

# Modeling of Phasor Measurement Unit (PMU) in Matlab

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## Abstract

With the arrival of PMUs, controlling and monitoring of energy transmission in real-time has become possible. Time-stamped measurements of voltage and current phasors are provided by the PMUs, in microseconds. Fast Fourier transform (FFT), Discrete Fourier transform (DFT) are phasor estimation techniques which can be used by PMUs to estimate phasors. In this paper, MATLAB/Simulink based models of PMU which uses DFT, have been developed to estimate phasors. There is different algorithm use for updating the phasor also discussed in this paper.

**Keywords:** Synchro-phasors, PMU, Phasor Estimation, DFT.

## I. INTRODUCTION

A very popular device in the field of a smart grid named phasor measurement unit (PMU) [1] is a combination of some devices which can use for synchronized measurement of the phasor, magnitude, and frequency, and the data can be transmitted to the higher order of hierarchy for monitoring and control. It has many advantages over the old SCADA [1].

For designing the PMU block in MATLAB, we are interested in computing the magnitude and phasor of the signal with the help of discrete Fourier transform.

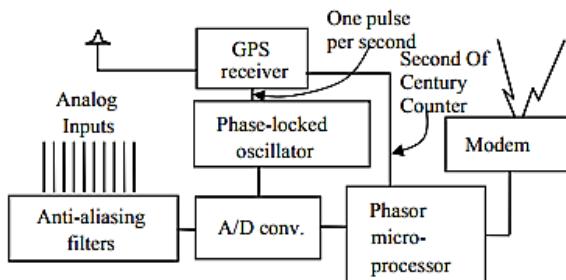
With the advancement of computing technology, it has become quite easier to achieve a quicker time response of PMUs. As earlier the computation will be performed, more control over the power system can be made. We must ensure the sampling clock is phase-locked with the GPS clock pulse for obtaining the real time-stamped data. Sampling rate increasing with the evolution of digital technology. With the crucial advancement in microprocessor and analog

to digital converters, nowadays the rate of sample per cycle in PMUs are 96 or 128 samples per cycle. In the future, it is quite possible to have PMUs of higher sampling speed which will help in more accurate estimation. One pulse per second signal is provided by GPS satellites to keep accurate clocks.

We must select the substation for installation of PMUs while the selection mainly depends upon measurement provided by the PMUs [1].

## II. MAJOR ELEMENTS IN PMU

Although there are different configuration and arrangement of the elements in the PMU depends on a different manufacturer. But here we see some principal elements require to obtain the essence of the PMU. In Figure (1) there is the configuration of the PMU built at Virginia Tech. It is the first PMU ever made.



**Fig.1 Major elements of the modern PMU.**

#### A. Anti-aliasing filters

The sampling rate is provided by the anti-aliasing filter which uses by A/D converter during the sampling process. Mostly analog-type filters are used with a cut-off frequency of less than half of the sampling frequency so it can satisfy the Nyquist criterion [1]. In most of the designs of the relay, it is possible to have a higher sampling rate which is called oversampling. After that, we lower the sampling rate by using a digital ‘decimation filter’ [1]. Then It provides a ‘digital anti-aliasing filter’ concatenated with the analog anti-aliasing filters.

#### B. Phase-Locked Oscillator

The phase-locked oscillator used to obtain the state of phase-locked between the sampling clock and the GPS clock pulse. After computation of the phasor, it must be stamped by the time so we can make the data suitable for further analysis and observation. The PMUs are provided with one pulse per second from satellite to phase-locking of output data of A/D converter.

#### C. Microprocessor

Microprocessors are used to calculate positive-sequence estimates for all the voltage and current signals by using different algorithms. It plays a vital role in the functioning of the PMUs and works as a computation device. There are more other estimates of interest available which measured locally such as frequency and rate of change of frequency, and these are also included in the output of the PMU.

#### D. Output Device

By using suitable modems timestamped output of PMUs transferred over the communication links. Before transmission, we should take care of output file structures’ specification to satisfying the need of the industry.

