TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

To find the solution and predict model for logistics by Traveling Salesman problem using graceful labeling and nearest neighbor algorithm.

Priyanka Bhalerao, M. Phil and Research Scholar
Research Scholar, Department of Mathematics, Shri Vaishnav Vidyapeeth University, Indore, MP, India
Dr. Seema Bagora,
Associate Professor, Department of Mathematics, Shri Vaishnav Vidyapeeth University, Indore, MP, India

Article Info
Volume 82
Page Number: 7914 - 7922
Publication Issue:
January-February 2020

Article History
***Article Received**: 18 May 2019*
***Revised**: 14 July 2019*
***Accepted**: 22 December 2019*
***Publication**: 05 February 2020*

Abstract
This research work deals with the travelling salesman problem using graceful labeling of k-ary trees. The aim of this research is to obtain the best possible route or the cheapest possible route to be taken during the distribution of product or work. This research presents an attempt to solve the problem of logistic companies having many regional offices and area office in different areas. We find solutions using TSP to obtain the information or data from regional offices and area offices with the best possible way of visiting all the offices and returning to the starting point that minimize the travel cost or travel distance. Here we use the nearest neighbor method of TSP to solve this problem.

Keywords: *Graceful Labeling, k-ary trees, Nearest Neighbor, Optimization, Travelling Salesman Problem.*

1 .INTRODUCTION:

1.1 Definitions:

1.1.1 Graph theory: One of the most active fields of current research in the subjects of discreet mathematics is the growing research areas within graph theory. For an excellent and recent dynamic survey on graph labeling we refer to Gallian.

1.1.2 Graph: A graph *G* is a finite non–empty set of objects called vertices together with a set of unordered pairs of distinct vertices of *G* called edges. The set of vertices and the set of edges *G* are denoted by *V(G)* and *E(G)* respectively.
The number of vertices in *G* is called the order of *G* and the number of edges in *G* is called the size of *G*.

1.1.3 Graph labeling: Graph labeling is the assignment of labels, where the vertices or edges or both are assigned real values subject to certain conditions, have often been motivated by their use in various applied fields and their intrinsic mathematical interest. Most graph labeling methods trace their origin to one introduced by Rosa in 1967 or given Graham and Sloane in 1980.Graph labeling was first introduced in mid 1960s.

1.1.4 Graceful labeling: Graceful labeling was first introduced by Rosa in 1967. However Rosa called this labeling as β-labeling or valuation. After several years Golomb studied the same type of labeling and called this labeling graceful labeling.

A Graph is known as graceful when its vertices are labeled from 0 to $|E|$. The size of graph and this labeling induces an edges labeling from 1 to $|E|$. Graceful labeling means that if there is more than one common point of two distinct edges, then it is not graceful labeling.

2. Tree:
In graph theory, a tree is an undirected, connected and acyclic graph. Alternatively, a connected graph that does not contain even a single cycle is called a tree. A tree represents hierarchical structure in a graphical form. The elements of trees are called their nodes and the edges of the tree are called branches. The tree with n vertices has *(n-1)* edges. Trees provide many useful applications as simple as a family tree to as complex as trees in data structures of computer science.

Let T be any arbitrary tree of m edges. If its vertices can be distinctly labeled using integers *0, 1. . .* m so that all the induced edge labels (vertices labeled i and j induce label $|i - j|$ on edge (i, j)) are also distinct, then the labeling is called graceful.

There is a famous conjecture named Graceful tree conjecture or Ringle Kotzig's conjecture that says "All trees are graceful" has been the focus of many papers.

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Graphs of different classes have been proven mathematically to be graceful or non graceful.

3. Classification of Trees

3.1 Balanced Tree: A balanced tree is defined as a tree in which every end point has the same level.

3.2 Rooted Tree: Rooted tree is a tree in which one vertex is distinguished and called a root.

3.3 K–ary tree: A k-ary tree is a rooted tree in which each vertex has at-most *k* children, 2-ary trees are often called binary tree, 3–ary are ternary tree, 4–ary are called quaternary tree. The binary tree and all k-ary trees are special cases of balanced tree. In building a binary tree, so that it falls within the definition of a balanced tree, it is noticed that the fixed vertex is always the root of initial trees.

3.4 Binary tree: Binary tree is a rooted tree where each internal vertex has at most two children: Left and Right. The number of vertices of a binary tree is always odd. Maximum number of vertices possible in k level binary tree is 2^k. Maximum possible height of an *n*-vertex binary tree is $(n-1)/2$.

3.5 Full Binary tree: Every node has exactly two children and all the leaves are on the same level. Maximum number of nodes at any level *'l'* is less than or equals to 2^l.
Where $l = 0,1,2,3\ldots\ldots\ldots\ldots\ldots l - 1$.Height of tree is the maximum distance from the root to any node.

Travelling Salesman Problem (TSP) is a widely studied combinatorial optimization problem and still it is challenged in operational research. Travelling Salesman Problem is (TSP) were studied in the 18th century by a mathematician from Ireland named Sir Willian Rowam Hamilton and by the British mathematician named Thomas Penyngton Krikman. Travelling salesman first gained fame in a book written by German Salesman B.F. Voiqt in 1832 on how to be a successful traveling salesman. He mentions the TSP, although not by that name, by suggesting that to cover as many locations as possible without visiting any location twice is the most important aspect of the scheduling of a tour.

