

# The effect of Hall Currents on the Double Diffusion Heat Transfer Flow of a Chemically Reacting Fluid Past a Stretching Sheet in the Presence of Constant Heat Sources

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#### Abstract

It is investigated that, the effect of hall currents on the double diffusion heat transfer flow of a chemically reaction fluid past a stretching sheet in the presence of constant heat sources. The equations determining the heat flow and transfer of mass are derived by employing a Galerkine finite element analysis with three nodded line segments. The velocity, temperature and concentration are solved for G, M, m, N, Sc, a and y. The rate of heat and mass transfer are numerically evaluated for different variations of parameters.

Keywords: Hall current, heat and mass transfer, heat transfer flow

# I. INTRODUCTION

Laminar boundary layer behavior over a continuously moving and stretching surface is a significant type of flow has considerable practical applications in engineering, electrochemistry. In particular, different metallurgical methods associate the cooling of continuous strips or filaments by drawing them through a quiescent fluid.

In 1961, Sakiadis (1) who developed a numerical solution for the boundary layer flow field over a continuous solid surface moving with constant speed. Chen and Cher (2) have studied the absorption and insertion on a linearly moving plate under uniform wall temperature and heat flux. In general, using a power law velocity and temperature distribution at the surface was studied by Ali (3), Magyari et al. (4) have reported analytical and computational solution when the surface moves with rapidly decreasing velocities using the self-similar method.

In the above references, the effect of buoyancy force was relaxed. The above authors carrying the issue of a polymer sheet extruded continuously from a dye. It is supposed that the sheet is in stretchable, but in the polymer industry in which it is necessary to handle a stretching plastic sheet, as noted by Crane (5). The study of heat producing or absorption in moving fluids is significant in the problems in relation with chemical reactions. Vajravelu and A. Hadjinicolaou (6) studied the heat characteristics in laminar boundary layer of a viscous fluid over a stretching sheet with viscous diffusion or frictional heating and internal heat generation.

The effect of chemical reaction on free convection of viscous flow and mass transfer of incompressible and electrically conducting fluid



over a stretching sheet was investigated by Afify (7) in the presence of transverse magnetic field. Anjalidevi and Kandaswamy (8) discussed that, the impact of a chemical reaction on the flow along a semi infinite horizontal plate in the presence of heat transfer.

Anjalidevi and Kandaswamy (9) have discussed that the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field. Raptis et al. (10), have studied the viscous flow over a non-linearly stretched sheet in the presence of a chemical reaction and magnetic field. In all these investigations the electrical conductivity of the fluid was assumed to be uniform. However, in an ionized fluid where the density is low over magnetic field is very strong. The conductivity normal to the magnetic field is reduced due to the rising of electrons and ions about the magnetic lines of force before collisions take place and a current induced in a direction normal to both the electric and magnetic fields.

The hall effect on MHD boundary layer flow over a continues semi-infinite flat plate moving with a uniform velocity in its own plane in an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field were investigated by Watanabe and Pop (11). Abo-Eldehbab (12) disussed that free-convective flows past a semiinfinite vertical plate with mass transfer. Samad et al. (13) have discussed that, the MHD heat & mass transfer of free convection flow along vertical stretching sheet in presence of magnetic field with heat generation. G.C Shit (14) has studied Hall effects on MHD free convective flow on mass transfer over a stretching sheet.

Configuration of the problem



# **II. FORMULATION**

We deal with the steady flow of an incompressible, viscous and electrically conducting fluid past a flat surface which is assuming from a horizontal slit on a vertical surface and is stretched with a velocity proportional to distance from a fixed origin O. We take a stationary frame of reference (x, y, z) such that x-axis is along the direction of motion of the stretching surface, y-axis is normal to this surface and z-axis is transverse to the xy-plane.