### III. PHASOR COMPUTATION TECHNIQUES

For evaluation of the phasor of nominal frequency signals let us take a constant frequency signal as input with the nominal frequency of the power system  $f$ .

$$x(t) = X_m \cos(2\pi ft + \phi) \quad (1)$$

#### a. Sampled Data and aliasing

The beginning of digital signal processing is a sampling of the analog input signal. So, for the evaluation of phasors of input signals begins with a sampling of the signal. The sampling is done at uniform intervals  $k\Delta T$ , ( $k = 0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$ ). The input signal  $x(t)$  which is yield sampled data  $x(k\Delta T)$  after sampling. For making the computation easier we should take the signal in the form of impulse. As we can write as

$$x'(t) = \sum_{k=-\infty}^{\infty} x(k\Delta T) \delta(t - k\Delta T) \quad (2)$$

Equation (2) shows the uniformly spaced impulse signals each with magnitude  $x(k\Delta T)$ . Now, we could determine the DFT of the sampled signal.

#### b. DFT calculation

Phasor can be estimated by using DFT [1] of a window of the sampled input signal. So, we need to obtain a sampled window data and then the DFT of the window will be obtained by using the formula given below

$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi m n}{N}} \quad (3)$$

In Equation (3) “m” represents the  $m^{\text{th}}$  frequency bin of the DFT and  $x(n)$  represents the  $n^{\text{th}}$  sample of  $x(t)$ . Here we are only interested in the phasor and magnitude calculation so we will only consider the fundamental frequency (i.e.  $m=1$ ) of the whole bin in DFT.

### c. Phasor Estimation

For fundamental frequency, we will put  $m=1$  in Eq. (3) and then computes the DFT by using the values of first  $N$  samples. We can use some algorithm for updating the phasor for further samples. However, the result will be obtained in the complex form (i.e.  $a+ib$ ) by using which we can determine phasor and magnitude as follows

$$\text{magnitude} = \sqrt{a^2 + b^2}$$

$$\text{phasor angle} = \arctan(b/a)$$

The magnitude and the angle represent the RMS value and the phase shift of the input signal respectively.

### A. Algorithms of phasor estimation

By using DFT, there are two algorithms available. We will take one by one from them.

#### a. Non-recursive updates

The phasor can be estimated by using the DFT of the sampled data. When we take the first  $N$  sample then the

phasor  $X^{N-1}$  is given by

$$X^{N-1} = \frac{X_m}{\sqrt{2}} e^{-j\phi} \quad (5)$$

In Equation (5), the superscript  $(N-1)$  is used to denote that the phasor as having the  $(N-1)^{\text{st}}$  sample as the last sample used in the phasor estimation. For mathematical computation with the sampling angle , $X^{N-1}$  can be computed with equation (6) and (7)

$$X^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x(n) e^{-jn\theta} \quad (6)$$

$$X^N = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x(n+1) e^{-jn\theta} \quad (7)$$

With the advancement of samples in every iteration the calculated phasor will show the magnitude of the input sample signal which uses fresh samples for each estimate and does not take the data from earlier estimates, so, this algorithm is known as a “non-recursive algorithm” for phasor estimation.

#### b. Recursive updates

Equation (6) and (7) are the formula for calculating the  $(N-1)^{\text{st}}$  and  $(N)^{\text{th}}$  phasor by non-recursive algorithm respectively. In the given summation multipliers are different for the same sample and the samples  $x(n)$ :  $\{n = 1, 2, 3, \dots, N-1\}$  are common in both windows. There is no  $x(0)$  in the second window so that it starts with  $x(1)$ , and it ends with  $x(N)$ , and  $x(N)$  did not exist in the first window. For save the computation time we need to keep the multipliers same for common sample in the two windows. For doing the same, we multiply both sides of the Equation (7) by  $e^{-j\theta}$ , we obtain the following result:

$$\hat{X}^{N+r} = e^{-jr\theta} X^{N+r-1} \frac{\sqrt{2}}{N} \{x(N+r) - x(r)\} e^{-jr\theta}$$

Phasor estimate will be the same for both the window ie old and new window when the constant sinusoid input signal is provided. In general, the recursive algorithm is numerically unstable. Nevertheless,because of the great computational efficiency of a recursive algorithm, it is the algorithm of choice in most applications. So, the final formula for recursive phasor update is illustrated in the equation (8)

