u_i \\ u_{i+1} \end{bmatrix} : i = 1, 2, \ldots, m-1, \alpha\right.\right.$

Where $A_i = \begin{bmatrix} a_{ij} \ldots a_{i(k-1)} \\ a_{(i+1)j} \ldots a_{(i+1)(k-1)} \end{bmatrix}$, $B_i = \begin{bmatrix} a_{ik} \ldots a_{i(l-1)} \\ a_{(i+1)k} \ldots a_{(i+1)(l-1)} \end{bmatrix}, 1 \le j \le k < l \le n+1$ (or) $1 \le j < k \le l \le n+1, \alpha \in \{here, out, in_t\}$ and u_i and u_{i+1} are of size $1 \times p$ with $p \ge 1$. (or) $\left\{\left(\left\{\varphi_r^i(C_i, E_i) = [u_i u_{i+1}] : i = 1, 2, \ldots, n-1\right\}, \alpha\right)\right\}$

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Where $C_i = \begin{bmatrix} a_{ji} & a_{j(i+1)} \\ \vdots & \vdots \\ a_{(k-1)i} & a_{(k-1)(i+1)} \end{bmatrix}, E_i = \begin{bmatrix} a_{ki} & a_{k(i+1)} \\ \vdots & \vdots \\ a_{(l-1)i} & a_{(l-1)(i+1)} \end{bmatrix}, 1 \le j \le k < l \le m+1$ (or) $1 \le j < k \le l \le m+1, \alpha \in \{here, out, in_t\}$ and u_i and u_{i+1} are of size $p \times 1$ with $p \ge 1$.

With respect to Π, the direct derivation is a binary relation $\Rightarrow$ on V^{**} defined as $X \Rightarrow_i Y$ where X, $Y \in V^{**}$ if and only if, $X = X_1 \ominus A \ominus B \ominus X_2$, $Y = X_1 \oplus A \oplus I_c \oplus B \oplus X_2$ (or) $X_3 \ominus A \ominus B \ominus X_4$, $Y = X_3 \oplus A \oplus I_c \oplus B \oplus X_4$ for some $X_1, X_2, X_3, X_4 \in V^{**}$ and I_c , I_r are the corresponding contexts inserted rows and columns.

A computation of this P system begins with the initial configuration consisting of the membrane structure with h membranes with labels $1, 2, \dots, h$. The skin membrane with label 1 is the outermost membrane acting as the output membrane. The system performs a step by step computation based on the row or column insertion rules I_i. At the end of each computation, the resulting array is sent to the membrane based on the choice of $\alpha \in \{here, out, in_t\}$. If the choice of α is '$here$' the resultant array is left in the same membrane. If the choice of α is 'out', then the resultant array is pushed to the next outer membrane. If the choice of α is 'in_t', then the resultant array is moved to the membrane with label t. If there are no further rules to be applied then the resultant array obtained after the last computation is either in the skin membrane or in any other membrane and the system halts. If the resultant array is in the skin membrane then it belongs to the language generated by this P system. The set of all arrays with symbols over T produced by the PCAIPS is denoted by $PCAIP(\Pi)$ or $L(\Pi)$.

The class of all array languages generated by PCAIPS with at most h membranes is denoted by $PCAIPS_h$.

III. FABRICATION OF THE COMPOSITE

Siromoney et al [3] have defined nine generative models to generate pictures using array grammars with array rewriting rules and have established a hierarchy between classes of pictures. The derivation rules started with the start symbol S and applying non-terminal rules without any restriction till all the non-terminals are replaced by making use of parenthesis, if necessary as the row and column catenation operator $\oplus \in \{\ominus, \oplus\}$ is not associative, and then all the intermediate symbols are to be replaced with the corresponding matrix languages subject to the rules of row and column catenation operator $\oplus$.