A uniform magnetic field in the presence of fluid flow induces the current  $(J_x, 0, J_z)$ . When the strength of the magnetic field is very large, we include the hall current so that the generalized Ohm's law (CF. Cowling(1975)) is modified to

$$\overline{J} + \omega_e \tau_e \overline{J} x \overline{H} = \sigma(\overline{E} + \mu_e \overline{q} x \overline{H})$$
(1)

where J is the current density vector,  $\omega_e$  is the cyclotron frequency,  $\tau_e$  is the electron collision time, q is the velocity vector, H is the magnetic field intensity vector, E is the electric field,  $\sigma$  is the fluid conductivity and  $\mu_e$  is the magnetic permeability.



The effect of hall current gives to a force in the z-direction which in turn produces a cross flow velocity in this direction and thus the flow becomes three-dimensional. To simplify the analysis, we consider that the flow quantities does not change along z-direction and this will be valid if the surface is of very width along the zdirection. Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field E=0, we have

$$j_x - m H_0 J_z = -\sigma \mu_e H_0 w \tag{2}$$

$$J_z + mH_0 J_x = \sigma \mu_e H_0 u \tag{3}$$

here  $m = \omega_e \tau_e$  be the hall parameter.

From the equations (2) & (3), we obtain

$$j_{x} = -\frac{\sigma \mu_{e}^{2} H_{h}^{2}}{1+m^{2}} (mu - w)$$
(4)

$$j_{z} = \frac{\sigma \mu_{e}^{2} H_{o}^{2}}{1 + m^{2}} (u + mw)$$
(5)

Here, u and w are the velocity components along x and z directions respectively.

The Continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

The equations of the Momentum are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \mu_e H_0 J_z - \rho \overline{g} \quad (7)$$

$$u\frac{\partial W}{\partial x} + v\frac{\partial W}{\partial y} = v\frac{\partial^2 W}{\partial y^2} + \mu_e H_0 J_x$$
(8)

The equation of the energy is

$$\rho C_p \left( u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} \right) = k_f \frac{\partial^2 T}{\partial y^2} + Q$$
(9)

The euation of a diffusion is

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_0(C - C_\infty)$$
(10)

The equation of state is

$$\rho - \rho_0 = -\beta(T - T_\infty) - \beta^{\bullet}(C - C_\infty) \tag{11}$$

Substituting  $J_x$  and  $J_z$  from equations (4) & (5) in equations (7) & (8), we obtain

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e H_0^2}{1+m^2}(u+mw)$$
(12)  
+  $\beta g(T-T_{\infty}) + \beta^{\bullet}g(C-C_{\infty})$   
$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial x} = v\frac{\partial^2 w}{\partial x^2} + \frac{\sigma\mu_e^2 H_0^2}{2}(m_0u-w)$$
(13)

 $u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial z} = v \frac{\partial w}{\partial y^2} + \frac{\partial \mu_e H_0}{1 + m^2} (m_0 u - w)$ (13) where, the temperature is T, the concentration in the fluid is C, the coefficient of thermal expansion

the fluid is C, the coefficient of thermal expansion is  $\beta$ ,  $\beta^{\bullet}$  is the volumetric expansion with concentration and Q is the strength of the heat source.

where, the boundary conditions are

$$u = bx, v = w = 0, T = T_w, C = C_w, at y = 0$$
(14)

$$u = w = 0, T = T_{\infty}, C = C_{\infty}, as \quad y \to \infty$$
(15)

where b > 0.

The boundary conditions on the velocity for (2.14) are the no-slip conditions at the surface at y = 0, while the boundary conditions on the velocity as  $y \rightarrow \infty$ , there is no flow far away from the stretching surface. The temperature and concentration are maintained at a prescribed constant values  $T_w$  and  $C_w$  at the sheet and are assumed to vanish far away from the sheet.

