

Research on Mathematical Model of Image Enhancement in Complex Environment Based on Wavelet Analysis

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Abstract

Based on wavelet analysis, the technology in the research of image enhancement mathematical model in complex environment has effectively solved the problem of low signal-to-noise ratio of the image through fuzzy enhancement operators. Other solutions developed for intra-pulse networks (such as modems) cannot effectively solve extreme problems. The successful development of the mathematical model of image enhancement in complex environments based on wavelet analysis will lead to a large number of derivation of images of complex environments in all directions. This mathematical model of image enhancement creates conditions for complex environments.

Keywords: Complex Environment Image Processing, Image Enhancement, Wavelet Transform, Enhancement Operator, Experimental Comparison;

1. Introduction

Complex environment images have the characteristics of high noise and low contrast. Therefore, the image needs to be enhanced to highlight the information of interest and obtain images that are more in line with the human visual effect^[1-2]. Traditional image enhancement methods for complex environments are divided into spatial domain and frequency domain^[3-4]. The spatial domain performs arithmetic processing on image pixels and spatial smoothing, and the frequency domain performs image spectrum transformation operations to achieve low-pass filtering and high-pass in frequency domain. In practical applications. Geological exploration, seismic data acquisition, etc. need to be used, but combined with the actual situation, the image will not be perfect and can not achieve the desired effect, so the efficiency of seismic interpretation is also reduced^[5-6]. Therefore, in this case, the image enhancement is necessary in complex environments. This paper uses wavelet analysis to establish a

mathematical model of complex image enhancement, uses wavelet to enhance and weaken the low frequency and high frequency, and then reconstruct the complex environment image to improve the signal-to-noise ratio.

2. Basic principles of wavelet image processing

The image enhancement technology can effectively improve the visual effect of the image, and strengthen the image interpretation and recognition effect. The correlation algorithm of the histogram is the basis of the image enhancement algorithm. Such as local histogram equalization, adaptive histogram equalization, local statistical special noise removal methods. This kind of method is intuitive and reversible, and the amount of calculation is small. Insufficiency There is no ability to select the processed data. After the image changes, the local detail image disappears. Wavelet analysis can describe local features well in both the time domain and the frequency domain. By refining the time domain step in the high-frequency domain, any part

of the image can be described. But in some cases, due to the discontinuities in the signal, the noise will appear pseudo Gibbs phenomenon. The fuzzy theory can solve the uncertainty of image signals through fuzzy sets, and solve the problem of edge detection that affects noise interference. However, the inverse function of the existing blur enhancement algorithm has a partially unsolvable situation, so that part of the gray information of the image is lost, which affects the image enhancement effect.

Let $f(x_1, x_2) \in L_2(\mathbb{R}^2)$ represent a two-dimensional signal, and x_1, x_2 are its abscissa and ordinate respectively. $\varphi(x_1, x_2)$ represents the two-dimensional basic wavelet, φ_a ; $b_1, b_2(x_1, x_2)$ represents the scale expansion and two-dimensional displacement of $\varphi(x_1, x_2)$, namely:

$$\varphi_{a,b_1,b_2}(x_1, x_2) = \frac{1}{a} \varphi\left(\frac{x_1 - b_1}{a}, \frac{x_2 - b_2}{a}\right) \quad (1)$$

The factor $\frac{1}{a}$ is a normalization factor introduced to ensure that its energy remains unchanged before and after wavelet expansion. Then the two-dimensional continuous wavelet transform is defined as:

$$WT_f(a; b_1, b_2) = \langle f(x_1, x_2), \varphi_{a,b_1,b_2}(x_1, x_2) \rangle = \frac{1}{a} \iint f(x_1, x_2) \varphi\left(\frac{x_1 - b_1}{a}, \frac{x_2 - b_2}{a}\right) dx_1 dx_2 \quad (2)$$

In the case of separability, the two-dimensional multi-resolution wavelet analysis is usually carried out in two steps: first, use two different resolution functions $\phi(x_1)$ and $\varphi(x_1)$ (corresponding to low frequency and high frequency respectively) along the x_1 direction. For analysis, $f(x, x)$ is decomposed into two parts of smooth approximation and detail along the direction of x_1 , and then the two parts are analyzed along the direction of x_2 with $\phi(x_2)$ and $\varphi(x_2)$ respectively. Among the four outputs obtained in this way, the output obtained by processing $\phi(x_1)\phi(x_2)$ is the first-stage low-frequency smoothing approximation of $f(x_1, x_2)$ to $A1f(x_1, x_2)$, and the remaining three outputs are $H1f(x_1, x_2)$, $V1f(x_1, x_2)$ and $D1f(x_1, x_2)$, which correspond to

the high-frequency detail functions in the horizontal, vertical, and diagonal directions, respectively. Similarly, the above analysis can be repeated for the smooth approximation $A1f(x_1, x_2)$ to obtain the wavelet coefficients after n -layer decomposition. The decomposition process is shown in Figure 1.