An array grammar (AG) is called $(\alpha:\beta)AG$ if the non-terminal rules are from α and at least one intermediate language is β, where $\alpha, \beta \in \{CS, CF, R\}$, where CS (CF, R) stands for the class of Context Sensitive (Context Free and Regular) languages in that order. Of these nine classes, (CF:CF) and (CS:CS) are two important classes of grammars which we consider for our paper.

By considering only the insertion operation of PCAIDPS[4], we get PCAIPS and prove that $\mathcal{L}(PCAIPS) \cap (CF:CF)AL \neq \emptyset$, and $\mathcal{L}(PCAIPS) \cap (CS:CS)AL \neq \emptyset$. This study is interesting since James et al [4] have given an example to show that $\mathcal{L}(PCAIPS) \cap (R:R)AL \neq \emptyset$ but not examined further connection with Siromoney Array Grammars there.

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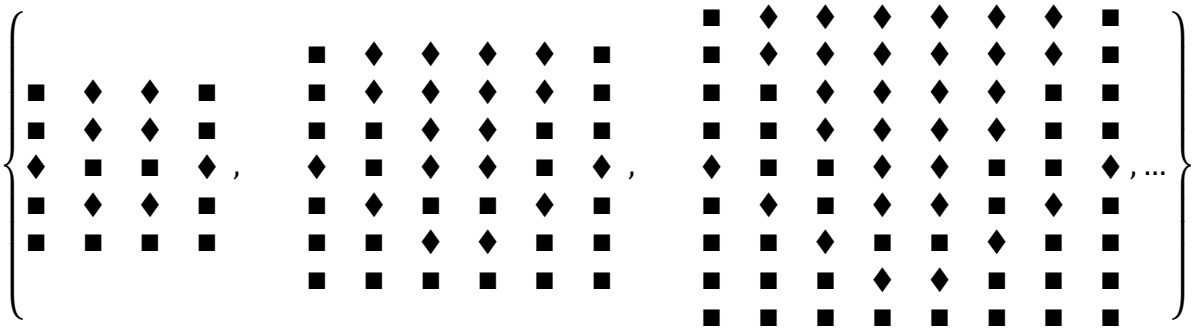
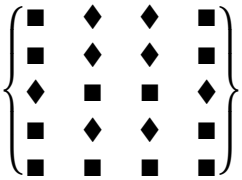
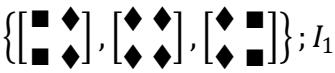
Theorem 1. $\mathcal{L}(PCAIPS_5) \cap (CF{:}\,CF)AL \neq \emptyset$.

Proof. Consider the array language $L_1 \in (CF{:}\,CF)AL$ given in example 2.6 of [3], namely, $\{N_n / n \geq 1\}$ is a set of matrices such that $N_n = \bar{M}_n \textcircled{\scriptsize 1} M_n, M_{n+1} = (X_1 \ominus M_n) \textcircled{\scriptsize 1} Y_1, M_1 =$  , $L_{X_1} = \left\{ \left(\begin{matrix} \blacklozenge \\ \blacklozenge \end{matrix} \right)^n : n \geq 1 \right\}$, and $L_{Y_1} = \left\{ \begin{matrix} (\blacksquare)_n \\ \blacklozenge \\ (\blacksquare)_n \end{matrix} : n \geq 1 \right\}$ and is given by



Where, each array in the language L_1 is of the form $Y \textcircled{\scriptsize 1} Z$ where Z is the mirror image of Y.

We now configure a P system to accept the above language as follows: Consider the P system $\Pi = (V, T, \mu, C, R, (M_1, I_1), \ldots, (M_5, I_5), \varphi_r^i, \varphi_c^i, 1)$ such that $L(\Pi) = L_1$,where $\mu = [_1[_2[_3]_3]_2]_1, [_4[_5]_5]_4]_1; M_2 = M_4 = M_5 = \phi;$ $M_3 =$  ; 1 is the output membrane; $C =$; $R =$  ; $I_1 = \phi;$