On introducing the similarity variables

$$\eta = \sqrt{\frac{b}{v}} y, \qquad u = bx f'(\eta),$$
$$v = \sqrt{bv} f(\eta) ,$$

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$$w = bx g(\eta), \ \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}},$$
  
$$\phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}$$
(16)

Equations (9), (10), (12) & (13) reduce to

$$f''' + f f'' - f'^{2} + G(\theta + N\phi - \frac{M^{2}}{1 + m^{2}}(f' + mg) = 0$$
 (17)

$$g'' + fg' - (f' + \frac{M^2}{1 + m^2})g + \frac{mM^2}{1 + m^2}f' = 0$$
(18)

$$\theta'' + P f \theta' + \alpha = 0 \tag{19}$$

$$\phi'' + Sc(\phi' f - \gamma \phi) = 0$$
 (20)

and the boundary conditions (14) & (15) are obtained from (2.16) as

$$f'(0) = 1, \ f(0) = 0, \ \theta(0) = \phi(0) = 0$$
 (21)

$$f'(\infty) = g(\infty) = \theta(\infty) = 0$$
(22)

### **III. VARIATIONAL FORMULATION**

The variational form with the equations (17) to (20) over a typical two nodded linear element  $(\eta_e, \eta_{e+1})$  is given by

$$\int_{\eta_e}^{\eta_{e+1}} w_1(f'-h) d\eta = 0$$
 (23)

$$\int_{\eta_e}^{\eta_{e+1}} [w_{12}(h'' + fh' - h^2 + G(\theta + N\phi) - \frac{M^2}{1 + m^2}(h + mg)] d\eta = 0$$

(24)

$$\int_{\eta_{e}}^{\eta_{e+1}} w_{3}(g'' + fg' - (h + \frac{M^{2}}{1 + m^{2}})h + \frac{mM^{2}}{1 + m^{2}}h)d\eta = 0$$
(25)
$$\int_{\eta_{e}}^{\eta_{e+1}} w_{4}(\theta'' + Pf\theta' + \alpha)d\eta = 0$$
(26)

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$$\int_{\eta_e}^{\eta_{e+1}} w_5(\phi'' + Sc(\phi'f - \gamma\phi)d\eta = 0 \qquad (27)$$

where the arbitrary test functions are  $w_1$ ,  $w_2$ ,  $w_3$ ,  $w_4$  and  $w_5$  and regarded as the variations in f, h, g,  $\theta$  and  $\phi$  respectively.

#### **III. (i) FINITE ELEMENT FORMULATION**

From the equations (23) to (27), by substituting finite element approximations of the form

$$f = \sum_{k=1}^{3} f_k \psi_k \quad , \qquad \theta = \sum_{k=1}^{3} \theta_k \psi_k$$
$$h = \sum_{k=1}^{3} h_k \psi_k \quad , \qquad \phi = \sum_{k=1}^{3} \phi_k \psi_k$$
(28)

with  $w_1 = w_2 = w_3 = w_4 = w_5 = \psi_i^j$  (*i*, *j* = 1,2,3) (29)

From (29), the equations (23) to (27) reduces to

$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{df}{d\eta} - h\right) \psi_i^j d\eta = 0$$
(30)

$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{d}{d\eta}\left(\frac{dh}{d\eta}\right) + f\left(\frac{dh}{d\eta}\right) - h^2 + G(\theta + N\phi) - \frac{M^2}{1 + m^2}(h + mg)\right)\psi_i^j d\eta = 0$$
(31)

$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{d}{d\eta} \left(\frac{dg}{d\eta}\right) + f\left(\frac{dg}{d\eta}\right) - \left(h + \frac{M^2}{1 + m^2}\right)g + \frac{mM^2}{1 + m^2}h\right)\psi_i^j d\eta = 0$$
(32)

$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{d}{d\eta} \left(\frac{d\theta}{d\eta}\right) + Pf\left(\frac{d\theta}{d\eta}\right) + \alpha\right) \psi_i^j d\eta = 0$$
(33)

(34)  
$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{d}{d\eta} \left(\frac{d\phi}{d\eta}\right) + Sc \left(\frac{d\phi}{d\eta}\right) f - \gamma \phi\right) \psi_i^j d\eta = 0$$

