TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 3224 - 3227

Priority Retrial Queues with *J* Working Vacations

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Article Info
Volume 82
Page Number: 3224 - 3227
Publication Issue:
January-February 2020

Article History
Article Received: *14 March 2019*
Revised: *27 May 2019*
Accepted: *16 October 2019*
Publication: *19 January 2020*

Abstract

A single server priority retrial queuing system with finite number of working vacations is described. By using the supplementary variable method technique (SVT), the steady state probability generating function of the orbit size and system size is obtained. Some analytic expressions like steady state probabilities, mean length of orbit and system are found.

Keywords: *retrial queue; modified working vacation; vacation interruption;*

I. Introduction

In queueingliterature, vacation queues and retrial queues are associated with various type of customers discussed by many authors (Artalejo and Corral [1]).The concept of priority customers and working vacations are discussed in detail by many authors. Recently, Wu and Lian[7], Gao[4], Chandrasekaran et al. [3]and Rajadurai [5]. This work has been extended from the work of Gao [4] by incorporating the concepts of finite number of working vacations and vacation interruption. This model finds the practical application of computer processing systems in particular CPU scheduling. A Process Scheduler schedules different processes to be assigned to the CPU based on particular scheduling algorithms. Priority scheduling is a method of scheduling processes based on priority.

II. Description of the model

Apriority retrial queueing system with finite number of (J)working vacations is considered. In this work, we extend the work of Sundararaman et al. [5] and Gao [4] with concept of priority models and WVs.

Where, the inter-arrival times has an arbitrary distribution $R(t)$ with Laplace Stieltijes Transform (LST) $R^*(\vartheta)$, the service time of ordinary/priority customers follows a general function $S_b(t)$ and $S_p(t)$ with LST $S_b^*(\vartheta)$ and $S_p^*(\vartheta)$ and the moments are $\beta_b^{(1)}$ and $\beta_b^{(2)}$, in working vacation period, the service time follows a general distribution function $S_v(t)$ with LST $A_4^*(\vartheta)$ and its moment $A_4^{*'}(\theta) = \int_0^\infty x e^{-\theta x} dA_4(x)$.

III. Steady state probabilities

The steady state equations and solutions are developed in this section.

3.1.The steady state equations

Here, assume that $A_i(0) = 0$, $A_i(\infty) = 1$are continuous at $x = 0$ and $y = 0$. Then the functions $a_i(x)$ (i = 1,2,3,4) the hazard rates for repeated trials, priority, ordinary and low rate service respectively.

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i.e., $a_i(x)dx = \frac{dA_i(x)}{1-A_i(x)}$;

The a bivariate Markov process $\{C(t), N(t),\ t \ge 0\}$ and the Markov chain $\{Z_n;\ n \in N\}$ is Ergodic, and then satisfies the condition if and only if $\rho < A_1^*(\lambda+\delta)$ for system to be stable, where

$$\rho = \lambda\left\{\delta\beta_p^{(1)}\overline{A_1^*}(\lambda+\delta) + \left(A_1^*(\lambda+\delta) + \lambda\overline{A_1^*}(\lambda+\delta)\right)\beta_b^{(1)}(1+\delta\beta_p^{(1)})\right\}$$

3.1 Steady state equations:

"Using SVT to get,

$$(\lambda+\delta)P_0 = \theta Q_{J,0} \tag{3.1}$$

$$(\lambda+\delta+\theta)Q_{1,0} = \int_0^\infty \Pi_{3,0}(x)a_3(x)dx + \int_0^\infty \Pi_{2,0}(x)a_2(x)dx + \int_0^\infty Q_{1,0}(x)a_4(x)dx \tag{3.2}$$

$$(\lambda+\delta+\theta)Q_{i,0} = \theta Q_{i-1,0} + \int_0^\infty Q_{i,0}(x)a_4(x)dx,\ (i=1,2,...,J) \tag{3.3}$$

$$\frac{d\Pi_{1,n}(x)}{dx} + (\lambda+\delta+a_1(x))\Pi_{1,n}(x) = 0,\ n \ge 1 \tag{3.4}$$

$$\frac{d\Pi_{2,n}(x)}{dx} + (\lambda+a_2(x))\Pi_{2,n}(x) = \lambda\Pi_{2,n-1}(x),\ n \ge 1 \tag{3.5}$$

$$\frac{\partial\Pi_{3,n}(x,y)}{\partial x} + (\lambda+a_2(x))\Pi_{3,n}(x,y) = \lambda\Pi_{3,n-1}(x,y),\ n \ge 1 \tag{3.6}$$

$$\frac{d\Pi_{4,n}(y)}{dy} + (\lambda+\delta+a_3(y))\Pi_{4,n}(y) = \lambda\Pi_{4,n-1}(y) + \int_0^\infty \Pi_{3,n}(x,y)a_2(x)dx,\ n \ge 1 \tag{3.7}$$

$$\frac{dQ_{i,n}(x)}{dx} + (\lambda+\theta+a_4(x))Q_{i,n}(x) = \lambda Q_{i,n-1}(x),\ \ n \ge 1 \tag{3.8}$$