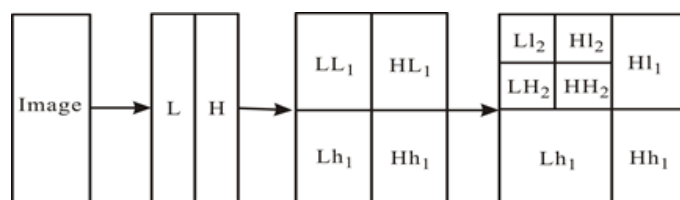


Figure 1. Schematic diagram of two-layer wavelet decomposition.

From the above analysis, it can be seen that wavelet analysis can be regarded as a mathematical tool for multi-level decomposition functions. The image signal (data) can be described by the coefficients of wavelet decomposition after wavelet transformation. The wavelet coefficients reflect the original image information (data) and properties, and the local characteristics of the image information (data) can be changed by processing the wavelet coefficients.

3. Algorithm implementation

The core of wavelet image processing algorithm is the processing algorithm for wavelet coefficients. The enhancement and attenuation of wavelet coefficients can usually be processed by linear or non-linear algorithms. The linear method selects a fixed parameter k as the adjustment ratio of wavelet coefficients. The algorithm is simple, but it is difficult to select the threshold segmentation point, and the local image processing effect is different. It can not adapt to the gray level changes of the image well. Moreover, according to the visual physiology and psychological principles of the human eye, objects with the same brightness give people different visual perception in different background areas, that is, the absolute brightness difference between the background and the target needs to be larger in the high-brightness area. The value of can

produce the same visual effect as the brightness difference of the low-brightness area. Based on this, this paper uses an enhancement operator to change the wavelet coefficients after image decomposition to highlight the characteristic information of the image. The principle of the algorithm is as follows:

The wavelet coefficients obtained by the decomposition of the image by the multi-scale wavelet transform correspond to the components on different scales (subband coefficients). The enhancement operator is used to increase the difference between the subband coefficients, which can not only effectively improve the difference between the components on each scale. Contrast, and the amplitude of enhancement can be controlled according to the value of the coefficient. At the same time, this enhancement process can also be used to obtain scale parameters to ensure the improvement of the overall contrast of the target image.

The specific algorithm is: first use the obtained wavelet coefficients of each scale to obtain the fuzzy membership function relative to the threshold T_n :

$$\mu_{ij} = T(a_{ij}) = (a_{ij} - T_n) / (a_{n\max} - T_n) \quad (3)$$

Secondly, the fuzzy membership function is adjusted nonlinearly, and the nonlinear function used is:

$$\mu'_{ij} = I(\mu_{ij}) = \begin{cases} \sqrt{1 - (1 + \mu_{ij})^2}, & -1 \leq \mu_{ij} \leq 0 \\ \mu_{ij}^2, & 0 < \mu_{ij} < r \\ \sqrt{1 - 2(1 - \mu_{ij})^2}, & r \leq \mu_{ij} \leq 1 \end{cases} \quad (4)$$

Then the inverse transformation of equation (3) is implemented on μ'_{ij} , and the nonlinearly adjusted wavelet coefficients of each scale can be obtained:

$$a'_{ij} = T^{-1}(\mu'_{ij}) \quad (5)$$

In the formula, a_{ij} and a'_{ij} are the wavelet coefficients at each scale before and after adjustment respectively; $a_{n\max}$ is the largest wavelet coefficient value on the n th scale before adjustment, which is determined by the result of wavelet decomposition.

The value of T_n ranges from 0 to 255, and its value can be determined by the histogram of the image. r is the coefficient factor, and its range is between 0 and 1. It is the piecewise gain function node in the membership function. It is used to control the strength of image enhancement, which can suppress small coefficients representing noise and background, and at the same time enhance the larger Coefficient, which can more effectively suppress noise and enhance target content information.

