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Detour Domination Number of More Graphs

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Abstract:
Let G = (V, E) be any graph. For a subset $S \subseteq V(G)$ we define a detour dominating set of G if S is a detour and dominating set of G. In this paper we find the detour domination number of some special graphs $C_n \odot C_m$, Middle graphs and Inflated graphs. Also we characterize graphs with particular values for detour domination numbers.

Keywords:*Detour, Detour domination, Detour domination number.*

I. INTRODUCTION

The graphs we consider here are finite graphs with no loops and multiple edges. In a connected graph G, for any two vertices u and v the detour distance $D(u,v)$ is the length of the longest $u-v$ path in G. A $u-v$ path of length $D(u,v)$ is called a $u-v$ detour. A vertex x is said to lie on a $u-v$ detour P if x is a vertex of a $u-v$ detour path P including the verticles u and v. A set $S \subseteq V$ is called a detour set if every vertex v in G lies on a detour joining a pair of vertices of S. The detour number $dn(G)$ is called a minimum order of a detour set and any detour set of order $dn(G)$ is called a minimum detour set of G In this paper , we investigate the detour domination number of Middle graphs and Inflated graphs.

Definition 1.1: A set $S \subseteq V(G)$is called a dominating set of G if every vertex in $V(G) - S$ is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum order of its dominating sets and any dominating set of order $\gamma(G)$ is called a γ-set of G. A detour domianating set is a subset S of $V(G)$ which is both a dominating and a detour set of G.

Definition 1.2: A detour dominating set S is said to be minimum detour dominating set of G if there exists no detour dominating set S' such that $|S'| < |S|$. The smallest cardinality of a detour dominating set of G is called the detour domination number of G. It is denoted by $\gamma_d(G)$. Any detour dominating set S of G of cardinality $\gamma_d(G)$ is called a (γ, d)-set of G.

Definition 1.3: The Middle graph $M(G)$ of a graph is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 1.4: Let G be a graph with $\delta(G) \geq 1$, a graph denoted by G_1 is obtained as follows: To each $u \in V(G)$, a clique A_u of order $deg_G u$ is obtained and a bijection $\Phi_u : N(u) \longrightarrow A_u$ is constructed. $\Phi_u(v)$is denoted by v' for all $v \in N(u)$, $V(G_1) = \cup_{u \in V(G)} A_u$ and
$E(G_1) = \in \cup_{u \in V(G)} E(A_u) \cup \{u'v' : uv\, E(G)\}$. $v' \in A_u$, $u' \in A_v$. The graph G_1 is known as the Inflated graph of G.

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Theorem 1.5: For the path $G = P_p$ $(p \geq 2)$,

$$\gamma_d(G) = \begin{cases} \left\lceil \frac{p-4}{3} \right\rceil + 2 & if\ p \geq 5 \\ 2 & if\ p = 2,3, or\ 4 \end{cases}.$$

Theorem 1.6: For the cycle $G = C_p$ $(p \geq 6)$, $\gamma_d(G) = \left\lceil \frac{p}{3} \right\rceil$.

II. Detour Domination Number of Middle Graphs

Theorem 2.1: For $n \geq 2, \gamma_d(M(P_n)) = \left\lceil \frac{n}{2} \right\rceil + 1$.

Proof:

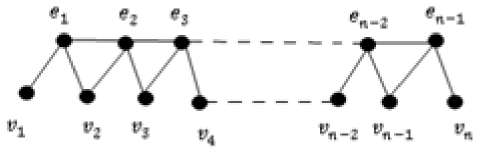

Figure 2.1

Let $n = 2$, $M(P_2) = P_3$, $\gamma_d(M(P_2)) = \gamma_d(P_3) = 2 = \left\lceil \frac{n}{2} \right\rceil + 1$.

Hence the result is true for $n = 2$.

Let $n \geq 3$. $V(M(P_n)) = \{v_1, v_2, \dots, v_{n-2}, v_{n-1}, v_n, e_1, e_2, \dots, e_{n-1}\}$where $v_1, v_2, \dots, v_{n-2}, v_{n-1}, v_n$ and $e_1, e_2, \dots, e_{n-1}$ are the vertices and edges of P_n respectively.