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Using "Galerkine weighted residual method" and "integration by parts method" to the equations (30) to (34), we have

$$\int_{\eta_e}^{\eta_{e+1}} \left(\frac{df}{d\eta} - h\right) \psi_i^j d\eta = 0$$
(35)

$$\begin{split} &\int_{\eta_{e}}^{\eta_{e+1}} \frac{d\psi_{i}^{j}}{d\eta} (\frac{dh}{d\eta}) d\eta - \int_{\eta_{e}}^{\eta_{e+1}} f(\frac{dh}{d\eta}) \psi_{i}^{j} d\eta + \int_{\eta_{e}}^{\eta_{e+1}} h^{2} \psi_{i}^{j} d\eta \\ &- G \int_{\eta_{e}}^{\eta_{e+1}} (\theta + N\phi) \psi_{i}^{j} d\eta + \frac{M^{2}}{1 + m^{2}} \int_{\eta_{e}}^{\eta_{e+1}} (h + mg) \psi_{i}^{j} d\eta \\ &= Q_{1,j} + Q_{2,j} \end{split}$$

where 
$$-Q_{1,jj} = \psi_j(\eta_e)(\frac{dh}{d\eta})(\eta_e)$$
  
 $Q_{21,jj} = \psi_j(\eta_{e+1e})(\frac{dh}{d\eta})(\eta_{e+1e})$ 
(36)

$$\int_{\eta_{e}}^{\eta_{e+1}} \frac{d\psi_{i}^{j}}{d\eta} \frac{dg}{d\eta} d\eta - \int_{\eta_{e}}^{\eta_{e+1}} f\left(\frac{dg}{d\eta}\right) \psi_{i}^{j} d\eta + \int_{\eta_{e}}^{\eta_{e+1}} \left(\left(h + \frac{M^{2}}{1 + m^{2}}\right)g + \frac{mM^{2}}{1 + m^{2}}h\right) \psi_{i}^{j} d\eta$$
$$= R_{1,j} + R_{2,j}$$

where 
$$-R_{1,j} = \psi_j(\eta_e) \frac{dg}{d\eta}(\eta_e)$$
  $R_{2,j} = \psi_j(\eta_{e+1}) \frac{dg}{d\eta}(\eta_{e+1})$   
(37)

$$\int_{\eta_{e}}^{\eta_{e+1}} \left(\frac{d\psi}{d\eta}\frac{d\theta}{d\eta}d\eta - P\int_{\eta_{e}}^{\eta_{e+1}} f(\frac{d\theta}{d\eta}) + \alpha\right)\psi_{i}^{j}d\eta = S_{1,j} + S_{2,j}$$
where
$$-S_{1,j} = \psi_{j}(\eta_{e})\frac{d\theta}{d\eta}(\eta_{e}) \qquad S_{2,j} = \psi_{j}(\eta_{e+1})\frac{d\theta}{d\eta}(\eta_{e+1})$$

(38)

$$\int_{\eta_{e}}^{\eta_{e+1}} \left(\frac{d\psi_{ij}}{d\eta} \frac{d\phi}{d\eta} - Sc \int_{\eta_{e}}^{\eta_{e+1}} \left(\frac{d\phi}{d\eta}\right) f - \gamma \phi\right) \psi_{i}^{j} d\eta = T_{1,j} + T_{2,j}$$
  
where  $-T_{1,j} = \psi_{j}(\eta_{e}) \frac{d\phi}{d\eta}(\eta_{e})$   $T_{2,j} = \psi_{j}(\eta_{e+1}) \frac{d\phi}{d\eta}(\eta_{e+1})$ 

(39)

