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Theorems on Weakly Standard Rings

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Abstract

If a ring holds flexible law and the commutators lie both in left and middle nucleus, then the ring is said to be weakly standard . Here we establish a prime non associative weakly standard ring is accessible. Further it will be proved that R is strongly (1,0) ring and also a strongly (-1,1) ring. Finally we conclude this paper by presenting an Algebra which is Weakly Standard ring

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INTRODUCTION

A"Weakly Standard ring (WSR)" is a non associative ring whichholds the flexible law

$(a,b,a)) = 0$, (1)

And the commutator lies in left and middle nucleus

$((a,b),R,R) = 0$, (2)

and$(R,(a,b),R) = 0$. (3)

A linearization of (1) yields the identity

$(a,b,c) = - (c,b,a)$ (4)

wherethe notation(a,b,c) is an associatorand is given as $(a,b,c) = ab.c - a.bc$and another notation(a,b) is a commutator and is given as $(a,b) = ab–ba$. From the definition of Weakly standard ring , obviously one obtains the identity $(y,z,(w,x)) = 0$. Hence *commutators lie in nucleus* implies that $[R,R] \subset N$, here N is a nucleus given by, for every element n in N ,for every element

$N = \{ n \in N , \forall x,y \in R / (n,x,y)=(x,n,y)=x,y,n)=0\}$.In this article Rrepresents a Weakly standard ring .Let$M$,Nbe two ideals of R such that $MN= 0$ and if R is prime then either $M =0$or$N = 0$.

It was well known that if a non associative ring satisfies$(R,(a,b,a)) = 0$and$[R,R] \subset N$, thenwe will say that

" Theorem 1 : If R is semi-prime , then R is isomorphic to a sub direct sum of a semi-prime associative ring and a semi-prime commutative ring."

"Theorem 2**:** If R is prime , then R is either associative or commutative."

"Theorem 3**:** If R is simple then R is either associative or commutative."

"Theorem 4**:**IfR is semi-simple, then R is isomorphic to a sub direct sum of a semi-simple associative ring and a semi-simple commutative ring."

The above four theorems are applicable to Weakly Standard ring since R satisfies $(R,(a,b,a)) = 0$*and* $[R,R] \subset N$.

Associativity or commutativity of a primitive Weakly Standard ring was proved in [1] .Primitive was replaced by simple in [2] , in that case R is either commutative or (-1,1) ring. In general Weakly Standard rings are not alternative.

Here we establish that a prime non-associative ringWSRis Strongly $(1,0)$ ring. A non associative ring R is referred as‘Strongly $(1,0)$ring’ if the left alternative law $(a,a,b)=0$ and $((a,b),c)=0 \ \forall\ a,b,c$€ R hold in a ring. Where asit is ‘Strongly $(-1,1)$’ if the right alternative law $(b,a,a)=0$andhold in a ring.Let us look at [3].Subhashini proved in [3] that a Simple (-1,1) ring is an Accesiible, when it is not associative as well as not commutative. In [4] it was established that a Simple (-1,1) ring of char $\neq 2,3$ is Strongly (-1,1).

Any arbitrary ringsatisfy the given below important equations

$(xy,z) = x(y,z) + (x,z)y + (x,y,z) +(z,x,y) - (x,z,y)$. (5)

Techmulleridentity :

$(wx,y,z) = (w,xy,z) + (w,x,yz)\ w(x,y,z) + (w,x,y)z$.(6)

and

$(x,y,z)+(y,z,x) +(z,x,y)= ((x,y),z) + ((y,z),x) +((z,x),y)$ (7)

MAIN RESULT :We now introduce a set $T= \{ t$ €$N/tR\subset N \}$

Lemma1**:**$T$is the ideal of R.

Proof: ∀ t€ T and ∀ x,y € R. Then$(t,x,y) = 0$ implies t$x.y = t.xy$. From the definition of T, t$,xy$ € N and then t$x.y$ € N. *Thus* $TR \subset T$.Also $xt = (x,t) + tx$ € N.

It is clear that $x.ty = (x,ty) + ty.x$ € N , Since it have been already observed that t$y.x$ € N. Moreover (x,ty) is a commutator and hence in N. Hence $x.ty$ € N.

Since t € N , in weakly standard ring $(x,t,y) = 0$ implies $xt.y = x.ty$. it is clear that the product $x.ty$is in N and hence $xt.y$is in N, so that $RT\subset T$ and then T becomes the ideal of R.

From now onwards R represents a Prime non associative Weakly Standard ring, then we have following lemma

Lemma*2***:**In R**,**Commutators lie in center.

Proof:Because$((a,b),c)$ ∀ a,b,c€ R is a commutator , obviously it is in N.Put $a= (x,y)$, $b = w$, $c = z$ in equation (5) then it is observed that $((x,y)w,z)-(x,y)(w,z)- ((x,y),z)w =0$, because the commutators are in nucleus i.e$[R,R] \subset N$.Moreover the product $(x,y)(w,z)$ *and* $((x,y)w,z)$ *are in* N.Hence $((x,y),z)w$ is in N and then $((a,b),c)$ *is in* T.

let$t$ € T .Put$w =t$ in equation (6) .Then we get the identity $(tx,y,z)-(t,xy,z)+(t,x,yz)= t(x,y,z)+(t,x,y)z$.

