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Immersion Method in Skyrme Model

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Abstract

This paper is an analysis of one of the soliton varieties. The Immersion method will be presented here, the result of which will be presented as a formal row. The idea of the method is the following: some parameter (μ) will be introduced into the initial equation. In this case, if this parameter is set to zero, the result will be a known problem that already has a solution. In our case, we will get rid of the parameter by tending it to one. As a result, we will get some members of the formal series.

Keywords: *Skyrme model, topological charge, homotopy groups*

INTRODUCTION

Skyrme model

In 1958, Tony Skyrme proposed a model of description of chiral topological solitons (Skyrmions) (3+1) -dimensional nonlinear sigma model with spontaneously broken chiral symmetry [1,2]. The main object of the model is the chiral field Uthat takes on the values of floor diversity $\Phi = S^3[\cong SU(2)]$ and parameterizable by the vector field. φ^a

$$U = exp[in^a\tau^a \ominus (x)] = \varphi^0 + i\tau\varphi, \qquad (1)$$

$$n^a = \frac{\varphi^a}{|\varphi|}, \qquad \ominus = \pm sin^{-1}\varphi, \qquad a = 1,2,3.$$

Fields (1) subject to natural boundary conditions at spatial infinity

$$U(x) \rightarrow I(\varphi^a(x) \rightarrow 0), at|x| \rightarrow \infty$$

every step at any fixed point in time t can be considered as reflections

$$U: (S^3 \rightarrow S^3)[\cong SU(2)] \qquad (2)$$

As a result, each mapping (2) can be compared with a certain value of the topological invariant - the degree of mapping $Q = deg(R^3 \rightarrow S^3)$, and to study the connectivity of the entire display space through the 3rdhomotopic group.

$$\pi_3(S^3) = \mathbb{Z}$$

Cited isomorphism with group $\mathbb{Z}$ indicative of model acceptability N-Soliton states, and its explicit implementation gives the following equation for the topological charge of the model:

$$Q = \frac{\varepsilon^{ijk}}{24\pi^2}\int dx\, Tr\,(I^i I^j I^k)$$

The dynamics of Skyrme model is set by the density of Lagrangian.

$$L = \frac{\epsilon^2}{16}Tr\left[I^i, I^j\right]^2 - \frac{1}{4\lambda^2}Tr(I^i)^2 \qquad (3)$$

The first term in (3) coincides with Weinberg-GürschieLagrangian, which in a tree approximation reproduces well the results of current algebra for low energy dynamics of peons [3-4].

Originally, "Skyrmions" were intended to describe baryons as extended localized structures with non-trivial topological chargeQ which was interpreted as a *baryon number*. At the same time, the Skyrme model proved to be a sufficiently successful and simple prototype of the effective meson theory, to which should be reduced the theory of strong

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interaction - quantum chromodynamics, in the low-energy limit.

In the framework of this fairly simple model, it is possible to describe in a satisfactory way both the spectroscopy of the basic states of hadrons and their interactions [5, 6, 7,8,9]. At the same time, the prediction capabilities of the Skyrme model are obviously limited by the fact that out of the whole observed variety of mesons it includes only peonies, and, therefore, various generalizations of the model are proposed to consider, for example, the influence of vector ρ- mesons on the nucleon structure. This is achieved by calibration generalization of the Skyrme model [10].

Another interesting modification of the Skyrme model is the so-called Skyrme-Manton model [11, 12], on the basis of which it is possible to obtain reasonable answers, both in relation to the high-energy physics of hadrons, and in describing dense nuclear matter.

But the most surprising quality of model Skyrme is the mechanism of construction fermion states (nucleons) from boson fields (mesons), the so-called phenomenon of *Fermi-Bose transmutation* realized for the first time within its limits. In this connection, the term "Skyrmions" has acquired an expanded meaning: so now are called any topological solitons arising as part of the purely boson field theories, but subject to the statistics of Fermi-Dirac after the quantum, i.e. characterized by a half-body back. Moreover, it is precisely the further development of this idea that showed that it is possible to obtain solitons with an arbitrary fraction of the spin.

In addition to these applications, the Skyrme model has found itself in astrophysicists. The main results of this application can be considered in the works of T. Li [13].

Envelop method: Setting the task

Let's write down the Skyrme energy functional on the "hedgehog" configuration –a Skyrmion with the topological charge. $Q = N$:

$$I_1(\ominus) = \frac{1}{2}\int_0^\infty dr(\ominus^{/2}\, r^2 +\ 4\,\ominus^{/2}\, sin^2\,\ominus + sin^2\,\ominus).$$

$$I_2(\ominus) = \int_0^\infty dr(\, r^{-2} sin^4\,\ominus).$$

$$I_{total}\,(\ominus) = I_1(\ominus) + I_2(\ominus) \qquad (4)$$

Skyrmion configurations with charge Q=N *meet the* following boundary conditions:

$\ominus\,(0) = N\pi,\ \ominus\,(\infty) = 0(5)$

However, for G_1 -variant configurations in the Skyrme model, essentially nonlinear differential equations in private derivatives, it is possible to reduce them to the ordinary differential equations and to prove, that they possess infinitely smooth or real-analytical soliton decisions. Unfortunately, so far none of the methods has resulted in finding accurate solutions for any of the 2nd order (4) homogeneous equations obtained as a result of reduction. At best, such solutions can be found only in the form of formal series [14, 15]. Therefore, the *study of the geometry of multi-variety solutions of* such homogeneous, which are called *soliton diversity* [16], seems quite justified.

