

Sharpe's Single Index Model: An Approach of Building Optimal Portfolio

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Abstract:

Holding a perfect investment portfolio is a cherished dream of all investors. A rational investor is constantly in search of optimal portfolio and popular measure is the risk-return trade-off. The study has used Sharpe's Single Index Model (SIM) to construct an optimal portfolio. Eight popular stocks which are regular constituents of BSE Sensex Index were picked for constructing a portfolio. Sharpe's optimal portfolio based on SIM selected only two of the securities, rejecting the other 6 securities. Daily returns of Individual securities are compared with a single index, which is BSE Sensex daily return over a period of 5 years. The paper is aimed at educating the current and prospective investors to construct an optimal portfolio based on Sharpe's SIM. The paper also intends to sensitize the investors about the advantage of SIM for constructing an optimal portfolio over the complex Markowitz's Optimal Portfolio Model.

Keywords:-Beta, Cut-off rate, Excess Return to Beta ratio, Optimal Portfolio, Sharpe's SIM, Single Index Model, Systematic Risk, Unsystematic Risk

I. INTRODUCTION

Creating and maintaining a portfolio is a natural instinct of human being. In earlier times people would invest in land, animals, crops, gold and jewelry in varying proportions. In modern times with evolution of paper currencies, financial institution's securities and derivatives, the investment options have only amplified. All investment options have varying degree of risk and return combinations. Further with growth of modern science and improved health care the average life expectancy across the world is increasing. As on an average a man is going to live longer and cost of living is constantly increasing, particularly in developing economies, people are in constant search of dependable investment avenues, to park their hard earned money to maximize the return with least risk. As in the earlier times and so in the current, people are creating portfolios to mitigate the risks. The portfolios may be a result of series of haphazard and unrelated decisions, or it may be a result of a thoughtful planning and analysis.

Growth of financial institution and varying range of investable assets, provides number of options but it has also compounded the complexity of correct decision making. A rational investor either an individual or institutional investor, construct portfolio to diversify the risk. Advancements in the

investment spectrum over the past half century has established that investor cannot construct optimum investment portfolio by cherry picking and combining the individual securities, meeting investors' desirable risk and return characteristics. Harry Markowitz's Modern Portfolio theory has its focus on factoring the relationship among the investment to build an optimum portfolio (Sharpe, 1963).

II. OBJECTIVE OF THE STUDY

The focus of the study carried out is to meet the understated objectives:

- To make learn the investment enthusiasts, current and prospective investors the process of construction of Optimal Portfolio as per Sharpe's Single Index Model
- To exemplify how optimum portfolio helps in risk diversification and optimizes the return

III. LITERATURE REVIEW

Pratibha Jenifer Andrade (2012):- used Sharpe's SIM to construct optimal portfolio of securities from information technology sector of India and IT stocks. 5-year data was used and stock of 5 companies were finally figured in optimal portfolio. In the optimal portfolio as per Sharpe's Single Index Model proposed the optimal weight of 39.45% to TCS, 33.13% to HCL, 18.04% to Infosys and marginal weight to remaining two stocks.

Niranjan Mandal (2013): - constructed optimal portfolio of 21 securities from India capital market, using Sharpe's SIM. Author observed that portfolio return is greater than the average return of the single stocks in optimal portfolio barring few exceptions. It was further found that the portfolio *beta* value was less than of market. Indicating market portfolio returns would fluctuate at lower rate in comparison to the fluctuation in market index return. Further it was observed that the total risk of portfolio was less than the risk of consistent securities barring few.

Apurva Chauhan (2014): constructed optimal portfolio using top 10 stocks of NIFTY. Fewer input variable requirement was the reason for the choice of the model. They took BES Sensex as proxy to market portfolio.

M. Muthu Gopalakrishnan (2014): studied 13 activity tracked IT stocks at NSE IT Index, and concluded that there exists a relationship between IT Index and S&P NIFTY. Author found that most of IT Stocks were undervalued and investors should consider IT stocks in their portfolio. The paper compared the traditional portfolio theory with modern portfolio theory. The paper aims at testing whether SIM (single index model) offers proper explanation of stock returns on IT stocks. The time series data from secondary sources for the period 2004-2008 has been used. Regressing excess return on market return, it is found that IT Index has a significant sensitivity over S&P CNX Nifty. Study revealed that out of 13 IT stocks, 4 stocks were having beta coefficient of more than one.

J. Francis Mary and G. Rathika (2015) : stated that the role played by the risk-return is crucial in any decision related to investment. Decision here refers to answer to the question such as

- Whether investment should be done or not?
- Which securities to be included in portfolio?