3.1. Hard and soft threshold functions and their disadvantages

The threshold function is related to the continuity and accuracy of the reconstructed signal, and has a great influence on image enhancement based on wavelet transform. At present, the choice of threshold mainly includes two methods: hard threshold and soft threshold. The expressions of the two threshold functions are as follows:

The hard threshold function is:

$$\bar{W}_{j,k} = \begin{cases} W_{j,k}, & |W_{j,k}| > \lambda \\ 0, & |W_{j,k}| \leq \lambda \end{cases} \quad (6)$$

In formula (1), $W_{j,k}$ is the wavelet coefficient after positive transformation, and $\bar{W}_{j,k}$ is the wavelet coefficient after estimation.

Disadvantages: The hard threshold function is discontinuous in the entire wavelet domain, and there are discontinuities between λ and $-\lambda$. In practical applications, we often need to derivate the threshold function, and the existence of discontinuities and the derivation are contradictory, which makes this method have certain limitations; at the same time, it only processes wavelet coefficients that are less than the threshold, and that is greater than the threshold. The wavelet coefficients are not processed, which is inconsistent with the noise interference in the wavelet coefficients larger than the threshold in the actual situation.

The soft threshold function is:

$$\bar{W}_{j,k} = \begin{cases} \text{sgn}(W_{j,k})(|W_{j,k}| - \lambda), & |W_{j,k}| > \lambda \\ 0, & |W_{j,k}| \leq \lambda \end{cases} \quad (7)$$

In formula (2), $\text{sgn}(n)$ is a symbolic function, namely:

$$\text{sgn}(n) = \begin{cases} 1, & n > 0 \\ -1, & n < 0 \end{cases} \quad (8)$$

Disadvantages: The increasing and decreasing trend of wavelet coefficients do not match.

3.2. Improved threshold and threshold function

Aiming at the characteristic of wavelet decomposition, this paper re-gives a threshold, namely $\lambda_j = \sigma \frac{\sqrt{2 \ln N}}{\ln(j+1)}$, where $(j \geq j+1)$ is the decomposition scale. Derivation of λ_j can get

$$\lambda'_j = -\sigma \frac{\sqrt{2 \ln N}}{(j+1)} \ln^2(j+1) < 0, \text{ that is, } \lambda_j \text{ decreases}$$

with the increase of the decomposition scale j , which is consistent with the feature that the wavelet coefficient decreases with the increase of the decomposition scale.

Since only $|W_{j,k}|$ is compared with λ_j to estimate the wavelet transform coefficient of a certain point of the image, those noise elements with wavelet transform coefficients greater than λ_j are also retained. In order to improve it, you can make

$$\delta_{j,k} = \sum_{m=-2M}^{2M} W_{j,k-m} \quad (9)$$

3.3. Enhance the wavelet coefficients

Image enhancement based on wavelet transform is based on the difference between the wavelet coefficients of the signal and noise, and the correspondingly processed wavelet coefficients. The enhancement algorithm needs to reduce noise and detail, so as to improve the visual effect of the image. The problem research should be in the high frequency area denoising and low frequency area histogram processing. Since the image is

decomposed by wavelet, the noise and details of the image are mostly in the high-frequency part, and the visual perception of the entire image is determined by the low-frequency information of the image. The high-frequency and low-frequency information decomposed after the text wavelet transform, the histogram equalizes the low-frequency information to achieve edge enhancement, the threshold method is used to denoise the high-frequency information, the wavelet high-frequency coefficients are mapped to the fuzzy feature surface, and the fuzzy enhancement operation is used to The high frequency coefficient is not strengthened.

After the above threshold and threshold function processing, the saved wavelet coefficients contain most of the information of the image, but if the processed wavelet coefficients are directly subjected to wavelet inverse transformation at this time, the image obtained is only the denoised image. It has not been enhanced. Based on the actual needs of seismic exploration, this paper performs wavelet inverse transformation after enhancing the wavelet coefficients, so that the wavelet coefficients are effectively amplified (that is, enhanced), which is helpful for seismic interpretation. After repeated comparisons of many enhancement methods, this paper proposes to use a single threshold enhancement method to enhance the image. The algorithm mainly uses the following formula:

$$W_{out} = \begin{cases} W_{in} + \lambda(G-1), & W_{in} > \lambda \\ W_{in} - \lambda(G-1), & W_{in} < -\lambda \\ GW_{in}, & W_{in} < \lambda \end{cases} \quad (10)$$

Experiments have shown that the algorithm has a significant enhancement effect and has a good enhancement to the image. effect.