Since TR is a subset of$T$ *and* T *is subset of* N then we can have $t(x,y,z) = 0$. Hence $TA = 0$.

We assumed that R is prime hence either $T = 0$ or $A = 0$.Since R is notassociative ,implies $A \neq 0$. If $T = 0$ then$((a,b),c)$€ $T = 0$. Hence Commutators lie in center.

Corollary1*:*From lemma2, equation (7) is reduced to $(x,y,z)+(y,z,x)+(z,x,y) = 0$. And from (1)this leads to $(x,y,z)+(z,x,y)-(x,z,y) = 0$.Along this and from (2) hence here we have the following theorem.

Theorem*5:*R isan Accessible ring.

Lemma3**:**InR,everyassociator is in center.

Proof:ApplyCorollary1, (5) is reduced to

$(xy,z)= x(y,z) + (y,z)x$ (8)

When elements u,v,w,x,y,z€ R.By using the definition of associator and repeated use of equation (8)

$((w,x,y),z)= ((wx.y - w.xy),z)$

$=(wx.y,z) - (w.xy,z)$

$=wx.(y,z) + w(x,z).y + (w,z)x.y -(w,z).xy - w.x(y,z) -w.(x,z)y$

$= (w,x,(y,z)) + ((w,z),x,y) + (w,(x,z),y) =0$

Hence ∀ x,y,z€ R, we have $((w,x,y),z)=0$ *i.e* associator is in center.

Corollary2:Substitutez=$(u,v)$and $z = (p,q,r)$ an arbitrary associator in Lemma3 , gives

$((R,R,R),(u,v))=0$

and $((R,R,R),(p,q,r)) = 0$

Lemma 4:In R $((y,y,x)v,w) = 0$.

*Proof:*Replace*w by* a commutator(v,x) in (6). And then use (8) ,(2) and (3), we get

$(v,x)(x,y,z) = ((v,x)x.y,z) = ((vx,x)-v(x,x)),y,z)=((vx,x),y,z) =0$. So

$(v,x)(x,y,z) =0.$ ………. (9)

We now linearize equation (9), which gives

$(v,w)(x,y,z) = - (v,x)(w,y,z)$ (10)

Put $x=y$ in (10)

$(v,w)(y,y,z) = -(v,y)(w,y,z),$from orollary1

$=(v,y)((y,z,w)+(z,w,y))$

$=(v,y)(y,z,w)+(v,y)(z,w,y))$

$=(v,y)(z,w,y)),$ From (9)

$=- (v,y)(y,w,z),$ From (1)

$(v,w)(y,y,z) = 0,$ From (9)

And hence from (1)

$(v,w)(y,y,z) = 0 = (v,w)(z,y,y).$ (11)

Replace x by (y,y,z) and$y,z$by v,w in (8),

$((y,y,z)v,w) = (y,y,z)(v,w) + (v,w)(y,y,z)=0.$

From (11) , we have $((y,y,z)v,w) = 0.$

We now define another set namely$M= \{ m€ R/ (R,v)=0=(R,Rv) \}$

Lemma 5*:M*is an ideal of *R*.

Proof**:** For v €R , from lemma3 $(z,(x,y,v))=0$ or $(z,xy.v) – (z,x.yv) = 0$. Then $(z,x.yv) =0$. Thus

Ym€ M.It is clear that$M$becomes a left ideal of Rym€ M,*for all y in R*.From the definition of M, $(y,m)=0$ie$ym-my=0$ implies $my=ym$ € M. And hence Mbecomesideal of R.

Let us define C which is a set of all finite sums of elements of the form (R,R) or $(R,R)R$.

Lemma6**:***C*is an ideal of *R* .

Proof**:**In general C is not an ideal. But in a prime non associative weakly standard ring C is an ideal by virtue of (10) and lemma3. Also $C\subset N$.

Theorem6:A prime non associative Weakly Standard ring is either a commutative or strongly (1,0) ring.

Proof:.For any element m *in* M and any *element* c *in* C,from (11) we have $cm=0$. Therefore $CM=0$.Here we assumed R is prime then from the definition of a prime ring we have either $C=0$ *or* $M=0$.If $C=0$ then$R$ becomes a commutative ring .Since $M$is an ideal of R, from lemma3 and lemma4, the left alternator (y,y,z) lies in M. When $M=0$ therefore $(y,y,z) = 0$.And hence from lemma2,R is Strongly *(1,0)*ring.

Corollary3: R is either a commutative or a Strongly (-1,1) ring.

*Proof***:** From (11),$(v,w)(z,y,y) = 0$.Then R isStrongly (-1,1) ring.

EXAMPLE **:** If e,u,v,w,z are the basis elements of an algebra A over a field R where the only non zero products among the basis elements are $e^2=e$, $eu=ue=v$, $ev=ve=u$, $we=ze=w$.

Here commutators lie in left and middle nucleus. However A is neither associative nor commutative. Moreover which is not alternative.

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