We prove the result in 2 cases.

Case 1: n is odd.

Here $n - 1$ is even.

Being the end vertices, $S_1 = \{v_1, v_n\}$ is contained in every detour dominating set of $M(P_n)$. Further, they dominate only the vertices v_1, v_n, e_1, e_{n-1}.

Further, $S_2 = \{e_2, e_4, e_6, \dots, e_{n-1}\}$ together with S_1 forms a γ_d- set .

Hence, $\gamma_d(M(P_n)) = |S_1| + |S_2| = 2 + \frac{n-1}{2}$

$$= \left\lceil \frac{n}{2} \right\rceil + 1.$$

Case 2: n is even.

Proceeding as before, $\{v_1, v_n, e_2, e_4, , \dots, e_{n-2}\}$ is a γ_d- set of $M(P_n)$.

$$\gamma_d(M(P_n)) = 2 + \frac{n-2}{2} = \frac{n}{2} + 2 - 1 = \left\lceil \frac{n}{2} \right\rceil + 1.$$

Illustration 2.2: $\gamma_d(M(P_{17})) = 10 = \left\lceil \frac{17}{2} \right\rceil + 1$.

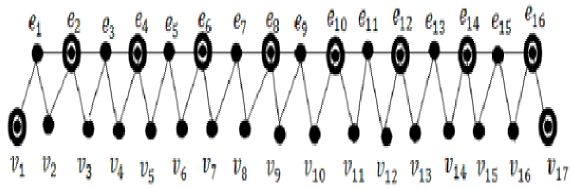

Figure 2.2

Here, $S = \{v_1, v_{17}, e_2, e_4, e_6, e_8, e_{10}, e_{12}, e_{14}, e_{16}\}$ is a γ_d- set of $M(P_{17})$.

Hence, $\gamma_d(M(P_{17})) = |S| = 10 = \left\lceil \frac{17}{2} \right\rceil + 1$.

Illustration 2.3:

$$\gamma_d(M(P_{12})) = 7 = \left\lceil \frac{12}{2} \right\rceil + 1.$$

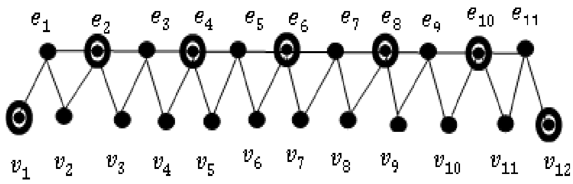

Figure 2.3

Here, $S = \{v_1, e_2, e_4, e_6, e_8, e_{10}, v_{12}\}$ is a γ_d- set of (P_{12}) .

Hence, $\gamma_d(M(P_{12})) = |S| = 7 = \left\lceil \frac{12}{2} \right\rceil + 1$.

Theorem 2.4: For $n \geq 3, \gamma_d(M(C_n)) = \left\lceil \frac{n}{2} \right\rceil$.

Proof:

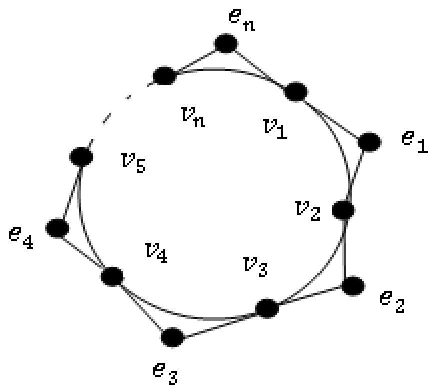

Figure 2.4

Let $n \geq 3$,
$V(M(C_n)) = \{v_1, v_2, \dots, v_{n-2}, v_{n-1}, v_n, e_1, e_2, \dots, e_n\}$ where v_i's and e_i's represent the vertices and edges of C_n respectively.

Obviously, $\{v_1, v_3, v_5, \dots, v_n\}$ and $\{v_1, v_3, v_5, \dots, v_{n-1}\}$ form the minimum detour dominating set of $M(C_n)$ according as n is odd or even.

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Therefore,

$$\gamma_d(M(C_n)) = \begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$$

$= \left\lceil \frac{n}{2} \right\rceil$ for all $n \geq 3$.