By showing " $f^k$ ,  $h^k$ ,  $\theta^k & \phi^k$ " in terms of local nodal values, then the equations (36) to (39), we have

$$\begin{split} &\sum_{k=1}^{3} h^{k} \int_{\eta_{e}}^{\eta_{e+1}} \frac{d\psi_{i}^{j}}{d\eta} (\frac{d\psi_{k}}{d\eta}) d\eta - \sum_{k=1}^{3} f^{k} \int_{\eta_{e}}^{\eta_{e+1}} \psi^{k} (\frac{dh}{d\eta}) \psi_{i}^{j} d\eta + \\ &\sum_{k=1}^{3} (h^{k}) \int_{\eta_{e}}^{\eta_{e+1}} \psi_{k}^{2} \psi_{i}^{j} d\eta - G \sum_{k=1}^{3} \int_{\eta_{e}}^{\eta_{e+1}} (\theta^{k} + N\phi^{k}) \psi_{k} \psi_{i}^{j} d\eta + \\ &\frac{M^{2}}{1+m^{2}} \sum_{k=1}^{3} \int_{\eta_{e}}^{\eta_{e+1}} (h^{k} + mg^{k}) \psi_{k} \psi_{i}^{j} d\eta = Q_{1,j} + Q_{2,j} \\ &\text{where} - Q_{1,jj} = \psi_{j}(\eta_{e}) (\frac{dh}{d\eta}) (\eta_{e}) \qquad Q_{21,jj} = \psi_{j}(\eta_{e+1e}) (\frac{dh}{d\eta}) (\eta_{e+1e}) \end{split}$$

(40)  

$$\sum_{k=1}^{3} g^{k} \int_{\eta_{e}}^{\eta_{e+1}} \frac{d\psi_{i}}{d\eta} \frac{d\psi_{k}}{d\eta} d\eta - \sum_{k=1}^{3} f^{k} g^{k} \int_{\eta_{e}}^{\eta_{e+1}} (\frac{d\psi_{k}}{d\eta}) \psi_{k} \psi_{i}^{j} d\eta + \\
+ \sum_{k=1} \int_{\eta_{e}}^{\eta_{e+1}} ((h^{k} + \frac{M^{32}}{1+m^{2}})g^{k} + \frac{mM^{2}}{1+m^{2}}h^{k}) \psi_{k} \psi_{i}^{j} d\eta = R_{1,j} + R_{2,j}$$
where  $R_{1,j} = \psi_{j}(\eta_{e}) \frac{dg}{d\eta}(\eta_{e})$   $R_{2,j} = \psi_{j}(\eta_{e+1}) \frac{dg}{d\eta}(\eta_{e+1})$ 
(41)

$$\sum_{k=1}^{3} \theta^{k} \int_{\eta_{e}}^{\eta_{e+1}} \left(\frac{d\psi}{d\eta} \frac{d\psi_{k}}{d\eta} d\eta - P \sum_{i=1}^{3} \int_{\eta_{e}}^{\eta_{e+1}} f^{k} \left(\frac{d\theta^{k}}{d\eta}\right) + \alpha\right) \psi_{k} \psi_{i}^{j} d\eta = S_{1,j} + S_{2,j}$$
  
where  $-S_{1,j} = \psi_{j}(\eta_{e}) \frac{d\theta}{d\eta}(\eta_{e}) k = 1, S_{2,j} = \psi_{j}(\eta_{e+1}) \frac{d\theta}{d\eta}(\eta_{e+1})$ 

$$(42) \\ \sum_{k=1}^{3} \phi^{k} \int_{\eta_{e}}^{\eta_{e+1}} (\frac{d\psi_{ij}^{j}}{d\eta} \frac{d\psi_{k}}{d\eta} - Sc \sum_{k=1}^{3} \phi^{k} f^{k} \int_{\eta_{e}}^{\eta_{e+1}} (\frac{d\psi_{k}}{d\eta}) \psi_{k} - \gamma \phi^{k}) \psi_{i}^{j} \psi_{k} d\eta = T_{1,j} + T_{2,j}$$
  