The objective of TSP is to find the shortest path, starting from the home city till the destination. The main difficulty of this problem is the immense number of possible tours (n-1)! /2 for n cities.

Definition: Let G=(V,E) be a graph where V a set of n vertices. E is a set of edges, and let S= (Sij) be a distance matrix connected with E. The TSP consists of designing a minimum distance circuit passing through each vertex once and only once. Such a circuit is known as tour. The most common practical understanding of TSP is that of a salesman finding the shortest path through n cities.

The TSP is to find the best possible way of visiting all the cities and returning the starting point that minimize the travel cost(or travel distance) given a set of cities and the cost of travel (or distance) between each possible pairs. Travelling Salesman Problem is a non deterministic polynomial time hard problem in combinatorial optimization studied in operational research and theoretical computer science. Travelling salesman problem is nothing but finding out Least Cost Hamilton Circuit.

The following three are the basics of TSP:

Complete graph: It is a graph where all vertices are joined by a single edge.

Weighted graph: It is a graph where each edge carries a value.

Hamiltonian circuit: It is a path which starts from one vertex and passes through all vertices on a graph only once and then return to the origin vertex.

Theoretical Approach of TSP:

- TSP can be modeled as an undirected weighted graph.
- Cities are the graph vertices.
- Paths are the graph's edges.
- TSP becomes Hamilton cycle.
- Optimal TSP tour is the shortest Hamilton cycle.

Problem statement of TSP: If there are 'n' cities and cost of travelling from any city to any other city is given.

- Then we have to obtain the cheapest round trip such that each city is visited exactly once and returning to starting city will complete the tour.
- Typically a travelling salesman problem is represented by weighted graph.

Complexity:- Given 'n' no of cities be visited the total number of possible routes covering all cities to be visited the total number of possible routes covering all cities can be given a set of feasible solutions of the TSP and is given as (n-2)!/2 .

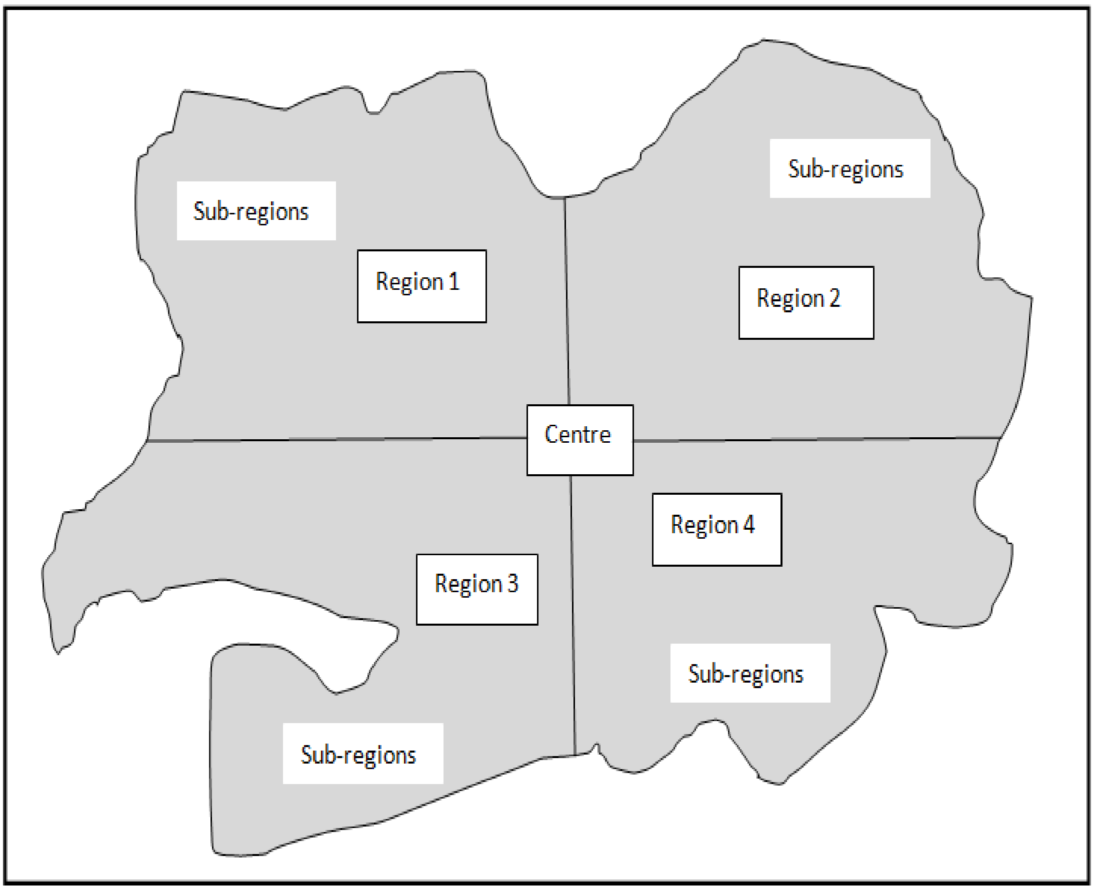
January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Solution Approach:
Given below is a graphical representation of the typical regional sales co-ordination problem faced by a travelling salesman. Consider the following five steps to represent the solution of this problem.