Illustration 2.5: $\gamma_d(M(C_8)) = 4 = \left\lceil \frac{n}{2} \right\rceil$.

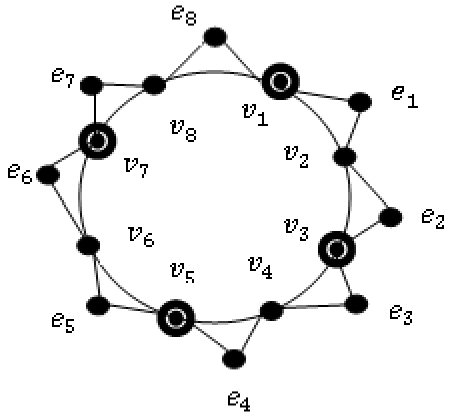

Figure 2.5

$\{v_1, v_3, v_5, v_7\}$ is a γ_d- set of $(M(C_8)$.

Therefore, $\gamma_d(M(C_8)) = 4 = \left\lceil \frac{8}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$.

Illustration 2.6:

$$\gamma_d(M(C_9)) = 5 = \left\lceil \frac{n}{2} \right\rceil.$$

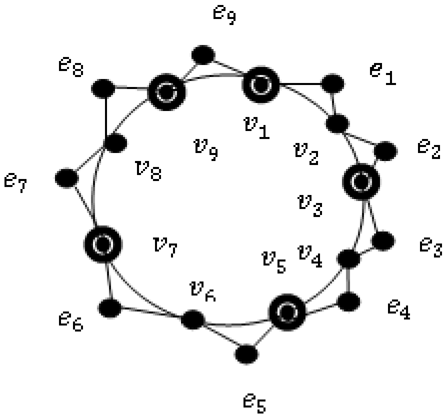

Figure 2.6

$\{v_1, v_3, v_5, v_7, v_9\}$ is a γ_d- set of $M(C_9)$.

Hence, $\gamma_d(M(C_9)) = 5 = \left\lceil \frac{9}{2} \right\rceil$.

Theorem 2.7: $\gamma_d\left(M(K_{1,n})\right) = n + 1$.

Proof:

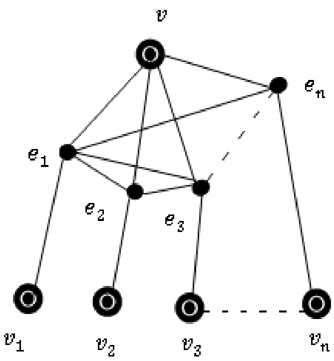

Figure 2.7

From the figure, it is clear that $S = \{v_1, v_2, \ldots, v_{n-2}, v_{n-1}, v_n\}$ being the set of end vertices is contained in every detour dominating set of $M(K_{1,n})$. Further, S dominates all the vertices of $M(K_{1,n})$ other than v. Hence, $S \cup \{v\}$ and $S \cup \{e_i\}$ are γ_d-sets of $M(K_{1,n})$.

Therefore, $\gamma_d\left(M(K_{1,n})\right) = |S| + 1 = n + 1$.

Theorem 2.8:

$\gamma_d((M(D_{m,n}))) = m + n + 1$.

Proof :

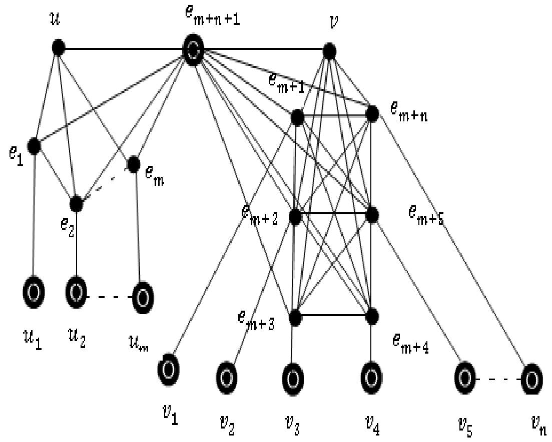

Figure 2.8

Figure 2.8 represents $M(D_{m,n})$.

Let $S = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$. Being the set of end vertices, S is contained in every detour dominating set of $M(D_{m,n})$.