The steady state boundary conditions are at $x = 0$ and $y = 0$ are

$$\Pi_{1,n}(0) = \int_0^\infty \Pi_{2,n}(x)a_2(x)dx + \int_0^\infty \Pi_{3,n}(y)a_3(y)dy + \sum_{i=1}^{J}\int_0^\infty Q_{i,n}(x)a_4(x)dx,\ n \ge 1. \tag{3.9}$$

$$\Pi_{2,n}(0) = \delta\int_0^\infty \Pi_{1,n}(x)dx,\ n \ge 1. \tag{3.10}$$

$$\Pi_{3,n}(0,y) = \delta\Pi_{2,n}(y),\ n \ge 0. \tag{3.11}$$

$$Q_{i,0}(0) = (\lambda+\delta)\sum_{i=1}^{J} Q_{i,0},(i=1,2,...,J) \tag{3.12}$$

$$\Pi_{b,n}(0) = \left(\int_0^\infty \Pi_{1,n+1}(x)a_1(x)dx + \theta\sum_{i=1}^{J}\int_0^\infty Q_{i,n}(x)dx + \lambda\int_0^\infty \Pi_{1,n}(x)dx\right),\ n \ge 0. \tag{3.13}$$

The normalizing condition is

$$P_0 + \sum_{i=1}^{J} Q_{i,0} + \sum_{n=1}^{\infty}\int_0^\infty \Pi_{1,n}(x)dx + \sum_{n=0}^{\infty}\left(\int_0^\infty \Pi_{2,n}(x)dx + \int_0^\infty \Pi_{4,n}(y)dy\right) + \sum_{n=0}^{\infty}\left(\sum_{i=1}^{J}\int_0^\infty Q_{i,n}(x)dx + \int_0^\infty\int_0^\infty \Pi_{3,n}(x,y)dxdy\right) = 1. \tag{3.14}$$

3.2.The steady state solutions of the model

To solve the above equations, we define the generating functions for $|z| \le 1$, as follows:

$$\Pi_i(x,z) = \sum_{n=0}^{\infty}\Pi_{i,n}(x)z^n;\ Q_i(x,z) = \sum_{n=0}^{\infty} Q_{i,n}(x)z^n;$$

Multiplying the equations (3.1) - (3.14) by z^n and summing over n, (n = 0,1,2...) and solving the differential equations we getthe limiting probabilities $\Pi_i(x,z)$ and $Q_j(x,z)$.

Results: If the system in stability condition, then

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(i) The PGF of orbit size when server is idle,

$$\Pi_1(z)=\int_0^\infty \Pi_1(x,z)dx=\frac{\overline{A_1^*}(\lambda+\delta)\left(zP_0\left(\lambda A_3^*\left(A_3(z)\right)+\delta S_2^*(\lambda(1-z))-(\lambda+\delta)\right)+\sum_{i=1}^{J} z(\lambda+\delta)Q_{i,0}\left[V(z)A_3^*\left(A_3(z)\right)+A_4^*\left(A_4(z)\right)-1\right]\right)}{z-\left(\delta z\overline{A_1^*}(\lambda+\delta)(A_2^*(A_{2p}(z))\right)-\left(A_1^*(\lambda+\delta)+\lambda z\overline{A_1^*}(\lambda+\delta)\right)A_3^*\left(A_3(z)\right)} \tag{3.15}$$

(ii) The PGF of orbit size when server is busy on priority customers,

$$\Pi_2(z)=\int_0^\infty \Pi_2(x,z)dx=\frac{(1-A_3^*\left(A_3(z)\right))\times\delta\left(P_0\left(z-A_1^*(\lambda+\delta)A_3^*\left(A_3(z)\right)\right)+(\lambda+\delta)z\sum_{i=1}^{J}Q_{i,0}\left[V(z)A_3^*\left(A_3(z)\right)+A_4^*\left(A_4(z)\right)-1\right]\right)}{A_3(z)\times\left\{z-\left(\delta z\overline{A_1^*}(\lambda+\delta)(A_2^*\left(A_2(z)\right))\right)-\left(A_1^*(\lambda+\delta)+\lambda z\overline{A_1^*}(\lambda+\delta)\right)A_3^*\left(A_3(z)\right)\right\}} \tag{3.16}$$