$I_2 = \left\{ \left(\left\{ \varphi_c^i \left[\begin{matrix} \lambda, \blacksquare \\ \lambda, \blacksquare \end{matrix} \right] = \left[\blacksquare \right], \varphi_c^i \left[\begin{matrix} \lambda, \blacksquare \\ \lambda, \blacklozenge \end{matrix} \right] = \left[\blacklozenge \right], \varphi_c^i \left[\begin{matrix} \lambda, \blacksquare \\ \lambda, \blacksquare \end{matrix} \right] = \left[\blacksquare \right] \right\}, in_5 \right) \right\};$

$I_3 = \left\{ \left(\left\{ \varphi_c^i \left[\begin{matrix} \blacksquare, \lambda \\ \blacksquare, \lambda \end{matrix} \right] = \left[\blacksquare \right], \varphi_c^i \left[\begin{matrix} \blacksquare, \lambda \\ \blacksquare, \lambda \end{matrix} \right] = \left[* \right], \varphi_c^i \left[\begin{matrix} \blacksquare, \lambda \\ \blacklozenge, \lambda \end{matrix} \right] = \left[\blacklozenge \right], \varphi_c^i \left[\begin{matrix} \blacklozenge, \lambda \\ \blacksquare, \lambda \end{matrix} \right] = \left[\blacksquare \right] \right\}, out \right) \right\};$

$I_4 = \left\{ \left(\left\{ \varphi_r^i [\lambda\ \lambda, \blacksquare\ \blacksquare] = \left[\begin{matrix} \blacksquare & \blacklozenge \\ \blacksquare & \blacklozenge \end{matrix} \right], \varphi_r^i [\lambda\ \lambda, \blacksquare\ \blacklozenge] = \left[\begin{matrix} \blacklozenge & \blacklozenge \\ \blacklozenge & \blacklozenge \end{matrix} \right], \varphi_r^i [\lambda\ \lambda, \blacklozenge\ \blacklozenge] = \left[\begin{matrix} \blacklozenge\blacklozenge \\ \blacklozenge\blacklozenge \end{matrix} \right], \varphi_r^i [\lambda\ \lambda, \blacksquare\ \blacksquare] = \right. \right.$
$\blacklozenge\blacklozenge\ \blacksquare\ \blacksquare, \varphi r i \lambda\ \lambda, * \blacksquare = \blacklozenge\blacklozenge\blacklozenge\blacklozenge, out;$ where $\alpha \in out, in3.$

$I_5 = \left\{ \left(\left\{ \varphi_r^i [\blacksquare\ \blacksquare, \lambda\ \lambda] = [\lambda\ \lambda] \right\}, out \right) \right\}.$

A sample computation is given below:

