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Energy Value of Friendship Graph using Laplacian Minimum Majority Domination Parameter

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Abstract
In this article we have introduced Laplacian Minimum majority domination energy graph. We defined Laplacian Minimum Majority dominating matrix and its energy values Friendship graph.

***Keywords:** Laplacian Majority Domination matrix, Laplacian minimum majority domination Energy value.*

I. Introduction:

In this paper $G=(V,E)$ means finite simple graph with p vertices and q edges. A set D of vertices of a graph G is dominating set if every vertex in $V-D$ is adjacent to some vertex in D. The domination number γ_M of G is the minimum cardinality of all dominating sets of G. A set $S \subseteq V$ is called a majority dominating set if atleast half of the vertices either in S or adjacent to the vertices S. That is $|N[S]| \le \left\lceil \frac{|V(G)|}{2} \right\rceil$. The minimum cardinality of a majority dominating set is called majority domination number $\gamma_M(G)$.

This concept was introduced by Jose line Monora Swamination [9],[16],[17],[18]. The concept of energy graph was introduced Ivan Gutman [4].

In this paper we have introduced Laplacian Minimum Majority Dominating energy graph.

2. Laplacian Minimum Majority Dominating Energy of a Graph:

Definition:1

Let G be a simple graph with vertex set $v=\{v_1,v_2,v_3 \ldots v_n\}$ and the edge set $E=\{e_1,e_2,e_3 \ldots e_n\}$ and D is the Laplacian Minimum Majority Dominating set. Let $D(G)$ be the diagonal matrix of vertex degrees of the graph G. Then $L_{MMD}(G)=D(G)-A_D(G)$ is called the Laplacian Minimum Majority Dominating Matrix of G.The Laplacian Minimum Majority matrix of the graph G,denoted by $L_{MMD}(G)=(L_{nm})$,is a square matrix of order n whose elements are defined as

$$L_{MMD}=\begin{cases} -1 & \text{if } \ v_n \textit{ and } v_m \textit{ are adjacent} \\ 0 & \textit{if } \ v_n \textit{ and } v_m \textit{ are adjacent} \\ d_n & \textit{if } \ m=n \\ d_n-1 & \textit{if } \ m=n \textit{ and } v_n \in D \end{cases}$$

Where d_n is the degree of the vertex v_n. Let $\mu_1,\mu_2,\ldots,\mu_n$ be the eigen values of $L_{MMD}(G)$

Definition:2

For any graph G the characteristic polynomial of L_D is denoted by

$$L_{MMD}(G,\mu)=\det(\mu I - L_{MMD}(G)).$$

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The Laplacian minimum majority dominating energy of the graph G is defined as

$$LE_{MMD}(G)=\sum_{i=1}^{n}\left|\mu_i-\frac{2m}{n}\right|.$$

Where m is the number of edges of G and $\frac{2m}{n}$ is the average degree of G.

2.1. Theorem

For the friendship graph F_n with $n>2$, the minimum majority dominating energy graph

$$E_{MMD}(F_n)=\frac{3}{n}\left\lceil\frac{n-2}{2}\right\rceil+\frac{3-2n}{n}\left\lfloor\frac{n-2}{2}\right\rfloor+2\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}$$

Proof :

Let $G=F_n$ friendship graph with the vertex set $V(G)=\{u,v_1,v_2,v_3\ ...v_{n-1}\}$. The minimum majority dominating set is $D=v_1$. Hence MMD matrix is

$$A_D(G)=\begin{bmatrix}1&1&1&1&1&\cdots&1&1\\1&0&1&0&0&\cdots&0&0\\1&1&0&0&0&\cdots&0&0\\1&0&0&0&1&\cdots&0&0\\1&0&0&1&0&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\cdots&\vdots&\vdots\\1&0&0&0&0&\cdots&0&1\\1&0&0&0&0&\cdots&1&0\end{bmatrix}$$

$$D(G)=\begin{bmatrix}n-1&0&0&0&0&\cdots&0&0\\0&2&0&0&0&\cdots&0&0\\0&0&2&0&0&\cdots&0&0\\0&0&0&2&0&\cdots&0&0\\0&0&0&0&2&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\cdots&\vdots&\vdots\\0&0&0&0&0&\cdots&2&0\\0&0&0&0&0&\cdots&0&2\end{bmatrix}$$

$$LD(G)=D(G)-AD(G)$$

$$L_{MMD}(G)=\begin{bmatrix}n-2&-1&-1&-1&-1&\cdots&-1&-1\\-1&2&-1&0&0&\cdots&0&0\\-1&-1&2&0&0&\cdots&0&0\\-1&0&0&2&-1&\cdots&0&0\\-1&0&0&-1&2&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\cdots&\vdots&\vdots\\-1&0&0&0&0&\cdots&2&-1\\-1&0&0&0&0&\cdots&-1&2\end{bmatrix}$$