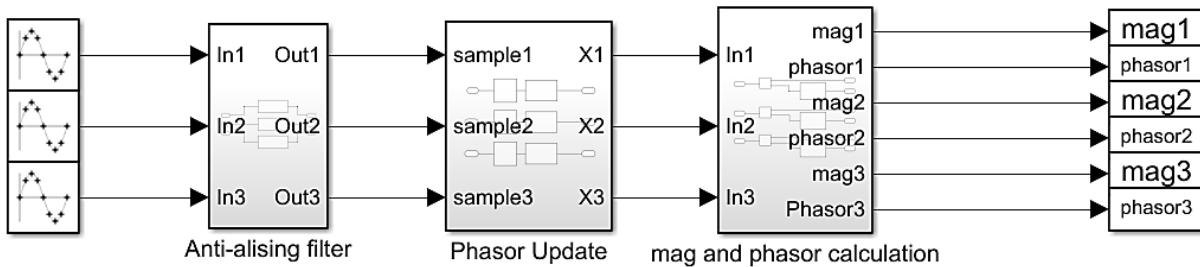
$$\hat{X}^{N+r} = \hat{X}^{N+r-1} \frac{\sqrt{2}}{N} \{x(N+r) - x(r)\} e^{-jr\theta} \quad (8)$$

#### IV. PMU MODEL IN MATLAB/SIMULINK

##### a. Input Signal(sine wave)

Here, we have taken a sine wave as an input signal which may have been obtained from the current or potential transformer [2]. All the input signals are symmetrical in nature (i.e. having  $120^\circ$  phase shift with each other). The input signals are in the form of the sample with the desired sampling frequency.

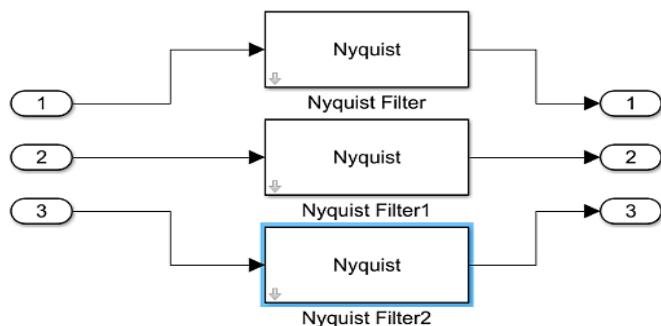
The sampling frequency must satisfy Nyquist criteria for more accurate estimation of phasors. In this model, we have taken the symmetrical phase line signal which has no harmonic component. Also, the signal has a constant frequency in steady-state. However, The change in frequency is not going to affect the value of the magnitude and phasor difference. Instead of that, the method will vary if the variable frequency signal is provided as input.



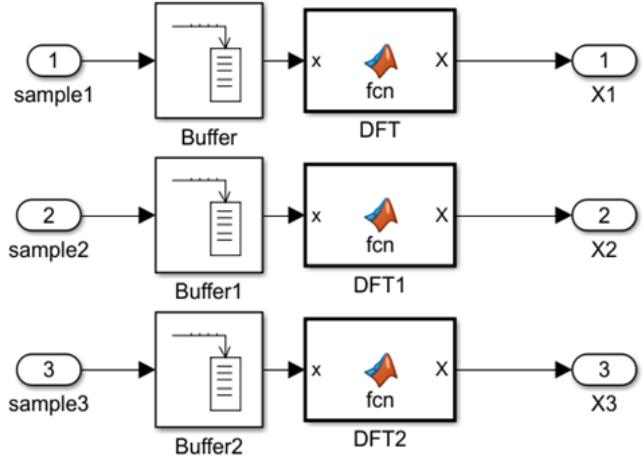
**Fig. 1. MATLAB/SIMULINK model of PMU**

##### b. Anti-aliasing Filter

The Anti-aliasing filter is used to satisfy the sampling theorem. Here, we used a Nyquist filter block which checks the minimum sampling frequency criteria according to Nyquist.



**Fig. 2.Nyquist filter as an anti-aliasing filter**



**Fig.3. Phasor Estimate using MATLAB function block**

##### c. Phasor Update

For phasor estimation, we used the DFT of the sampled signal. we have been obtained the estimated phasor by using both non-recursive and recursive methods in this block.

The resulting phasor will be complex in nature. Buffer is used for moving the window of sampled data. The DFT block is embedded MATLAB function which use one of the methods described earlier for updating the phasor.