(iii) The PGF of orbit size when server is busy on preemitivepriority customers,

$$\Pi_3(z)=\int_0^\infty \Pi_3(x,z)dx=\frac{\left(1-A_2^*\left(A_2(z)\right)\right)\left(1-A_3^*\left(A_3(z)\right)\right)\times\delta\left(P_0\left(z-A_1^*(\lambda+\delta)A_3^*\left(A_3(z)\right)\right)+(\lambda+\delta)z\sum_{i=1}^{J}Q_{i,0}\left[V(z)A_3^*\left(A_3(z)\right)+A_4^*\left(A_4(z)\right)-1\right]\right)}{A_3(z)\times\left\{z-\left(\delta z\overline{A_1^*}(\lambda+\delta)(A_2^*\left(A_2(z)\right))\right)-\left(A_1^*(\lambda+\delta)+\lambda z\overline{A_1^*}(\lambda+\delta)\right)A_3^*\left(A_3(z)\right)\right\}} \tag{3.17}$$

(iv) The PGF of orbit size when server is busy on ordinary priority customers,

$$\Pi_4(z)=\int_0^\infty \Pi_4(x,z)dx=\frac{(1-A_3^*(A_3(z)))\times\left(\begin{array}{l}(\lambda+\delta)\sum_{i=1}^{J}Q_{i,0}V(z)\left\{A_3^*(A_3(z))\left(z-\left(A_1^*(\lambda+\delta)+\lambda z\overline{A_1^*}(\lambda+\delta)\right)\right)-\delta A_2^*(A_2(z))\overline{A_1^*}(\lambda+\delta)-\left(A_4^*(A_4(z))-1\right)\right\}\\ +P_0A_1^*(\lambda+\delta)\left(1-\delta A_2^*(A_2(z))-\left(A_4^*(A_4(z)))-1\right)\right)\end{array}\right)}{\left\{z-\left(\delta z\overline{A_1^*}(\lambda+\delta)A_2^*(A_2(z)\right)-\left(A_1^*(\lambda+\delta)+\lambda z\overline{A_1^*}(\lambda+\delta)\right)S_3^*\left(A_3(z)\right)\right\}} \tag{3.18}$$

(v) The PGF of orbit size when server ison working vacation

$$Q_i(z)=\left\{\frac{(\lambda+\delta)V(z)}{\theta}Q_{i,0}\right\},\quad for\ (i=1,2,\dots,J) \tag{3.19}$$

(vi) The probability that server is idle,

By using this relation, $P_0+\sum_{i=1}^{J}Q_{i,0}+\sum_{i=1}^{4}\Pi_i(1)+\sum_{i=1}^{J}Q_i(1)=1.$

$$A(P)_0+B\left(\sum_{i=1}^{J}Q_{i,0}\right)=A_1^*(\lambda+\delta)-\rho\quad \rho=\lambda\left\{\delta\beta_p^{(1)}\overline{A_1^*}(\lambda+\delta)+\left(R^*(\lambda+\delta)+\lambda\overline{A_1^*}(\lambda+\delta)\right)\beta_b^{(1)}(1+\delta\beta_p^{(1)})\right\}$$

$$\bar{A}_2^*(A_2(z))=\frac{1-A_2^*(A_2(z))}{A_2(z)};\ \bar{A}_4^*(A_4(z))=\frac{1-A_4^*(A_4(z))}{A_4(z)};\ \bar{A}_3^*(A_3(z))=\frac{1-A_3^*(A_3(z))}{(A_3(z))};\quad A_2(z)=\lambda(1-z);\ A_4(z)=\lambda(1-z)+\theta;$$