Obviously, $S' = S \cup \{ e_{m+n+1} \}$ is the unique γ_d-set of $M(D_{m,n})$.

Hence, $\gamma_d(M(D_{m,n})) = |S'| = |S| + 1$
$= m + n + 1$.

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III. DETOUR DOMINATION NUMBER OF INFLATED GRAPHS

Theorem 3.1: $\gamma_d(I(P_n)) = \left\lceil \frac{2n}{3} \right\rceil$.

Proof:

Inflated graph of P_n is again a path on $2n-2$ vertices.

Therefore, by theorem 1.5,

$$\gamma_d(I(P_n)) = \gamma_d(P_{2n-2}) = 2 + \left\lceil \frac{2n-2-4}{3} \right\rceil$$

$= 2 + \left\lceil \frac{2n-6}{3} \right\rceil = 2 + \left\lceil \frac{2n}{3} \right\rceil - 2$

$\gamma_d(I(P_n)) = \left\lceil \frac{2n}{3} \right\rceil$.

Remark 3.2:

For a path, the geodetic number is equal to the detour number which is equal to 2. Any (G, D)-set of P_n is also a detour dominating set and vice versa. Hence, $\gamma_d(P_n) = \gamma_G(P_n)$ for all n.

Remark 3.3:

Inflated graph C_n is again a cycle on $2n$ vertices.

Hence, by theorem 1.6, $\gamma_d(I(C_n)) = \gamma_d(C_{2n})$

$= \left\lceil \frac{2n}{3} \right\rceil$.

Theorem 3.4: $\gamma_d(I(K_n)) = n - 1$.

Proof:

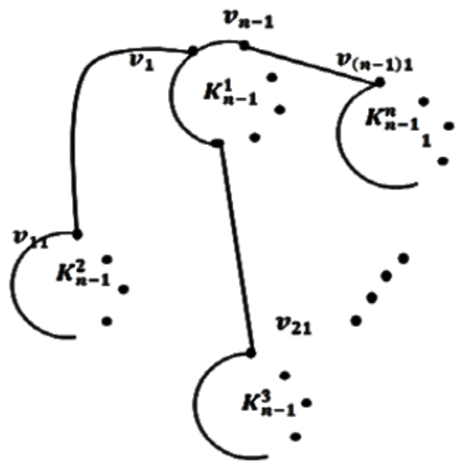

Figure 3.1

K_n is a regular graph of degree $n-1$. Hence, $I(K_n)$ contains n cliques with $n-1$ vertices. Let them be ${K_{n-1}}^1, {K_{n-1}}^2, {K_{n-1}}^3, \dots, {K_{n-1}}^n$. Consider one of the cliques with $n-1$ vertices, say ${K_{n-1}}^1$. Label the vertices of this clique as $v_1, v_2, \dots, v_{n-1}$. Now, label the vertices adjacent to $v_1, v_2, \dots, v_{n-1}$ in the remaining $n-1$ cliques as $v_{11}, v_{21}, \dots, v_{(n-1)1}$ ……………………… (1) as in figure 3.1. These $n-1$ vertices detour dominate all the vertices of $I(K_n)$. Further, no set of less than $n-1$ vertices dominate all the vertices of $I(K_n)$. Therefore, $S = \{v_{11}, v_{21}, \dots, v_{(n-1)1}\}$ is a detour dominating set of minimum cardinality. So, $\gamma_d(I(K_n)) = |S| = n-1$.

Illustration 3.5: $\gamma_d(I(K_7)) = 6$.