X – is the (row or column) context inserted

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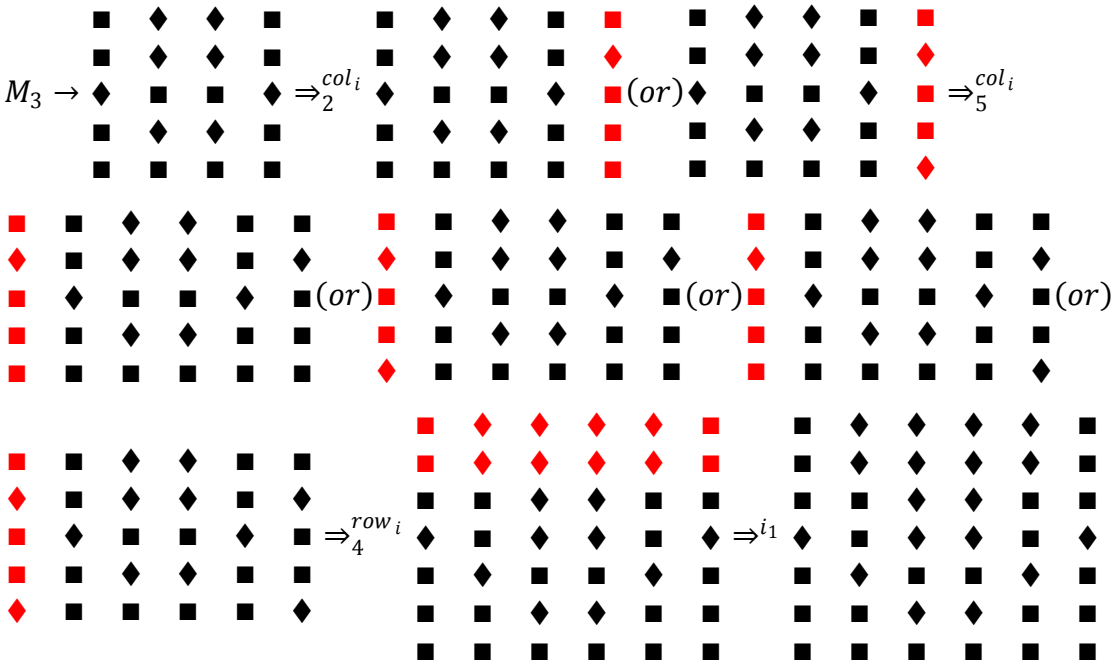
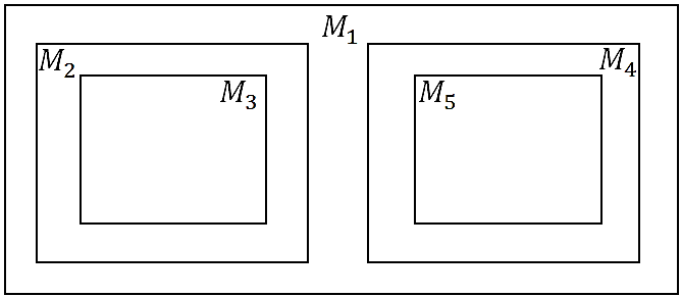
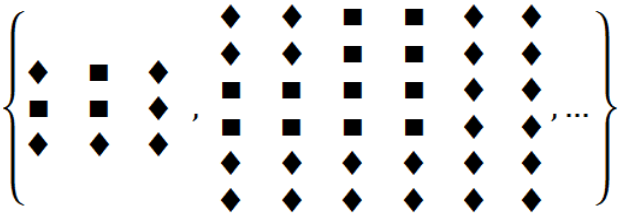


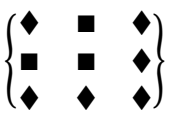
The structure of the P system is given by



Theorem 2. $\mathcal{L}(PCAIPS_5) \cap (CF{:}CF)AL \neq \emptyset$.

Proof. Consider the language $L_2 \in (CS{:}CS)AL$ given in 2.10 of [3]. Pictures generated by L_2

are given by 

We construct a P system to accept the language L_2 as follows: Consider $\Pi = (V, T, \mu, C, R, (M_1, I_1), \ldots, (M_5, I_5), \varphi_r^i, \varphi_c^i, 1)$ where $\mu = [_1[_2[_3]_3]_2]_1, [_4[_5]_5]_4]_1; M_2 = M_4 = M_5 = \phi; M_1 = M_3 =$  ; 1 is the output membrane.

$C =$  ;

$R =$  ;

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$I_2 = \{(\{\varphi_c^i [\blacksquare, \blacklozenge] = [\blacklozenge], \varphi_c^i [\blacksquare, \blacklozenge] = [\blacklozenge], \varphi_c^i [\blacklozenge, \blacklozenge] = [\blacklozenge]\}, in_5)\};$