They stated in their study that using the single index (beta) model so proposed by Sharpe efficient portfolio can be determined with an asset class (e.g. – stocks)

Syed Mohammad Faisal & Omar Abdul Al Aboud (2017): Use stocks of 10 security listed in MASDAO to construct the optimal portfolio. They used Sharpe's SIM and their optimal portfolio selected 5 securities in final selection they paper reflected upon the risk diversification by choosing Sharpe's SIM.

Tanuj Nandan & Nivedita Srivastava (2017): constructed optimal portfolio using Sharpe's SIM. They preferred Sharpe's SIM over Harry Markowitz's optimal portfolio model as requires fewer input and is easier to compute. They consider NIFTY 50 stocks as stocks represent majority of market capitalization of NSE. Out of NIFTY 50 stocks only 24 stocks fulfilled the selection criterion and were selected in optimal portfolio. Majority of the stock which found place in the optimal portfolio were from the Banking sector. This can be credited to the speedy growth in the financial sector.

IV. MARKOWITZ'S MPT AND SHARPE'S SIM

In the Harry Markowitz's Modern Portfolio Theory (MPT), the covariance among the securities' returns takes the center stage in the construction of optimal portfolio. Markowitz provided the set of analytical tools for construction of optimal portfolio based on quantification of risk and returns. The MPT is vastly popular among the investment managers as it helps portfolio optimization facilitating higher return at constant risk or maintaining similar return at reduced risk. Still, despite of immense popularity of MPT the

accomplishment of select portfolio, hinge on the quality of estimates of securities returns and its covariance. Further the input list requires $[n(n-1)/2]$ estimates and this presents a formidable task before the investment manager for portfolio construction. Sharpe's Single Index Model attempts optimal portfolio creation based on single factor. The SIM rest on the premise that the positive covariances among stock returns ascend from common economic forces that affect the riches of most firms. The rate of return on broad market index of securities, is considered as a valid proxy for the common macroeconomic factor. The detailed methodology of the Sharpe's Single Index Model is discussed over the next section of the paper.

V. CONSTRUCTION OF SHARPE'S SINGLE INDEX MODEL: METHODOLOGY

Fundamental Analysis is popular mechanism of Stock picking for constructing investment portfolio.

Bottom up approach is followed for:

- selecting eight securities listed in Bombay Stock Exchange (BSE); and
- constituent of BSE Sensex which is free float market weighted stock market index, consisting of 30 securities.

Top down approach of fundamental analysis of stock picking, involves:

- Macro analysis of economy, sector or industry; and
- Finally the hand full of narrowed down companies are considered for portfolio.

The paper took contrarian view of following bottom up approach where stocks of companies which are constituent of BSE Sensex index are handpicked to form an initial portfolio. This approach is quite objective, as constituent companies in BSE Sensex index are well established and financially sound companies whose stocks are actively traded. The selected 8 companies are leaders and representative of various industrial sectors. The historical daily returns of the 8 stocks over the period of 5 years (July 2013 to August 2018) are taken along with that of the BSE Sensex for the matching period. Thereafter excess daily return have been worked out. For this 7.5% has been considered as annual risk free rate.

Selected securities daily excess returns are regressed against BSE Sensex daily excess return for the period under the study. Regression results spitted out beta (β) values, which captures the sensitivity of selected stock return due to macroeconomics market forces, i.e. systematic risk.

The ratio of excess return to beta (β) value is worked out for all securities in the initial portfolio. There after all securities are ranked on the basis of their excess return to beta ratio in descending order. The table 1 illustrate the ranking of the securities on the basis of the excess return to beta value.

Table 1: Stock Ranking - based on excess return to beta ratio

Security	Mean Return (%)	Ri- Rf	Beta (β)	(Ri- Rf)/ Beta	Rank
MARUTISUZ	40.16	32.66	0.99	33.14	1
HDFCBANK	24.52	17.02	0.93	18.23	2
RIL	13.61	6.11	1.09	5.63	3
TCS	9.21	1.71	0.47	3.66	4
IOC	5.46	-2.04	0.83	-2.45	5
ITC	0.45	-7.05	0.86	-8.23	6
ONGC	-5.67	-13.17	1.14	-11.57	7
INFOSYS	-4.14	-11.64	0.6	-19.45	8

The ratio of excess return to beta measures the premium i.e. additional return on a security per unit of systematic risk.