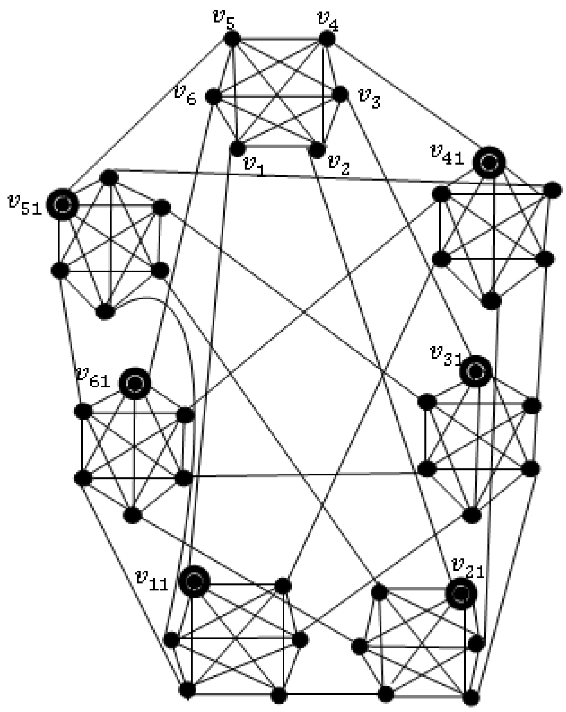

Figure 3.2

From the figure 3.2, we have, $\gamma_d(I(K_7)) = 6 = 7 - 1$.

Theorem 3.6: $\gamma_d\left(I(K_{1,n})\right) = n$.

Proof: Let $V(K_{1,n})u, = \{u, v_i / \, i = 1 \; to \; n\}$ where v is the root vertex.

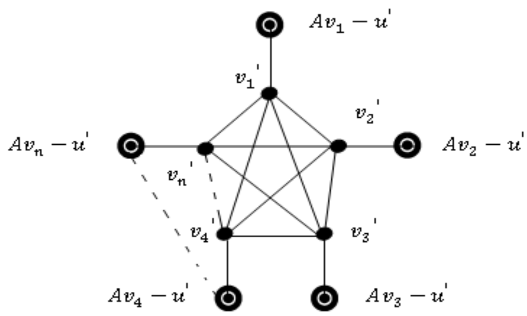

Figure 3.3

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Now, label the vertices of $I(K_{1,n})$ along with the clique name as in figure 3.3.

Here, $S = \{Av_i - u' / i = 1 \; to \; n\}$ be the end vertex set of $I(K_{1,n})$.

Therefore, S is contained in any γ_d-set of $I(K_{1,n})$. Further, S dominates all the vertices of $I(K_{1,n})$. Therefore, S is a γ_d-set of $I(K_{1,n})$.

Therefore, $\gamma_d\left(I(K_{1,n})\right) = |S| = n$.

Theorem 3.7: Let $D_{m,n}$ denote the Double star. Then, $\gamma_d\left(I(D_{m,n})\right) = m + n + 1$.

Proof:

Let $V(D_{m,n}) = \{u, u_1, \ldots, u_m, v, v_1, v_2, \ldots, v_n\}$ with u and v as the central vertices.

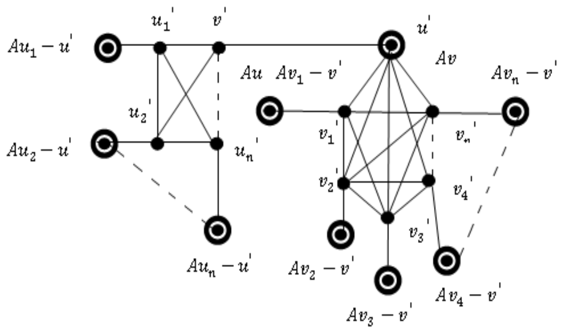

Figure 3.4

Assume that $m < n$. As in previous theorem refer the vertices along with the clique name in which they appear.

Let $S_1 = \{Au_1 - u', Au_2 - u', \ldots, Au_m - u'\}$ and $S_2 = \{Av_1 - v', Av_2 - v', \ldots, Av_n - v'\}$

$S_1 \cup S_2$ be the end vertex set of $I(D_{m,n})$. Therefore, $S = S_1 \cup S_2$ is contained in every detour dominating set of $I(D_{m,n})$. Clearly, $S \cup \{u'\}$ and $S \cup \{v'\}$ are γ_d-sets of $I(D_{m,n})$. Hence, $\gamma_d\left(I(D_{m,n})\right) = m + n + 1$.

IV. CONCLUSION

In this paper we have analysed the detour domination number of Middle graphs and Inflated graphs. It is interesting to investigate further the detour domination number of many other special classes of graphs that are widely used in other areas of research in graph theory.

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