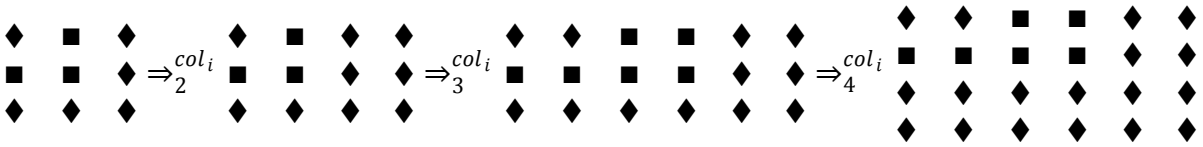
$I_3 = \{(\{\varphi_c^i [\blacklozenge, \blacksquare] = [\blacklozenge \ \blacksquare], \varphi_c^i [\blacksquare, \blacksquare] = [\blacksquare \ \blacksquare], \varphi_c^i [\blacksquare, \blacksquare] = [\blacksquare \ \blacksquare], \varphi_c^i [\blacklozenge, \blacklozenge] = [\blacklozenge \ \blacklozenge], \varphi_c^i [\blacklozenge, \blacklozenge] = [\blacklozenge \ \blacklozenge]\}, out)\};$

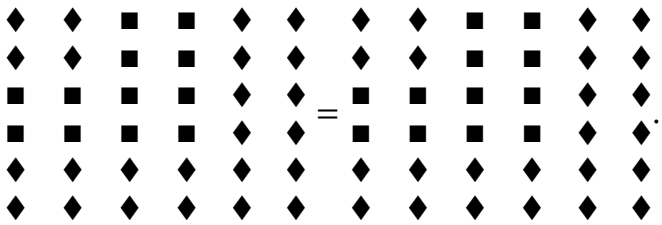
$I_4 = \{(\{\varphi_r^i [\blacksquare \ \blacksquare, \blacklozenge \ \blacklozenge] = [\blacklozenge \ \blacklozenge], \varphi_r^i [\blacksquare \ \blacklozenge, \blacklozenge \ \blacklozenge] = [\blacklozenge \ \blacklozenge], \varphi_r^i [\blacklozenge \ \blacklozenge, \blacklozenge \ \blacklozenge] = \blacklozenge\blacklozenge\blacklozenge\blacklozenge, out;$ where $\alpha \in out, in3.$

$I_5 = \{(\{\varphi_r^i [\blacklozenge \ \blacksquare, \blacksquare \ \blacksquare] = [\blacklozenge \ \blacksquare], \varphi_r^i [\blacklozenge \ \blacklozenge, \blacksquare \ \blacksquare] = [\blacklozenge \ \blacklozenge], \varphi_r^i [\blacksquare \ \blacksquare, \blacksquare \ \blacksquare] = [\blacksquare \ \blacksquare], \varphi_r^i [\blacksquare \ \blacklozenge, \blacksquare \ \blacklozenge] = [\blacksquare \ \blacklozenge], \varphi_r^i [\blacklozenge \ \blacklozenge, \blacklozenge \ \blacklozenge] = [\blacklozenge \ \blacklozenge]\}, out)\}.$

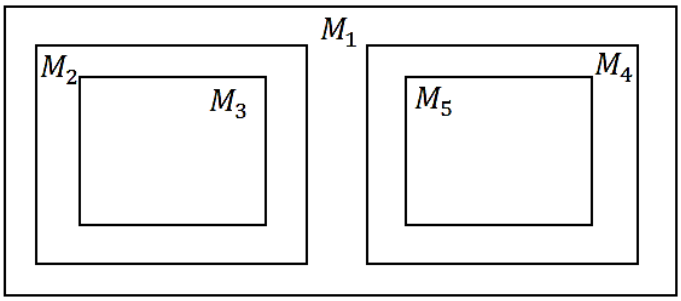
Clearly, $L(\Pi) = L_2$. A sample computation is given below:

X – is the (row or column) context inserted.

$M_3 \rightarrow$  $\Rightarrow_2^{col_i}$ $\Rightarrow_3^{col_i}$ $\Rightarrow_4^{col_i}$

$\Rightarrow_4^{col_i}$  .

The structure μ of the P system is given by



CONCLUSION

We believe that $(R:R)AL, (CF:CF)AL\ and\ (CS:CS)AL$ are properly contained in $\mathcal{L}(PCAIPS_h)$.This is worth investigating since these languages have close connection with *Kolam* patterns[17] and the same would be considered in our future work.

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