1) Raw Unstructured Representation of the Sales Area Network
2) Structured Representation of the Sales Area Network
3) Quaternary Tree Representation of the Sales Area Network
4) Gracefully Labeled Quaternary Tree Representation of the Sales Area Network
5) Using Nearest Neighbor Method of TSP to find the solution of the Sales Area Network

Raw Unstructured Representation of the Sales Area Network

Consider a **raw unstructured diagram of model** which is a random representation of a region where we need to work on sales network and efficiency. Given below is a random representation of an area having a central sales office denoted by 'Centre' and having 'n' sub-regions as shown below.



In the above random representational diagram of an arbitrary sales area network, we identify a central office and four distinct regional offices. Each of the regional offices is directly and uniquely connected to the central office. Further going into individual sales areas, each regional office behaves as a regional office for that particular sales region and each sub-regional office is connected to the regional office in a unique and distinct way.

Thus the random sales area network can be represented as a structured sales area network as shown in the next figure.

Problem: The central office (Head office) is faced with a problem of how to carry out a physical inspection tour of Regional offices to obtain information, and all area officers also submit their information to Regional offices, so that there is no overlaps of meeting during this whole process so that Head sales manager stationed at Head office has to travel every regional office once a month.

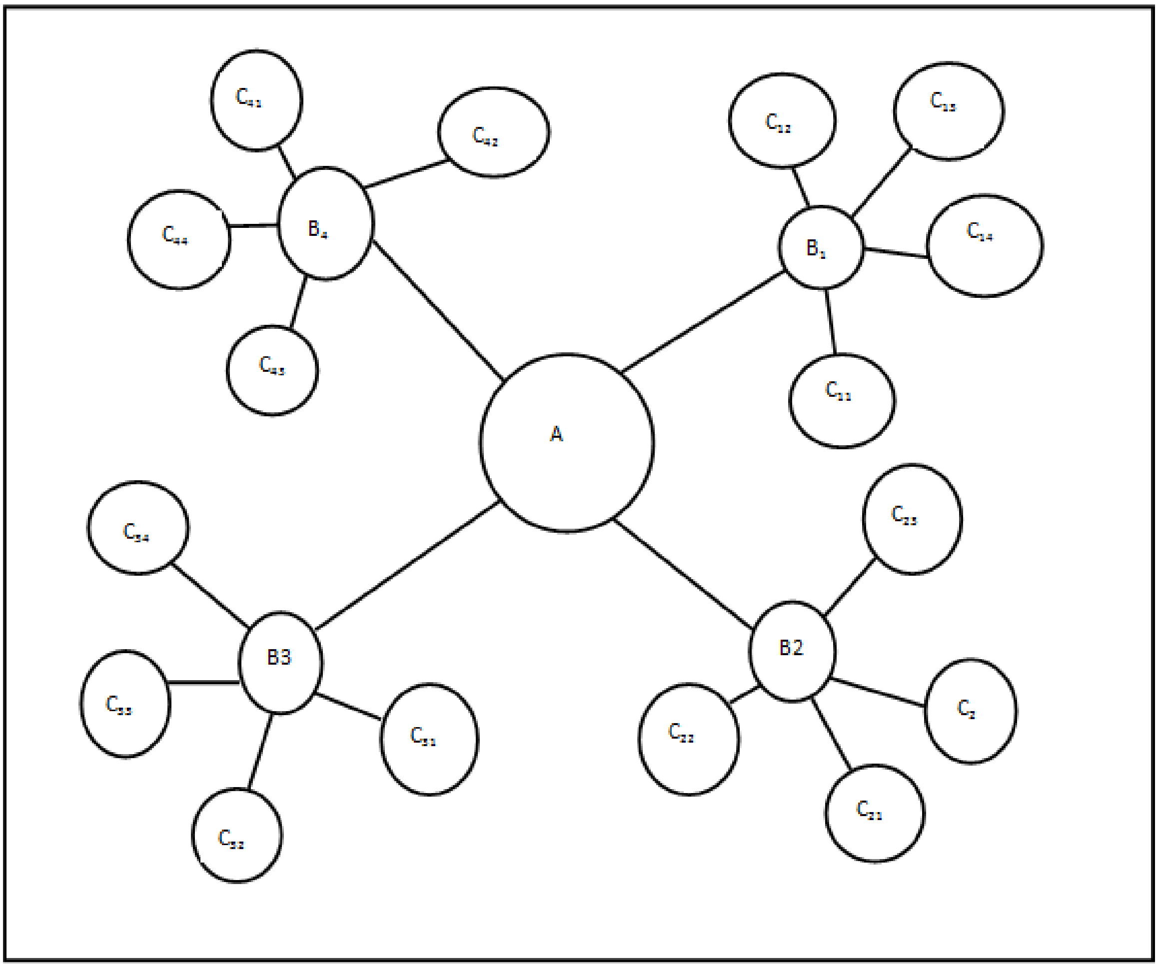
TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