4. Experimental results and analysis

The histogram equalization method can produce an image with uniform gray-scale distribution. After the histogram equalization, the image becomes blurred, but the local characteristics are blurred. How to

improve the histogram equalization method on the basis to achieve the local characteristic enhancement effect of the relevant area is research. In order to verify the effect of the algorithm, a set of 320×240 pixel infrared non-destructive testing image sequences are used, and the common regularization method, polynomial fitting method, correlation analysis method and the wavelet image enhancement algorithm proposed in this paper are used to analyze the original image. The target content is the defect part to be enhanced. The coefficient factor $r=0.4$ is set in this algorithm, and the corresponding experimental results are given.

The 100mm×50mm×2mm rectangular thin plate specimen to be tested is made of clad2024-T3, an aluminum alloy material for civil aviation aircraft, and a flat-bottomed blind hole is processed on the back of the specimen to simulate corrosion defects. The distribution and size of the blind hole are shown

in Figure 2. , The specific parameters of the corrosion zone are shown in Table 1.



Figure 2. The size and distribution of the simulated corrosion zone of the tested piece.

Table 1. Parameters of the corrosion zone.

Corrosion zone number	Diameter/m	Depth/mm	Material damage/%
1	16	1.2	60
2	16	0.8	40
3	Outer circle 20	0.4	26
	Inner circle 16	0.6	
4	10	1.2	60
5	10	0.8	40
6	10	0.4	20

Figure 3 shows the clearest frame (the fifth frame) of the acquired original complex environment image, which clearly reflects the characteristics of high background and low contrast unique to complex environment images, and the difference between the target (defect part) and the background The contrast is very poor, and the content information of the target image (including edge and internal texture information) is not obvious, which is manifested as the target edge is blurred, the internal texture gray difference is small, and the corrosion area with small area and shallow depth on the specimen is not observed on the original image Or not clear.

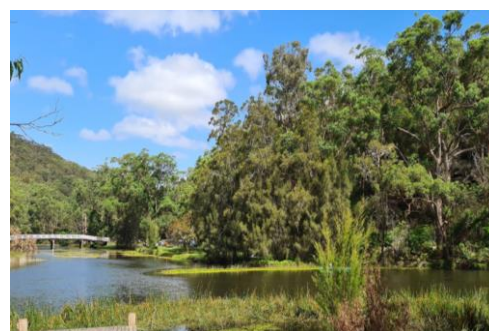


Figure 3. Original complex environment image.

Figure 4 shows the enhancement results of Figure 3 using regularization, polynomial fitting and correlation analysis methods. Their enhancement effects are different: the regularization method improves the signal-to-noise ratio compared with the original image, but the subjective visual effect is not

improved. Obviously; the processing result of the second-order polynomial fitting method is poor, the signal-to-noise ratio is reduced, and some abnormal image grayscale changes appear in the corroded area, which is easy to cause misjudgment visually; under the appropriate conditions for the reference point and image frame number selection, The correlation coefficient graph in the correlation analysis can improve the signal-to-noise ratio, enhance the contrast of the corrosion area, and show the slighter corrosion in the specimen. However, the correlation analysis method requires manual selection of reference points, which is not convenient for automatic processing, and the selection of the reference point also has a greater impact on the processing effect.



Figure 4. Image processing results of several commonly used algorithms.

Figure 5 shows the enhanced processing results of the algorithm in this paper. It can be seen that the outline of the defect area of the enhanced image is clearer, the contrast is enhanced, and it is easy to recognize; compared with the previous methods, the overall outline and background of the enhanced image target The inter-contrast has been significantly improved, achieving the expected enhancement effect.



Figure 5. Image processing results of the algorithm

in this paper.

5. Conclusion

On the basis of traditional image enhancement, this paper uses wavelet high and low frequency transform to study image enhancement, and improves the traditional method. The improved algorithm can solve the image enhancement in complex environment. Through experiments, using wavelet transform to suppress high-frequency noise and blurred background, it can effectively enhance, and is more conducive to reality.

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