#### d. The magnitude and phasor calculation

The updated phasor obtained using DFT is always a complex number. In this block, we use a buffer for controlling the input of sample in such a way that it would only provide a sample of a period at first and after that, the input window starts moving by taking just next sample and dropping the last of the previous window. After getting the sample DFT [1] block computes the phasor either by the recursive or non-recursive method. In this block, we obtained the magnitude of the complex which is RMS value of the sampled set of the signal. The phase angle of the complex denotes the phase shift of the given sampled set of the signal. In the calculation of the angle, we would have obtained the angle in radian. The angle is then converted into the degree by using the MATLAB function. At the output port, we would have been found a magnitude actually RMS

value of the signal and angle as a phase shift of the signal in degree.

#### e. Result

The results have been obtained from Complex to Magnitude-Angle block. we can evaluate the magnitude and phasor of the given sample which is time-stamped. Thus, the signal can be used for real-time control and of the parameters of the transmission line. The results had stored in the MATLAB workspace for further analysis.

### V. CASE STUDIES

Let us consider a single-phase 230V signal represented by expression  $x(t)=230 \cos(100\pi t + \pi/4)$  at frequency 50Hz which is sampled at a sampling frequency of 600 Hz i.e. 12 samples per cycle. The phasor output with their magnitude and phase for recursive and non-recursive estimates has been given in Table.1. And, for the second and third phase transmission line, you can see the values in Table .2 and Table .3 respectively. However, In Non-recursive phasor estimation, the phase angle advanced by  $30^\circ$  which is desired as the sample angle rotation is  $2\pi/N$ .

**TABLE I: OUTPUT OF PHASOR CALCULATED USING NON-RECURSIVE AND RECURSIVE ALGORITHM IN THE MATLAB FOR THE FIRST PHASE**

Sample No.	Sample $x(n)$	Non-recursive estimate of magnitude	Non-recursive estimate of phasor(degree)	Recursive estimate of magnitude	Recursive estimate of phasor(degree)
0.	162.6346	-	-	-	-
1.	59.5284	-	-	-	-
2.	-59.5284	-	-	-	-
3.	-162.6346	-	-	-	-
4.	-222.1629	-	-	-	-
5.	-222.1629	-	-	-	-

6.	-162.6346	-	-	-	-
7.	-59.5284	-	-	-	-
8.	59.5284	-	-	-	-
9.	162.6346	-	-	-	-
10.	222.1629	-	-	-	-
11.	222.1629	-	-	-	-
12.	162.6346	162.6346	45.0000	162.6346	45.0000
13.	59.5284	162.6346	75.0000	162.6346	45.0000
14.	-59.5284	162.6346	105.0000	162.6346	45.0000
15.	-162.6346	162.6346	135.0000	162.6346	45.0000
16.	-222.1629	162.6346	165.0000	162.6346	45.0000
17.	-222.1629	162.6346	-165.0000	162.6346	45.0000
18.	-162.6346	162.6346	45.0000	162.6346	45.0000
19.	-59.5284	162.6346	75.0000	162.6346	45.0000

**TABLE II: OUTPUT OF PHASOR CALCULATED USING NON-RECURSIVE AND RECURSIVE ALGORITHM IN THE MATLAB FOR THE SECOND PHASE**

Sample No.	Sample x(n)	Non-recursive estimate of magnitude	Non-recursive estimate of phasor(degree)	Recursive estimate of magnitude	Recursive estimate of phasor(degree)
0.	-222.1629	-	-	-	-
1.	-222.1629	-	-	-	-
2.	-162.6346	-	-	-	-
3.	-59.5284	-	-	-	-
4.	59.5284	-	-	-	-
5.	162.6346	-	-	-	-
6.	222.1629	-	-	-	-
7.	222.1629	-	-	-	-

8.	162.6346	-	-	-	-
9.	59.5284	-	-	-	-
10.	-59.5284	-	-	-	-
11.	-162.6346	162.6346	165.0000	162.6346	165.0000
12.	-222.1629	162.6346	-165.0000	162.6346	165.0000
13.	-222.1629	162.6346	-135.0000	162.6346	165.0000
14.	-162.6346	162.6346	-105.0000	162.6346	165.0000
15.	-59.5284	162.6346	-75.0000	162.6346	165.0000
16.	59.5284	162.6346	-45.0000	162.6346	165.0000
17.	162.6346	162.6346	-15.0000	162.6346	165.0000
18.	222.1629	162.6346	15.0000	162.6346	165.0000