where,  $-T_{1,j} = \psi_{j}(\eta_{e}) \frac{d\phi}{d\eta}(\eta_{e})$   $T_{2,j} = \psi_{j}(\eta_{e+1}) \frac{d\phi}{d\eta}(\eta_{e+1})$   
(43)

selecting different  $\psi_{ij}^{\ j}$  corresponding to each element  $\eta_e$  in the equation (40) yields a local stiffness matrix of order 3x3 in the form

$$(f_{i,j}^{k})(u_{i}^{k}) - G(\theta_{i}^{k} + NC_{i}^{k}) + \frac{M^{2}}{1 + m^{2}}(g_{i}^{k}) = (Q_{1,j}^{k}) + (Q_{2,j}^{k})$$

(44)

Likewise the equations (41), (42) & (43) give rise to stiffness matrices

$$(g_{i,j}^{k}) + (f_{i}^{k}) - \frac{mM^{2}}{1+m^{2}}(u_{i}^{k}) = (R_{1,j}^{k}) + (R_{2,j}^{k})$$
(45)

$$(e_{i,j}^{k})(\theta_{i}^{k}) - P(u_{i}^{k}) + \alpha = (S_{1,j}^{k}) + (S_{2,j}^{k})$$
(46)

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$$(l_{i,j}^{k})(\phi_{i}^{k}) - Sc(u_{i}^{k0})(\phi_{i}^{k}) = (T_{1,j}^{k}) + (T_{2,j}^{k})$$
(47)

where,  $(f_{i,j}^{k}), (g_{i,j}^{k}), (\theta_{i,j}^{k}), (\phi_{i,j}^{k}), (e_{i,j}^{k}), (l_{i,j}^{k})$  are 3x3 matrices and

 $(Q_{1,jj}^k), (Q_{2,jj}^k), (R_{1,jj}^k), (R_{1,jj}^k), (S_{1,jj}^k), (S_{1,jj}^k), (S_{1,jj}^k), (T_{1,jj}^k) and .(T_{1,jj}^k)$ 

are matrices of order 3 X 1 and such stiffness matrices (44) to (47) are local nodes to get the coupled global matrices of the global nodal values of "h, f, g,  $\theta$  and  $\phi$ ".

In case, we select n quadratic elements then the global matrices are of order 2n+1. To find the unspecified global values of velocity, temperature and concentration in the fluid region, we solve the ultimate coupled global matrices.

An iteration method adopted to solve these equations to involve the boundary effects in the porous medium.

The shape functions are

$$\begin{split} \psi_i^i &= \frac{(y-8)(y-16)}{128} \quad \psi_2^1 = \frac{(y-24)(y-32)}{128} \quad \psi_3^1 = \frac{(y-40)(y-48)}{128} \\ \psi_i^{2i} &= \frac{(y-4)(y-8)}{32} \quad \psi_2^2 = \frac{(y-12)(y-16)}{32} \quad \psi_3^2 = \frac{(y-20)(y-24)}{128} \\ \psi_i^3 &= \frac{(3y-8)(3y-16)}{128} \quad \psi_2^3 = \frac{(3y-24)(3y-32)}{128} \quad \psi_3^3 = \frac{(3y-40)(3y-48)}{128} \\ \psi_i^4 &= \frac{(y-2)(3y-4)}{8} \quad \psi_2^4 = \frac{(y-6)(y-8)}{8} \quad \psi_3^4 = \frac{(y-10)(y-12)}{8} \\ \psi_i^5 &= \frac{(5y-8)(5y-16)}{128} \quad \psi_2^5 = \frac{(5y-24)(5y-32)}{128} \quad \psi_3^5 = \frac{(5y-40)(5y-48)}{128} \end{split}$$

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#### **IV. STIFFNESS MATRICES**

The global matrix for  $\theta$  is

 $A_3X_3 = B_3$ 

The global matrix for  $\phi$  is

 $A_4X_4 = B_4$ 

The global matrix for h is

 $A_5X_5 = B_5$ 

The global matrix for f is

 $A_6X_6 = B_6$ 

The global matrix for g is

 $A_7X_7 = B_7$ 

## V. DISCUSSION OF THE NUMERICAL RESULTS

The system of ordinary differential equations (17) to (20) subject to the boundary conditions (21) and (22) are solved numerically by employing finite element analysis with three nodded line segments.