CENTRAL OFFICE	Regional office 1	Area Office 1
		Area Office 2
		Area Office 3
		Area Office 4
	Regional office 2	Area Office 1
		Area Office 2
		Area Office 3
		Area Office 4
	Regional office 3	Area Office 1
		Area Office 2
		Area Office 3
	Regional office 4	Area Office 1
		Area Office 2
		Area Office 3
		Area Office 4

Simplifying the Raw Unstructured diagram of the area with the various regions, we get a structured diagram with a distinct central office represented by A, four distinct regional offices represented by B1, B2, B3 and B4 and 16 sub-regional offices C_{11}, C_{12}, C_{13} and C_{14} attached to B_1, similarly, C_{21}, C_{22}, C_{23} and C_{24} attached to B_2, similarly, C_{31}, C_{32}, C_{33} and C_{34} attached to B_3 and C_{41}, C_{42}, C_{43} and C_{44} attached to B_4.

Structured Representation of the Sales Area Network



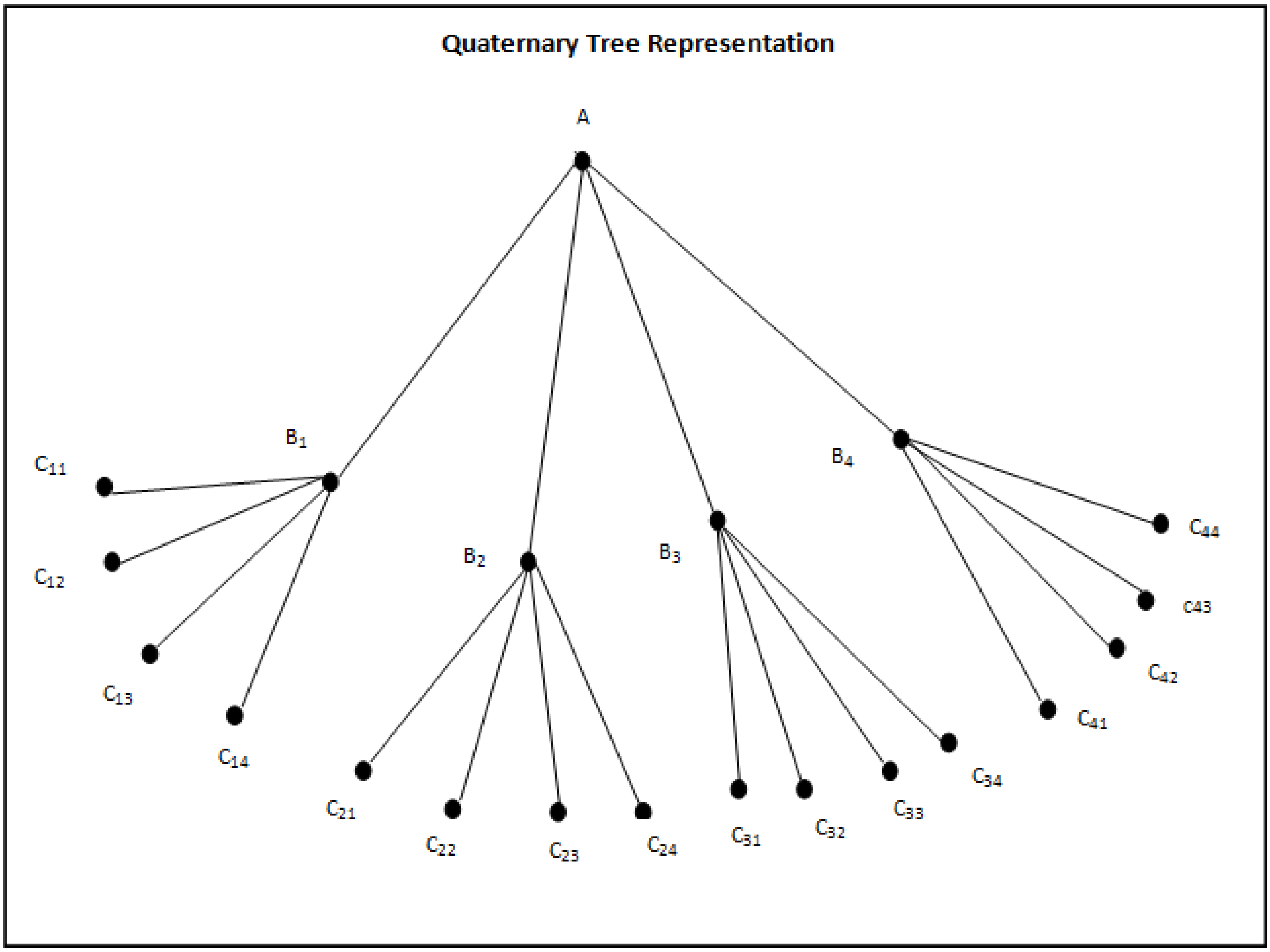
January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Each region will have several sub-regional offices which would be further represented in the next diagram. Such a typical sales region can be further structured as below by representation of it as a Symmetric Tree (which is called as a k-ary tree). Consider the following example below as the case of k-ary tree (k=4), hence a Symmetric Quaternary Tree.