$$A_3(z)=A_2(z)+\delta(1-A_2^*(A_2(z)))$$

$$A=\left(\begin{array}{l}\left[\delta\beta_p^{(1)}+\lambda\beta_b^{(1)}(1+\delta\beta_p^{(1)})\right]\overline{A_1^*}(\lambda+\delta)\\ +\left[1+A_1^*(\lambda+\delta)\lambda\beta_b^{(1)}(1+\delta\beta_p^{(1)})\right]\overline{A_2^*}(\lambda+\delta)\\ +A_1^*(\lambda+\delta)(\delta\beta_p^{(1)}-1)\beta_b^{(1)}+A_1^*(\lambda+\delta)(\delta\beta_p^{(1)}-1)\beta_b^{(1)}\beta_p^{(1)}\end{array}\right)\quad B=(\lambda+\delta)\left\{\begin{array}{l}\beta_b^{(1)}\beta_p^{(1)}+\left(\delta+\beta_b^{(1)}\right)\left[\left(A_1^*(\lambda+\delta)+\lambda\bar{A}_1^*(\lambda+\delta)\right)\lambda A_4^{*'}(\theta)\right.\\ +\left(1-A_4^*(\theta)\right)\left[\begin{array}{l}\left\{\begin{array}{l}\left[\frac{\lambda}{\theta}+\lambda A_4^{*'}(\theta)\right]\\ (1-\delta\bar{A}_1^*(\lambda+\delta))-\lambda\delta\beta_p^{(1)}\bar{A}_1^*(\lambda+\delta)\end{array}\right\}\\ -\lambda\bar{A}_1^*(\lambda+\delta)\end{array}\right]\left.\vphantom{\int}\right]\end{array}\right\}$$

(vii) The PGF of the number of customers in the system

$$K_s(z)=P_0+\sum_{i=1}^{J}Q_{i,0}+\Pi_1(z)+\sum_{i=2}^{4}z\Pi_i(1)+\sum_{i=1}^{J}Q_i(1)=1. \tag{3.20}$$

(viii) The PGF of the number of customers in the orbit

$$K_o(z)=P_0+\sum_{i=1}^{J}Q_{i,0}+\Pi_1(z)+\sum_{i=2}^{4}\Pi_i(1)+\sum_{i=1}^{J}Q_i(1)=1. \tag{3.21}$$

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ISSN: 0193 - 4120 Page No. 3224 - 3227

IV. Performance measures

From eqns. (3.15) – (3.21), put $z \to 1$, we get the probabilities for idle, busy and on working vacations.

$$P = P(1);\ \Pi_1 = \Pi_1(1);\ \Pi_b = \Pi_b(1);\ \Pi_2 = \Pi_2(1);\ \Pi_{vi} = \sum_{i=1}^{J} \Pi_{vi}(1).$$

The mean orbit size (L_q) is $L_q = K_o'(1) = \lim_{z\to 1} \frac{d}{dz} K_o(z)$

The mean system size (L_s) is $L_s = K_s'(1) = \lim_{z\to 1} \frac{d}{dz} K_s(z)$

The mean waiting time in system (W_s) and orbit (W_q) are" $L_s = \lambda W_s$ and $L_q = \lambda W_q$.

V. Special cases

***Case* (i):**Let δ=0 and $J = \infty$. This model reduced to the results of ofGao [4].

***Case* (ii):** Let $(\delta, \theta) \to (0, 0)$ and $J = \infty$. This model reduced to the results of Arivudainambiet al. [2].

VI. Numerical Examples

We presented some numerical examples using MATLAB software for some system performances. We considered the arbitrary values to the parameters such that the steadystate condition is satisfied. Figure 1show that the queue length (L_q) increases, if the value of arrival rates δ and λ is increasing. In figure 2, the server's idle (P_0) increases for increasing the value of μ_v and a.

VII. Conclusion

In this work, an M/G/1 retrial queueing system with priority customer under J working vacation is considered. The PGFs of the number of customers in the system are found by using the SVT. The average queue length of orbit and system also obtained. System performances are validated with the help of numerical examples. The novelty of this work is both single WV (J=1) and multiple WV (J=∞) in presence of priority retrial queueing system. Practical application of this model is in a priority scheduling algorithm and Wired Network.

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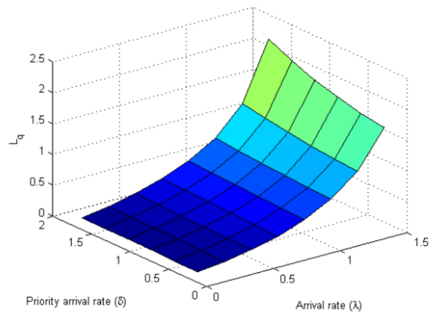

Figure 1.L_q versus λ and δ

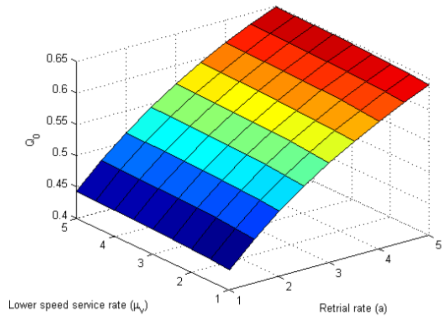

Figure 2.Q_0 versus μ_v and a