In the table 1, stock of Maruti-Suzuki shows highest excess return to beta ratio of 33.14 and therefore it has been ranked 1st, it is followed by HDFC Bank with excess return to beta value of 18.23. Therefore the HDFC Bank has been accorded second rank. This ratio is least for Infosys and according it gets the least rank of 8th position.

$$\text{Beta } (\beta) = \frac{\text{Covariance of Security Return \& Market Index Return}}{\text{Variance of Market Return}}$$

This is followed by computation of unsystematic risk of each of security in the initial portfolio. Unsystematic risk

represents firm specific surprises and not depends on the fickleness of market.

$$\sigma_{ei}^2 = \sigma_i^2 - \beta_i^2 \sigma_m^2$$

Where σ_i^2 = Variance of security's return
 σ_{ei}^2 = Variance of security's return
 σ_m^2 = Expected variance of market index
 β_i = Beta value of individual security

Since the daily return data has been used. The annualized unsystematic risk (%) value are worked out. The excess return of each security in initial portfolio is divided by the corresponding measure of unsystematic risk and multiplied by the corresponding beta (β) value.

Table 2: Unsystematic risk and other computation

Security	Unsystematic Risk	Unsystematic Risk Annualised	Unsystematic Risk Annualised Percent	Computation	Cumulative values
	$\sigma^2(ei)$	$\sigma^2(ei)$	$\sigma^2(ei) (\%)$	$[(Ri - Rf) / \sigma^2(ei)] \times \beta$	$[(Ri - Rf) / \sigma^2(ei)] \times \beta$
MARUTISUZ	0.0002	0.0411	4.1075	7.8334	7.8334
HDFCBANK	0.0001	0.0188	1.8772	8.4657	16.2991
RIL	0.0003	0.0867	8.6728	0.7658	17.0649
TCS	0.0004	0.0981	9.8126	0.0819	17.1468
IOC	0.0007	0.1860	18.6012	-0.0907	17.0560
ITC	0.0003	0.0659	6.5927	-0.9154	16.1406
ONGC	0.0003	0.0820	8.2012	-1.8282	14.3124
INFOSYS	0.0006	0.1510	15.0960	-0.4610	13.8514

Last column of table 2 provides the cumulative values where all securities are ranked in descending order of Sharpe's reward-to-risk ratio. It is observed that IOC has highest value of unsystematic risk whereas HDFC bank has least value of unsystematic risk. In the next step the 'C_i' value are computed.

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^N \left[\frac{R_i - R_f}{\sigma_{ei}^2} \right] \beta_i}{1 + \sigma_m^2 \sum_{i=1}^N \frac{\beta_i^2}{\sigma_{ei}^2}}$$

C_i = Cut off rate

R_i = Return on stock i

R_f = Risk free return

R_i - R_f = Excess return

σ_{ei}^2 = Variance of security's return

σ_m^2 = Expected variance of market index

β_i = Beta value of individual security

For this ($\frac{\beta_i^2}{\sigma_{ei}^2}$) value are worked out for individual securities

in the initial portfolio. Computed value of 'C_i' are tabulated in the table T-3. Cumulative 'C_i' values are plotted in T-3, column 5, which are gradually increasing for 10.25 to 13.27 but starts declining thereafter. Therefore, the value 13.27 is the cutoff point. Securities up to cut off point will be considered in the Sharpe's optimal portfolio and securities ranked after cut off point are not considered for the optimal

portfolio construction. From Table 3, only Maruti Suzuki and HDFC bank are the two stocks which are making it to optimal portfolio based on Sharpe's SIM.

Table 3: Computation of cut - off points for securities in the Initial Portfolio

Rank	Security	β^2 / σ^2 (ei)	Cumulative of β^2 / σ^2 (ei)	Ci
1	MARUTISUZ	0.24	0.24	10.26
2	HDFCBANK	0.46	0.70	13.27
3	RIL	0.14	0.84	12.51
4	TCS	0.02	0.86	12.37
5	IOC	0.04	0.90	11.98
6	ITC	0.11	1.01	10.52
7	ONGC	0.16	1.17	8.45
8	INFOSYS	0.02	1.19	8.07

Finally, the proportion of investment in Maruti Suzuki stock & HDFC Bank stock are worked out using the computations

Table 4, column 9, states the proportion of investment that is recommended by Sharpe's SIM.

Table 4: Proportions of securities in Optimal Portfolio								
Rank	Names	β^2 / σ^2 (ei)	Cumulative of β^2 / σ^2 (ei)	C _i	β / σ^2 (ei)	$(R_i - R_f) / \beta_i$	Z _i	X _i
1	MARUTISUZUKI	0.24	0.24	10.26	0.24	33.14	4.7	0.6
2	HDFCBANK	0.46	0.7	13.27	0.5	18.23	2.4	0.3
3	RIL	0.14	0.84	12.51				
4	TCS	0.02	0.86	12.37				
5	IOC	0.04	0.9	11.98				
6	ITC	0.11	1.01	10.52				
7	ONGC	0.16	1.17	8.45				
8	INFOSYS	0.02	1.19	8.07				
							ΣZ_i	7.23

Accordingly, 65.89% of investment is recommended in the stock of Maruti Suzuki, where as 34.01% of investment shall be made in stock of HDFC Bank. Table 5, illustrates the Sharpe's optimal portfolio's risk and return statistics.