Characteristic polynomial is

$$\begin{vmatrix}\mu-2&1&1&1&1&\cdots&1&1\\1&\mu-2&1&0&0&\cdots&0&0\\1&1&\mu-2&0&0&\cdots&0&0\\1&0&0&\mu-2&1&\cdots&0&0\\1&0&0&1&\mu-2&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\cdots&\vdots&\vdots\\1&0&0&0&0&\cdots&\mu-2&1\\1&0&0&0&0&\cdots&1&\mu-2\end{vmatrix}$$

The characteristics equation is

$$(\mu-1)^{\left\lfloor\frac{n-2}{2}\right\rfloor}(\mu-3)^{\left\lceil\frac{n-2}{2}\right\rceil}\left[\mu^2-(n-1)\mu-1\right]=0$$

The minimum majority dominating Eigen values are

$$\mu=3\ \left(\frac{n-1}{2}\ \ times\right),\ \mu=1\ \left(\frac{n-3}{2}\ \ times\right),\ \mu=\frac{n-1}{2}\pm\sqrt{\left(\frac{n-1}{2}\right)^2+1}$$

Number of vertices = n, Number of edges = $\frac{3n-3}{2}$

$\therefore$ Average degree = $\frac{3(n-1)}{n}$

Hence Laplacian Minimum Majority Dominating energy,

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$$LE_{MMD}(F_n)=\left|3-\frac{3(n-1)}{n}\right|\left\lceil\frac{n-2}{2}\right\rceil+\left|1-\frac{3(n-1)}{n}\right|\left\lfloor\frac{n-2}{2}\right\rfloor+\left|\left\lfloor\frac{n}{2}\right\rfloor+\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}-\frac{3(n-1)}{2}\right|+\left|\left\lfloor\frac{n}{2}\right\rfloor-\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}-\frac{3(n-1)}{2}\right|$$

$$=\frac{3}{n}\left\lceil\frac{n-2}{2}\right\rceil+\frac{3-2n}{n}\left\lfloor\frac{n-2}{2}\right\rfloor+\left|\left[\left\lfloor\frac{n}{2}\right\rfloor-\frac{3(n-1)}{2}\right]+\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}\right|+\left|\left[\left\lfloor\frac{n}{2}\right\rfloor-\frac{3(n-1)}{2}\right]-\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}\right|$$

$$=\frac{3}{n}\left\lceil\frac{n-2}{2}\right\rceil+\frac{3-2n}{n}\left\lfloor\frac{n-2}{2}\right\rfloor+2\sqrt{\left\lfloor\frac{n}{2}\right\rfloor^2+1}$$

2.2. Theorem

Let G be a graph with order n, size m, If $LMMD(G,\lambda)=a_0\mu^n+a_1\mu^{n-1}+a_2\mu^{n-2}+.....+a_n$ be the Characteristic polynomial of Laplacian minimum majority of dominating matrix of G then $(i)\ a_0=1$ $(ii)\ a_1=-\mu_M(G)$

2.3. Theorem

For any graph G with vertex set $V(G)=\{v_1,v_2,v_3\ ...v_n\}$, edge set E and λ_M -set $D=\{u_1,u_2,u_3\ ...u_k\}$ .If $\mu_1,\mu_2,.....,\mu_n$ are the Eigen values of the matrix $LMMD(G)$ then

(i) $\sum_{i=1}^{n}\mu_i=|MMD|$

(ii) $\sum_{i=1}^{n}\mu_i^2=2|E|+|MMD|$

II. CONCLUSION:

In this article we have introduced the concept of Laplacian minimum majority dominating energy graph and its energy value. We have obtained Laplacian minimum majority dominating energy value for some classes of graph.

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