Solution to the problem

Let us use equation (4) and boundary conditions (5). We'll replace the variables:

$$\begin{Bmatrix} \ominus = N\pi, & 0 \le x \le 1, \\ 2r^{-1} = t, & 0 \le t \le \infty, \end{Bmatrix} \to L(x.\dot{x},t)$$

Let's write down the Lagrangian in plain sight:

$L(x,\dot{x},t) = \int_0^\infty dt\,[N^2\pi^2\dot{x}^2(1 + t^2 sin^2 N\pi x) +$
$2t{-}2sin2N\pi x{+}{+}8t{-}4sin4N\pi x$

We'll use the Hamilton-Jacoby method:

$$\partial_{\dot{x}} L = 2N^2\pi^2\dot{x}(1 + t^2 sin^2 N\pi x)$$

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That way, the Hamiltonian will be recorded:

$$H + L = p\dot{x},\ \ \partial_t S + H(x,p,t) = 0$$

We will consider only those options that make a major contribution, and this happens when $t \gg 1$

$$S_{x,t} = -t^{-1}sin^2N\pi x - 2^{-1}t\ sin^4N\pi x + Z_{x,t}$$

From the Hamilton-Jacobi equation, we get the equation for Z.

$$Z_x^2 = 4N^2\pi^2t^{-2}sin^4N\pi x(1+t^2sin^2N\pi x)^2 + (4\mu N\pi t^{-1}sinN\pi x\,cos\,N\pi x Z_x - 4\mu N^2\pi^2 Z_t) * (1+t^2sin^2N\pi x) \quad (6)$$

We'll look for solutions in a row:

$$Z_{x,t,\mu} = \sum_{n=0}^{\infty}\mu^n Z_n\ (x,t)N\pi$$

In this case, you can immediately record Z_{0x}^2

$$Z_{0x}^2 = 4t^{-2}N^2\pi^2sin^4N\pi x(1+t^2sin^2N\pi x)^2$$

As we have already noticed, the solution will be sought in series, with private derivatives fromZ will look like this:

$$Z_x = \sum_{n=0}^{\infty}\mu^n Z_{nx}\ N\pi,\ \ Z_t = \sum_{n=0}^{\infty}\mu^n Z_{nt}\ N\pi.$$

Besides, we'll be interested in the function Z_x^2, that we're going to record as

$$Z_x^2 = N^2\pi^2\sum_{n=0}^{\infty}\mu^n\sum_{k=0}^{\infty}\mu^n Z_{n-k,x}\,Z_{k,x}$$

On the other hand, this equation can be rewritten using (6) where it comes from:

$$N^2\pi^2\sum_{k=0}^{\infty}\mu^n Z_{n-k,x}\,Z_{k,x} = 4t^{-2}N\pi^2sin^4N\pi x(1+t^2sin^2N\pi x)^2 + 4N\pi t^{-1}sinN\pi x\,cos\,N\pi x Z_x\,(1 + t^2sin^2N\pi x) * \sum_{n/=0}^{\infty}N\pi\mu^{n/+1}Z_{n/,x} - 4N^2\pi^2(1 + t^2sin^2N\pi x)\sum_{n/=0}^{\infty}N\pi\mu^{n/+1}Z_{n/,x}$$

Let's consider a case where $N = 1$,

$$\sum_{k=0}^{1}N^2\pi^2\mu^n Z_{1-k,x}\,Z_{k,x} = 4t^{-1}N\pi sinN\pi x\ \ cos\,N\pi x\,(1+t^2sin^2N\pi x) * N\pi\,Z_{0x} - 4N\pi(1+t^2sin^2N\pi x) * N\pi\,Z_{0t}$$

Of course, it's worth noting that

$$Z_{0x} = 2t^{-1}N\pi sin^2N\pi x(1+t^2sin^2N\pi x)$$

wherefrom by integration by x, we get the equation for Z_0.

$$Z_0 = N\pi x(0.75t - 4t^{-1}) - 2^{-1}N\pi\ sin2N\pi x\ (t+t^{-1}) + 4^{-2}\ N\pi\ sin4\,N\pi x + f_0(t)$$

For further investigation, it is necessary to find a private derivative from Z_0 at $t$which has the following appearance:

$$Z_{0t} = N\pi x(0.75 - 4t^{-2}) - 2^{-1}N\pi sin2N\pi x\ (1-t^{-2}) + 4^{-2}\ N\pi sin4\,N\pi x + f_o^{/}(t)$$

It should be noted that the need for the convergence of the series for our solution stems from the continuity of functions.Z. So, the solution will be presented as 2 branches, which will be sewn at some point t_0. Let's look at the site.$(0 \le t \le\ t_0)$.In this case.$f_{0-}(t) = 0$. Accordingly, the solution for the right branch will be calculated in the future without any additional function.