Quaternary Tree Representation of the Sales Area Network:

Description of above diagram:

- Let the above graph represent a combinatorial structure of the sales network of a company.
- Let A be the Central Office (Head Office) of a company . Let it have n Regional Offices and every Regional Office has K Area Offices under it.
- Let $B_1, B_2, B_3 \ldots\ldots\ldots B_n$ be the Regional Offices of the company.
- Let the Area Offices be
  $C_{n1}, C_{n2}, C_{n3} \ldots\ldots\ldots\ldots\ldots\ldots C_{nk}$
  $C_{(n+1),}1$, $C_{(n+1),}2$,$C_{(n+2)},3 \ldots\ldots\ldots C_{(n+1),}k$
  $C_{(n+2),}1$,$C_{(n+2),}2$,$C_{(n+2)},3 \ldots\ldots\ldots\ldots C_{(n+2),}k$..



Now, we can further work on the above Quaternary Tree and label it gracefully. This is shown in the following diagram.

Hence at level 2, we have different area offices under regional offices. Suppose this area offices are:

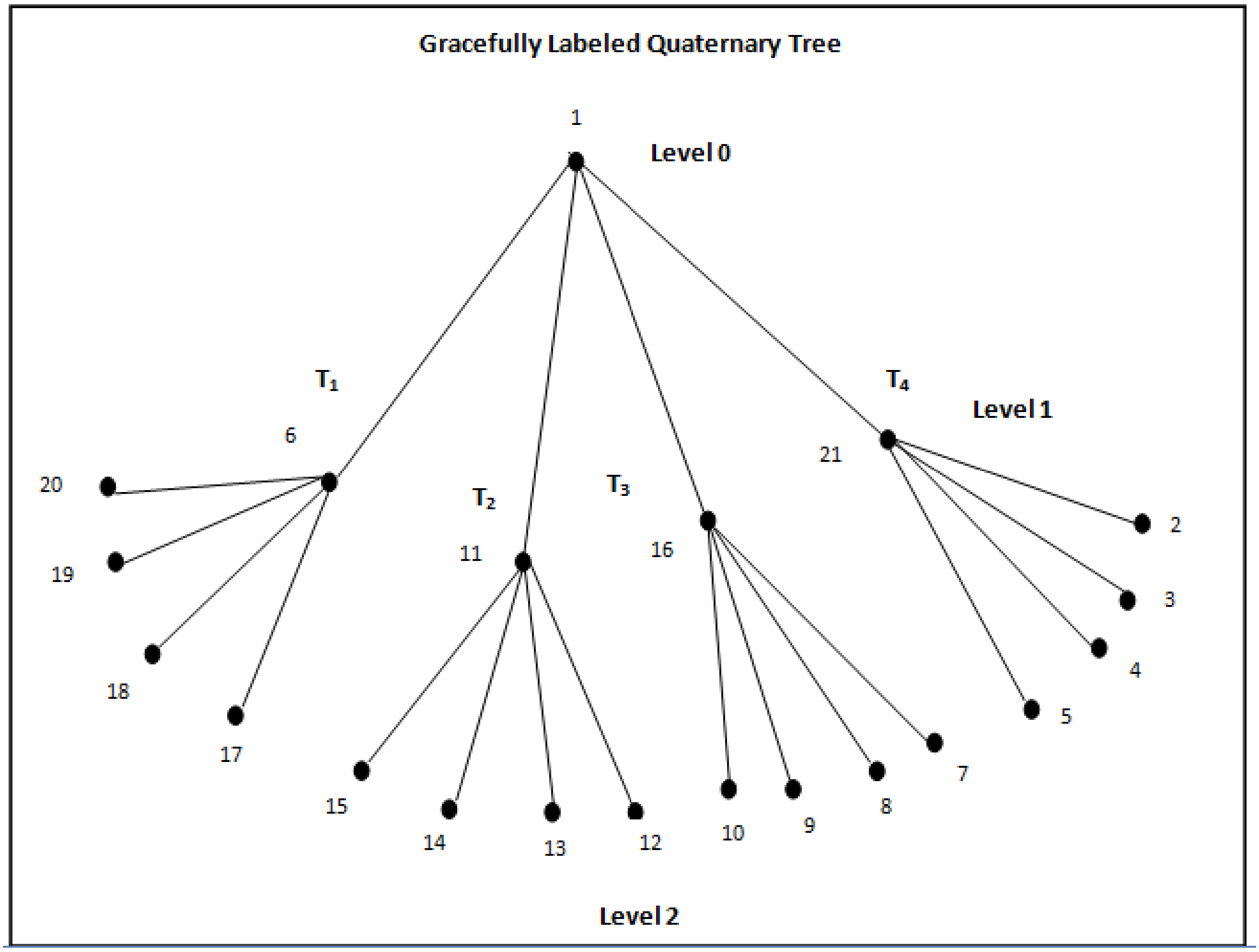
- For Regional office B_1 (sub tree T_1) we have$c_{11}, c_{12}, c_{13}, c_{14}$.
- For Regional office B_2 (sub tree T_1) we have $c_{21}, c_{22}, c_{23}, c_{23}$.
- For Regional office B_3 (sub tree T_3) we have $c_{31}, c_{32}, c_{33}, c_{34}$.