For numerical computations, the following values of the physical parameters have been considered according the data used in (Afify 2004).

$$P_r = 0.71, G = 10^2, N = 1, 2, -0.5, -0.5,$$
  
Sc = 0.24 - 2.01, n = 1, 2, 3, M = 0 to 5, m = 0 to 3,

$$\gamma = 0.1, 0.5, 1.0$$







Figure 1 : Variation of h with M, m

	Ι	II	III	IV	V	VI	
Μ	2	4	6	10	2	2	
m	1.5	1.5	1.5	1.5	0.5	2.5	

From fig. 2, the change of f'with buoyancy ratio N shows that, the molecular buoyancy force dominates over the thermal buoyancy force .



Figure 2 : Variation of h with N

	Ι	II	III	IV
N	1	2	-0.5	-0.8

From fig. 3, the change of f'with heat source parameter  $\alpha$  shows that f'enhances with increase in  $\alpha$ >0 and reduces with  $|\alpha|$ 



Figure 3 : Variation of h with α Published by: The Mattingley Publishing Co., Inc.

	Ι	Π	III	IV	V	VI
α	2	4	6	-2	-4	-6

From Fig. 4, the effect of chemical reaction  $\gamma$  on f'is shown. The axial velocity f'increases in the degenerating chemical reaction and reduces in the generating case.

Fig. 4 :	Variation	of h	with	γ
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	Ι	II	III	IV	V	VI
γ	0.5	1	1.5	2.5	-0.5	-1.5

From Fig. 5, we know that, for higher the Lorentz force larger f  $(\eta)$  and for higher Lorentz force lesser f.



	Figure 5 : Variation of f with M, m								
	Ι	II	III	IV	V	VI	VII		
Μ	2	4	6	10	2	2	2		
m	0.5	0.5	0.5	0.5	1	1.5	2.5		



From Fig. 6, we know that the transverse velocity increases with increase in N > 0 and decreases with N < 0 in the entire flow region



Figure 6 : Variation of f with N

	Ι	II	III	IV
N	1	2	-0.5	-0.8

Fig. 7 represents that, change of f with M and m. We seen that g increases with  $M \le 4$  and decreases with  $M \ge 6$ . And g increases with increase in the Hall parameter  $m \le 1.5$  and decreases with  $m \ge 2.5$ .



Figure 7 : Variation of g with M, m

	Ι	Π	III	IV	V	VI	VII
Μ	2	4	6	10	2	2	2
m	0.5	0.5	0.5	0.5	1	1.5	2.5

From Fig. 8, the change of g with N shows the cross flow velocity decreases with N > 0 and increases with |N| < 0.



Figure 8 : Variation of g with N



From Fig. 9, enhance in the strength of the heat source increases the cross flow and also higher  $|\alpha| \ge 6$ , get a reduction in |g| in the entire flow region.



Figure 9 : Variation of g with  $\alpha$ 

	Ι	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

From Fig. 10, we know that, the cross flow increases with increase in  $|\gamma| \le 1.5$  and for higher  $|\gamma| \ge 2.5$ , the cross flow velocity decreases in the flow region.





γ

From Fig. 11, the temperature increases with  $\alpha \le 4$  and decreases with  $|\alpha| \ge 6$ , it decreases in the case of heat source and increases in the case of heat source.







From Fig. 12, we know that, the temperature increases in the degenerating chemical reaction case and depreciate in the generating case.





I II III IV V VI γ 0.5 1 1.5 2.5 -0.5 -1.5

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