**TABLE III: OUTPUT OF PHASOR CALCULATED USING NON-RECURSIVE AND RECURSIVE ALGORITHM IN THE MATLAB FOR THE THIRD PHASE**

Sample No.	Sample x(n)	Non-recursive estimate of magnitude	Non-recursive estimate of phasor(degree)	Recursive estimate of magnitude	Recursive estimate of phasor(degree)
0.	59.5284	-	-	-	-
1.	162.6346	-	-	-	-
2.	222.1629	-	-	-	-
3.	222.1629	-	-	-	-
4.	162.6346	-	-	-	-
5.	59.5284	-	-	-	-
6.	-59.5284	-	-	-	-
7.	-162.6346	-	-	-	-
8.	-222.1629	-	-	-	-
9.	-222.1629	-	-	-	-
10.	-162.6346	-	-	-	-

11.	-59.5284	162.6346	-75.0000	162.6346	-75.0000
12.	59.5284	162.6346	-45.0000	162.6346	-75.0000
13.	162.6346	162.6346	-15.0000	162.6346	-75.0000
14.	222.1629	162.6346	15.0000	162.6346	-75.0000
15.	222.1629	162.6346	45.0000	162.6346	-75.0000
16.	162.6346	162.6346	75.0000	162.6346	-75.0000
17.	59.5284	162.6346	105.0000	162.6346	-75.0000
18.	-59.5284	162.6346	135.0000	162.6346	-75.0000

## VI. CONCLUSION

In this paper, we built a DFT based model of PMU in MATLAB/Simulink by using two algorithms for phasor update. We have also observed a case study using the PMU models. Out of the two methods, recursive estimation is less complex and more efficient due to which it is more popular. This model provides a real-time evaluation of the phasor which will help engineers to control the power system in real-time. The time response of the PMUs now days is around 10 microseconds. So, with such a fast evaluation and analysis, we can avoid the grid failure or blackout condition. The aim of this paper is to make a software-based model of PMUs which can work under nominal frequency operation. In the present work, we use the Discrete Fourier transform of sampled signal for phase and magnitude. The sampled signal was obtained from the analog signal which may be obtained from Current transformer (CT) or voltage transformer (PT). After that, we obtained the time-stamped signal from the PMU with the help of GPS [1] satellite. GPS satellite will provide one pulse per second for the time stamp. Once the time-stamped signal has been obtained, we transmit it to the Phasor Data Centre (PDC) [1] for analysis, observation, and control of the power.

## REFERENCES

- [1] A. G. Phadke and J. S. Thorp, "Synchronized phasor measurements and their applications," New York, Springer, 2008.
- [2] J. D. L. Ree, V. Centeno, J. S. Thorpand, and A. G. Phadke, "Synchronized phasor measurement applications in power systems," IEEE Transactions on Smart Grid, vol. 1, no. 1, June 2010.
- [3] V. Krishna, R. S. Ashok, and M. G. Krishnan, "Synchronized phasor measurement unit," IEEE International Conference on Power, Signals, Controls and Computation, Kerala, pp. 1-6, January 2014.
- [4] S. V. Hareesh, P. Raja, and M. P. Selvan, "An effective implementation of phasor measurement unit (PMU) by using non-recursive DFT algorithm," in Proc. International Conference on Condition Assessment Techniques in Electrical Systems (CATCON), Bangalore, India, pp. 195-199, December 2015.
- [5] G. Sanchez-Ayala, J. R. Aguero, D. Elizondo, and M. (Dino) Lelic, "Current trends on applications of PMUs in distribution systems," IEEE Innovative Smart Grid Technologies, Washington, DC, pp. 1-6, February 2013.
- [6] D. Ghosh, T. Ghose, and D. K. Mohanta, "Communication feasibility analysis for smart grid with Phasor measurement units," IEEE Transactions on Industrial Informatics, vol. 9, no. 3, pp. 1486-1496, August 2013.
- [7] Y. Yang and S. Roy, "PMU placement for optimal three-phase state estimation performance," in Proc.

IEEE International Conference on Smart Grid Communications, Vancouver, Canada, pp. 342-347, October 2013

- [8] Phadke, A.G. and Thorp, J.S., "Computer relaying for power systems", Research Studies Press Ltd., John Wiley & Sons, Inc., 1994, pp 127-129.