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Foe Regional office B_4 (sub tree T_4) we have c_{41},c_{42},c_{43},c_{44}

1. Here initial vertex i=1 we now named it "A" at level 0. Suppose "A" be the central office (Head office) of a company.
2. At level 1 as quaternary tree we have 4 sub tree T_1,T_2,T_3,T_4.Now suppose this sub tree as Regional offices B_1,B_2,B_3 and B_4.

Gracefully Labeled Quaternary Tree Representation of the Sales Area Network:



Above is the business model of a company which we structure this model as a quaternary tree, has three levels that is level 0,level 1,level 2..It is gracefully labeled quaternary tree that is each edge and each vertices has distinct labels.

Model in Business Logistics:

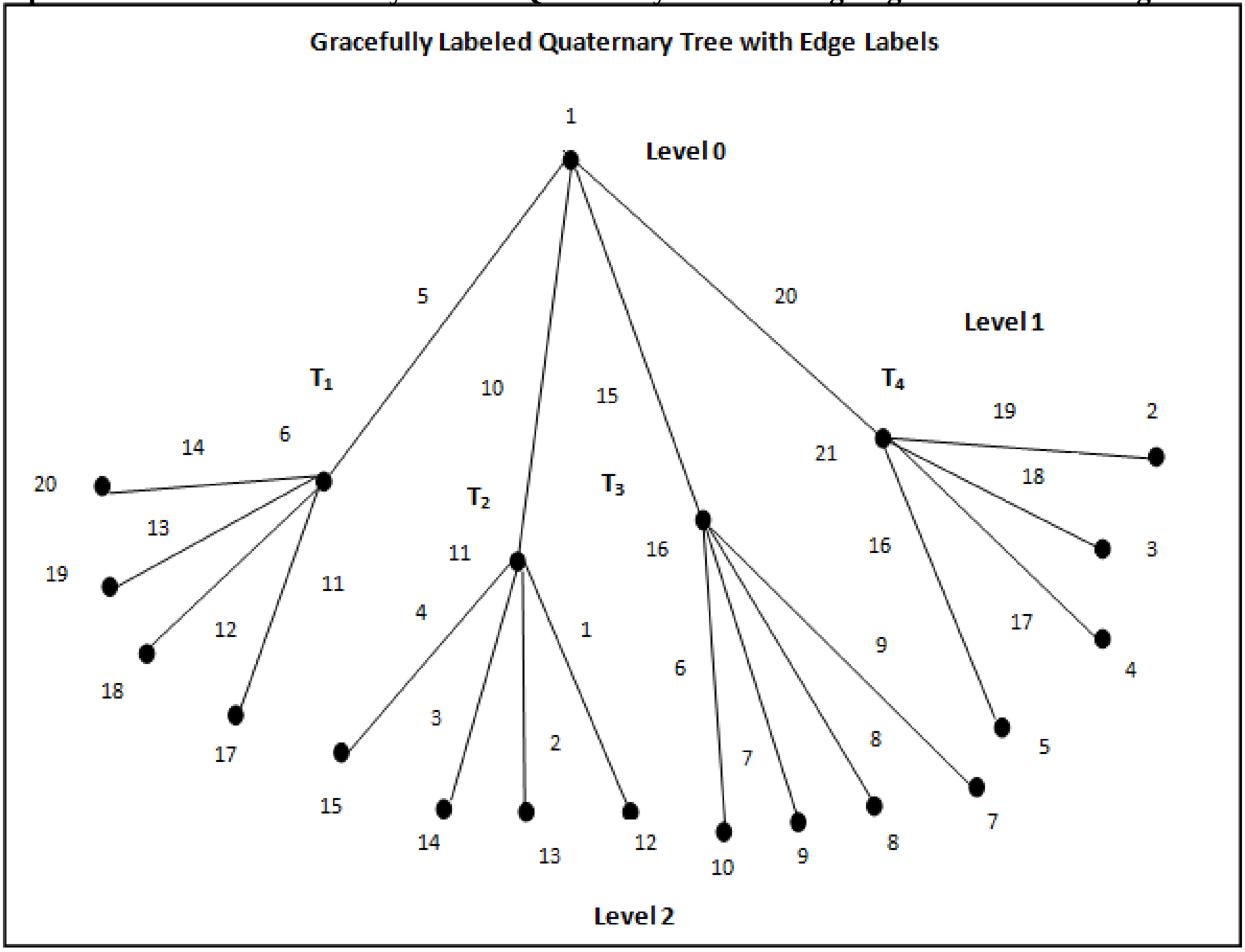
1. A model of distribution process which shows the distribution of goods at various stages and various problems that arise during distribution.
2. To make distribution proper is to remove the problems which arise during this process such as lack of communication, transportation problem, lack of responsibility by agents, lack of coordination etc.
3. We are proposing a business logistic model where the distance travelled by goods and merchandise is minimized along with the allotment of minimum time for each activity.
4. We are using Travelling salesman problem for this and use nearest neighbor method to solve this TSP.
5. We proposed this model in form of quaternary tree, which is gracefully labeled.

We solve this route problem in a form of tree, which can be solved using TSP. The Input for the TSP algorithm are the pair wise distances of a sequence and output is a circular tour through the optimal tour method can be used .

TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Representation of the Gracefully Labeled Quaternary Tree showing Edge Graceful Labeling



Method of solving TSP:

Nearest Neighbor Method:

The Nearest Neighbor algorithm is one of the first methods to be used to find a solution for the travelling salesman problem. It selects a starting point and then always selects the nearest city to be added to the tour, it then “walks” to that city and repeats by choosing a new non-selected city, until all cities is in the tour. In order to complete the tour, we add an edge between the last selected city and the starting city. The algorithm was introduced by J.G. Skellam and it was continued by the F.C. Evans and P.J Clark. In the nearest neighbor algorithm, we randomly choose a city as the starting city and then traverse to all the cities closest to the staring city that does not create any cycle. This process continues till all cities are visited once.

Solution Methodology:

1) The unique labeling of the gracefully labeled symmetric tree enables us to distinctly monitor the solution of network sales problem by systematically naming the vertices and edges.
2) Consider each sub-region of the tree as a distinct sales area network. The independent sub-regions will be used for implementing Traveling Salesman Method.
3) Suppose the salesman has to traverse sub-region B_1.
4) Select any node as starting node.

   a) Select any vertex which would be our current vertex for sub-tree T1.

   b) This vertex moves towards the minimum distance edge.

   c) Then this is our current vertex, again this vertex moves towards the edge which is of least distance.

   d) Keep changing the current vertex in such a way that it covers the least distance.

   e) Thus the regional sales manager can complete his tour for the sub-tree T1.

   f) This can be continued for all the sub-trees so that the tour can be completed.
5) Look for the neighbors of the starting node that have not been visited yet preferring the smallest edge-label and should not be repeated. Then we choose the next closest node to the starting node having the minimum distance.

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

6) Repeat this procedure until all the nodes in the tour have been visited exactly once.
7) Check if all the nodes are visited at least once. If yes, then return back to the starting node which will give the complete path.
8) Finally calculate the total distance of the tour.

We apply the method of nearest neighbor to solve route problem in this network. Here we have four sub trees namely T_1 T_2 T_3 T_4 which represent regional offices and area sales offices.

Now to apply nearest neighbor method, we take sub tree T_1 to which we apply this method.

Algorithm of nearest method:

For sub tree T_1, taking starting node i=6

Then, select nearest node j = 17 closed to i.

C_{ij} = 11(minimum distance between i and j).

Now form a tour T = 6-17-6.

Find a node K not in the sub tour that is closed to any of the sub tour nodes (that is for which distance to the sub tour is small).

Now insert K = 18, T = 6-17-18-6, $c_{jk} = 1$

$\Delta f = c_{ik}+c_{kj}-c_{ij}$, where $c_{ik} = 12$, $c_{kj}= 1$, $c_{ij}= 11$.

Δf = 12+1-11 = 13-11 = 2.(smallest)

Now insert L = 19 in tour, T = 6-17-18-19-6, $c_{kl} = 1$

$\Delta f = c_{il} +c_{lk}-c_{ik}$

Δf = 15+1-12

Δf = 16-12 = 4.

Now insert m=20 in tour, T = 6-17-18-19-20-6, c_{ml}=1.

$\Delta f = c_{im}+c_{ml}-c_{il}$

Here $c_{im} = 14$, $c_{ml}=1$,$c_{il}=13$

Δf = 14+1-13

Δf = 15-13 = 2 (smallest).

Similarly, for the second sub tree T_2 we can apply the same method,

Taking starting node i=11

Then, select nearest node j = 12 closest to i.

C_{ij} = 1(minimum distance between i and j).

Now form a tour T = 11-12-11.

Find a node K not in the sub tour that is closed to any of the sub tour nodes (that is for which distance to the sub tour is small).

Now Insert K = 13, T = 11-12-13-11, $c_{jk} = 1$

$\Delta f = c_{ik}+c_{kj}-c_{ij}$, where $c_{ik} = 2$, $c_{kj}= 1$, $c_{ij}= 1$.

Δf = 2+1-1= 2

Now insert L = 14 in tour, T = 11-12-13-14-11, $c_{kl} = 1$

$\Delta f = c_{il} +c_{lk}-c_{ik}$

Δf = 3+1-2

Δf =4-2=2

Now insert m=15 in tour, T = 11-12-13-14-15-11, c_{ml}=1.

$\Delta f = c_{im}+c_{ml}-c_{il}$

Δf = 4+1-3

=5-3

=2

Similarly, for the third sub tree T_3 we can apply the same method,

Taking starting node i=16

Then, select nearest node j = 10(closest to i).

C_{ij} = 6(minimum distance between i and j).

Now form a tour T = 16-10-16.

Find a node K not in the sub tour that is closed to any of the sub tour nodes (that is for which distance to the sub tour is small).

Now Insert K = 9, T = 16-10-9-16, $c_{jk} = 1$

$\Delta f = c_{ik}+c_{kj}-c_{ij}$, where $c_{ik} = 7$, $c_{kj}= 1$, $c_{ij}=6$.

Δf = 7+1-6=2(smallest)

Now insert L = 8 in tour, T = 16-10-9-8-16, $c_{kl} = 1$

$\Delta f = c_{il} +c_{lk}-c_{ik}$

Δf = 8+1-7=9-7=2.