As for the left branch and the site($t_0 < t \le +\infty$), this function will contribute here. Finding the function $f_{0+}(t)$is determined from the condition of continuity of functionW_{1x}. After calculating, we got that $\left(f_{o+}^{/}(t) = -0.75 + t^{-2}\right)$which gives$(f_{0+}(t) = -0.75t + t^{-1})$. So, we got a stapling point $t_0 = \frac{2}{\sqrt{3}}$.

Now, we can now write down equations for Z_{1x} for the right and left branches.

$$Z_{1x}^{-} = t^{-1}sin2N\pi x(1+t^2sin^2N\pi x) - t\ sin^{-2}N\pi x\{N\pi x(0.75-t^{-2}) + N\pi\ sin2N\pi x(t^{-2}-1) + 4^{-2}N\pi sin4N\pi x\}$$

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and

$$Z_{1x}^{+} = t^{-1}sin2N\pi x(1+t^2sin^2N\pi x) - t\, sin^{-2}N\pi x\{N\pi x - 1)(0.75-t^{-2}) + N\pi\, sin2N\pi x(t^{-2}-1) + 4^{-2}N\pi sin4N\pi x.\}.$$

After integrating each of the branches, we get the following equations:

$$Z_1^{-} = \frac{tcos4N\pi x}{16N\pi} - \frac{\cos 2N\pi x}{2N\pi}(0.25t+t^{-1}) - \frac{N\pi x}{2} - x\, cotN\pi x(0.75t-t^{-1})$$

$$Z_1^{+} = \frac{tcos4N\pi x}{16N\pi} - \frac{cos2N\pi x}{2N\pi}(0.25t+t^{-1}) - \frac{N\pi xt}{2} - (x-1)cotN\pi x(0.75t-t^{-1})$$

Let's consider a case where $N = 2$

$$\sum_{k=0}^{2} N^2\pi^2 Z_{2-k,x}\, Z_{k,x} = = 4t^{-1}N\pi sinN\pi x\ cos\, N\pi x\,(1 + t^2sin^2N\pi x) * (N\pi Z_{1x} + N\pi Z_{0x}) - 4N\pi(1+t^2sin^2N\pi x) * (N\pi Z_{0t} + N\pi Z_{1t})$$

Or, as a result of transformations and simplifications, this equation can be rewritten:

$$Z_{2x}Z_{0x} = Z_{1x}Z_{0x} - 2^{-1}Z_{1x}Z_{0x} + t^{-1}sin2N\pi x(1+t^2sin^2N\pi x)Z_{1x} - 2N\pi(1+t^2sin^2N\pi x)Z_{1t}$$

Now, the resulting equation will be integrated by x, and we'll get the following:

$$Z_2 = Z_1 + \int\frac{cotN\pi x Z_{1x}}{N\pi} - \int\frac{tZ_{1t}}{sin^2N\pi x} - \int\frac{Z_{1x}}{2Z_{0x}}.$$

We will only calculate this row member for the left branch, so we will be interested in the following equation Z_1^{-}that's gonna be recorded in the form

$$Z_{1t}^{-} = \frac{cos4N\pi x}{16N\pi} - \frac{cos2N\pi x}{2N\pi}(0.25-t^{-2}) - \frac{N\pi x}{2} - x\, cotN\pi x(0.75-t^{-2})$$

Let's write down each of the add-ons separately:

$$\int\frac{tZ_{1t}dt}{sin^2N\pi x} = \frac{x}{2N\pi}(0.75t-t^{-1}) - \frac{tsin4N\pi x}{16N^2\pi^2} - \frac{tx\ cotN\pi x}{2} + \frac{tx\ cot^2N\pi x}{2N\pi}(0.75t+t^{-1}) + \frac{t}{2N\pi}ln(sinN\pi x)$$

$$\int\frac{Z_{1x}\ cotN\pi x}{N\pi} = \frac{sin2N\pi x}{2tN^2\pi^2} - \frac{tsin4N\pi x}{16N^2\pi^2} - \frac{tsin2N\pi x}{2N\pi} - \frac{x\ cot^2N\pi x}{2N\pi}(0.75t+t^{-1}) + \frac{cot\ N\pi x}{2N^2\pi^2}(0.75t+t^{-1}) - \frac{cot\ N\pi x}{N\pi}(0.75t+t^{-1}) - x(0.75t+t^{-1}) + \frac{x}{N\pi}(0.25t+t^{-1})$$

Unfortunately, the 3rd addendum in this sum is not integrable, so in the future we will leave it in quadrature.

So, we got the equations for the first three sentences of our formal series. We believe that these combinations make a major contribution to the structure.

CONCLUSION

In this paper,we have recorded the equation for Skyrmion in the form of the Hamilton-Jacobi equation. For this obtained equation, the Immersion method was applied, by means of which the first three terms of the formal series were obtained

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