Table 4: Risk and Return Statistics of Sharpe's SIM based Optimal Portfolio

Composition of Sharpe's Optimal Portfolio				
Security	Proportion	Stand Alone Return in %	Return on Portfolio	Portfolio Risk
Maruti Suzuki	0.66	40.16	26.46	
HDFC Bank	0.34	24.52	8.36	
Total Portfolio Return	1		34.82	19.55

given below. Here, X_i and Z_i are to be determined to know how much funds need to be invested in each constituent security of the optimal portfolio.

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2} \left[\frac{R_i - R_f}{\beta_i} - C^* \right]$$

$$X_i = \frac{Z_i}{\sum_{i=1}^N Z_i}$$

X_i = Proportion of each selected stock in the portfolio

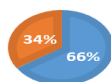
C^* = Cut off point

So finally, the securities of two companies are selected in Sharpe's optimal portfolio.

Fig- 1: Proportion of Investment in constituent securities in Sharpe's Optimal Portfolio

**PROPORTION OF INVESTMENT
PROPOSED IN SHARPE'S OPTIMAL
PORTFOLIO**

■ Maruti Suzuki ■ HDFC Bank



Optimal Portfolio Return is coming out to be 34.82% with the portfolio risk of 19.55%.

I. ASSUMPTIONS

- The period under the study is uniform and not witnessed by any unusual market volatility.
- The firm specific residual values of each security in the initial portfolio has zero expected values.
- The firm specific residual values are not correlated with the return on market portfolio.
- The firm specific residual values are not correlated with each other.

II. LIMITATION OF THE STUDY

For the construction of an optimal portfolio, eight securities were selected from the securities listed in BSE Sensex. For selecting these securities the top-down approach was followed which is not as robust as the bottom-up approach. The stated limitations will be considered while taking up further research in this area.

III. FINDINGS

Out of eight securities in the initial portfolio only two securities (Maruti Suzuki and HDFC Bank) are selected in the Optimum Portfolio. The computed value of the proportion of investments in these two securities are 65.90% (in Maruti Suzuki Stock) and 34.10% (in HDFC Bank Stock). The optimal portfolio is expected to give return of 34.82% with the portfolio risk of 19.55%. Investor can develop the optimal portfolio following the step explained in the paper. Nevertheless, the final investment decision shall be made only after carefully studying all the macroeconomic factors along with the factors intrinsic to the industry as well as specific to the companies whose stocks are considered to be part of portfolio.

IV. CONCLUSION

The study establishes that portfolio selection based on Sharpe's model is optimal. Prospective investors can design and construct their investment portfolio based on this model which will optimize the return alongside risk reduction.

REFERENCES

- Andrade, P. J. (2012). Construction of Optimal Portfolio of Equity, using Sharpe's Single Index Model :

A Case Study of IT Sector. *International Journal of Applied Financial Management Perspectives*, 86-88.

- Andrade, P. J. (2012). Construction of Optimal Portfolio of Equity, using Sharpe's Single Index Model : A Case Study of IT Sector. *International Journal of Applied Financial Management Perspectives*, 86-88.
- Apurva, & Chauhan, A. (2014). A Study on Usage of Sharpe's Single Index Model In Portfolio Construction With Reference To Cnx Nifty. *Global Journal of Research Analysis*, 3(10), 92-94.
- Faisal, S. M., & Aboud, O. A. (2017). Sharpe Index Model and its Utility in Portfolio Optimisation and Allocation of Funds in Stock. *International Journal of Economics , Commerce and Management*, 176-182.
- Gopalakrishna, M. M. (2014). Optimal Portfolio Selection Using Sharpe's Single Index Model. *Indian Journal of Applied Research*, 4(1), 286-288.
- Mandal, N. (2013, March). SHARPE'S SINGLE INDEX MODEL AND ITS APPLICATION TO CONSTRUCT OPTIMAL PORTFOLIO: AN EMPIRICAL STUDY. *Great Lakes Herald*, 7(1), 1-20.
- Mary, F. J., & Rathika, G. (2015, January). THE SINGLE INDEX MODEL AND THE CONSTRUCTION OF OPTIMAL PORTFOLIO WITH CNXPHARMA SCRIP. *International Journal of Management (IJM)*, 6(1), 87-96.
- Nandan, T., & Srivastava, N. (2017). Construction of Optimal Portfolio Using Sharpe's Single Index Model: An Empirical Study on Nifty 50 Stocks. *Journal of Management Research and Analysis*, 4(2), 74-83.
- Sharpe, W. F. (1963). A Simplified Model for Portfolio Analysis . *Management science*, 277-293.

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