Now insert m=7 in tour, T =16-10-9-8-7-16, c_{ml}=1.

$\Delta f = c_{im}+c_{ml}-c_{il}$

Δf =9+1-8

Δf = 10-8 = 2 (smallest).

Similarly, for the fourth sub tree T_4 we can apply the same method,

Taking starting node i=21

Then, select nearest node j = 5(closest to i).

C_{ij} = 16(minimum distance between i and j).

Now form a tour T = 21-5-21.

Find a node K not in the sub tour that is closed to any of the sub tour nodes (that is for which distance to the sub tour is small).

Now Insert K = 4, T = 21-5-4-21,$c_{jk} = 1$

$\Delta f = c_{ik}+c_{kj}-c_{ij}$, where $c_{ik} = 17$,$c_{kj}= 1$,$c_{ij}=16$.

Δf = 17+1-16 =2(smallest)

Now insert L = 3 in tour, T = 21-5-4-3-21, $c_{kl} = 1$

$\Delta f =c_{il}+c_{lk}-c_{ik}$

Δf = 18+1-17

Δf = 19-17 =2.

Now insert m=2 in tour, T =21-5-4-3-2-21,

$\Delta f = c_{im}+c_{ml}-c_{il}$

Δf =19+1-18

Δf = 20-18=2 (smallest).

January - February 2020
ISSN: 0193 - 4120 Page No. 7914 - 7922

Now apply to tree 'S'

Taking starting node i=1, then, select nearest node j = 6 closest to i.

C_{ij} = 5 minimum distance between i and j).

Now form a tour T =1-6-1

Find a node K not in the sub tour that is closed to any of the sub tour nodes (that is for which distance to the sub tour is small).

Now Insert K = 11, T = 1-6-11-1,

$\Delta f = c_{ik}+c_{kj}-c_{ij}$, where c_{ik} =15, c_{kj}=5, c_{ij}= 5

Δf = 15+10-5= 15

Δf = 20

Now insert L = 16 in tour, T = 1-6-11-16-1.

$\Delta f = c_{il} + c_{lk} - c_{ik}$

Δf = 15+5-10

Δf = 10

Now insert m=21 in tour, T =1-6-11-16-1-21.

$\Delta f = c_{im}+c_{ml}-c_{il}$

Δf =20+5-15

Δf =25-15=10.

Conclusion:

In the above model, we applied the nearest neighbor method to the 1st, 2nd, 3rd and 4th sub tree, and the increase in tour length $\Delta f = 2$ (the smallest value).

In the whole business model for the sub tree S, the increase in tour length $\Delta f = 10$ (the smallest value).

In this way we applied the nearest neighbor method of travelling salesman problem to symmetrical tree (to be molded as a business model) with the aim to find a route with minimum distance and cost or time.

This algorithm takes input as the number of cities and coordinates as the distances for cities which is represented in the form of adjacency matrix. If the starting node is specified it continues with it, otherwise starting node is chosen at random. The algorithm gives a sequence of all the visited vertices. The shorter routes can be detected by human insight easily. So, the nearest neighbor algorithm gives a feasible solution in case of gracefully labeled symmetric k-ary tree that can be used to increase the efficiency of a typical sales area network which would be beneficial for the business.

References:

1. Arora K, Agrawal S, Tanwar R, *Solving TSP using Genetic Algorithm and Nearest Neighbor Algorithm and their Comparison*, International Journal Of Scientific & Engineering Research, 7(2016).
2. Cesar R, Dorabela G, Glover F, Osterman C, *Travelling Salesman Problem heuristics leading methods: implementations and latest advances*, European Journal of operational Research(Elsevier),211(2011),427-441.
3. Chouhan C, Gupta R, Pathak K, *Survey of Methods of Solving Travelling Salesman Problem along with its implementation using dynamic programming approach*, International journal of computer applications ,52(2012).
4. Gurpreet S, Vinay C, *An analysis of various techniques to solve travelling salesman problem: A Review,* International Journal Of Advanced Research In Computer Science,3(2012).
5. Ismail Z, Wan Rohaizad wan Ibrahim, *Travelling Salesman Approach for solving petrol distribution using simulated Annealing* ,American Journal of Applied Sciences,5(2008),1543-1546.
6. Khan A. A, Agrawal H, *A Comparative Study Of Nearest Neighbor Algorithm and Genetic Algorithm In Solving Travelling Salesman Problem*, International Research Journal of Engineering and Technology(IRJET), 3(2016).
7. Kirutikaa US.,Dr. S.S. Dhenakaran, *A Method OF Solving Travelling Salesman Problem,* International Journal of innovative Research in computer and communication engineering, 4(2016).
8. Malik W, Rathinam S, Darbha S, *An approximation algorithm for a symmetric generalized multiple depot multiple TSP*,Department of Mechnical Engineering Texax A &M university USA, Elsevier 35(2007) 747-753.
9. Rai K, Madan L, Anand K, *Research paper on Travelling Salesman Problem and its solution using Genetic Algorithm,* IJIRI 1(2014), ISSN 2349-6002.
10. Rayon J, O'Neil, Haffman K,*Exact Methods for solving Travelling Salesman Problems with pickup and delivery in Real Time* ,Sept 2,2018