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Multi-Objective Stochastic Goal Mixed Integer Programming in Gauging the Optimum Portfolio Selection in the Amman Stock Exchange towards Financial Sustainability

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Abstract:

The paper attempted to gauge the optimum portfolio selection in the Amman Stock Exchange by using multi-objective stochastic goal mixed-integer programming. This paper presents a solution approach for a complex portfolio selection problem that captures the uncertainty in financial markets. Data were collected from every stock that got listed and continuously traded in the ASE from January 2010 to December 2014. The findings confirmed that the pure stock portfolio, the combination of stock and bond portfolio and a large number of managing constraints allow the portfolio to hold a strategy that beat the benchmark portfolio in certain stages. The result revealed the SGMIP dynamic general portfolio for a single scenario compared to the benchmark portfolio and show that the portfolio achieved a 24% of the return. The SGMIP portfolio achieved 29.2% total return which is greater than the total return of the index portfolio. The portfolio results in a loss of 9.8% in total return from investing in security and achieves a profit of 5.1% from bonds. The SGMIP algorithm has a strong effect on solution speed. The SGMIP model managed to reduce the risk of both portfolios and outperform the performance of the benchmark in two-stage pure portfolio and in the first stage of stock and bond portfolio. This study offers new insights by investigating the dynamic role of multi-objective stochastic goal mixed integer programming to gauge optimum portfolio selection in the Amman stock exchange. This study is limited because its only applicable and useful to countries with similar policies and regulations. Further studies can explore a comparative study between developed and developing economies' stock exchange.

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I. INTRODUCTION

Historically, the financial markets use stochastic models to represent the seemingly random behavior of assets such as stocks. The stochastic portfolio was firstly introduced by Robert Fernholz that aims at flexibly analyzing the performance of certain investment strategies in stock markets relative to benchmark indices and developed impressively by numerous different researchers. This theory gives reasonably intense methods to deal with an uncertain environment.

In light of utilizing empirical and experimental data for displaying and modeling a real-world problem, a deterministic mathematical model cannot represent a realistic problem perfectly. Few approaches were found to deal with such phenomena, for example, stochastic models, fuzzy techniques, and interval analysis, which vary by their favorable circumstances and drawbacks (Nguyen et al., 2012). In numerous practical circumstances, the uncertainties are not of the interval or statistical type; it depends on the experts' perception or human judgment. Therefore, a stochastic situation of different scenarios can be arranged and every value and parameter can be predicted (Solaymani Fard and Ranezanzadeh, 2017).

Dealing with uncertainty, investors find themselves obliged to rebalance their portfolios. To examine the dynamic portfolio, a Mixed-Integer Programming (MIP) employed to select the optimal portfolio configured with an objective and restrictions that account for various financial variables and significant portfolio attributes encompassing investment decisions. Extensive researches were directed to research the size of the portfolio as a main constraint (e.g. Chang et al., 2000); Jobst et al., 2001; Crama and Schyns, 2003; Yiu, 2004; Shaw et al., 2008; Stoyan 2009). The Stochastic Goal Mixed-Integer Programming (SGMIP) model created by Stoyan, (2009) presents liquidity and transaction cost as additional constraints in addition to portfolio size.

The Amman Stock Exchange (ASE) witnesses an uncertainty situation demonstrated by the ASE indicators that recorded the decrease in liquidity flows, market value, and Gross Domestic Product (GDP) by 75.5%, 37.6%, and 71.3%, respectively in 2013 as compared to 2007. It is also noticed that there was an increase of 72.6% in the total deposits in banks for the same period. Responding to this situation, some investors withdrew from the market, while others refrained from trading and/or become more careful in selecting their investments. This situation creates a motive to hedge against the uncertainty by holding a passive stock-bond portfolio and uses the SGMIP model to solve the portfolio selection problem. As a result, investors trading in such conditions within ASE are interested in selecting their optimal portfolio in order to minimize the portfolio risk. Satisfying this need, the multi-assets portfolio selection problem is presented to find the best combination that suits the investor’s preferences.

Even for moderate size problems, portfolio selection models require modeling strategies and solution techniques to oblige for factors, for example, multiple portfolio goals, uncertainty and portfolio rebalancing. There are many factors or practical constraints that generate interest of investors to include them in their portfolios to improve the decision in selecting the optimal portfolio such as limiting the portfolio size and transaction cost, at the same time, encouraging liquidity and diversity. Regarding modeling, Goal Programming (GP) might be utilized to encourage portfolio objectives, and Stochastic Programming (SP) can represent uncertainty issues along with asset price. This paper displays a stock–bond portfolio selection model that invests initially in stocks while taking into account bond investment. The model captures uncertainty and diverse traits associated with stock and bond investments utilizing a Stochastic-Goal Programming (SGP) approach (Ji et al., 2005; Ibrahim, 2008; Stoyan, 2009; Stoyan and Kwon, 2010; Stoyan and Kwon, 2011; Brown and Smith, 2011; Moallemi and Saˇglam, 2017; He and Qu, 2014). Moreover, uncertain conditions require periodic rebalancing of the portfolio or adapting with new circumstances (Valian, 2009).

Few financial researches such as Ji et al. (2005) used a Stochastic Goal Linear Programming approach. The portfolio designed to maximize a target value involving a rebalancing strategy over risky assets and a risk-free asset. The problem involved only one goal, a target wealth constraint, in which they included an investigation on the size of the relaxation parameters associated with this goal. Ballestero (2005) provided the SGP to construct a mean-variance optimization model. SGP methods have been generalized and examined in various publications (Heras and Aguado, 1999; Sengupta and Calif, 1979; Van Hop, 2007). Other researches approached the SP using different techniques (Muhlemann et al., 1978; Kallberg and Ziemba,

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1983; Abdelaziz et al., 2007; Stoyan and Kwon, 2010, 2011). Some of the researches satisfied one goal)Mulvey and Vladimirou, 1992; Golub et al., 1995; Gaivoronski and Stella, 2003; Hibiki, 2006; Escudero et al., 2007; Tapaloglou et al., 2008). The SP model with multi-objectives was developed by Muhlemann et al. (1978) with two portfolio goals. It was one of the first portfolios examining the uncertainty and multiple objectives, but with a small size. Brown and Smith (2011) introduced a model with a large number of assets. It was not required to be capable of running time software, but it also did not reach the optimal portfolio.

As opposed to portfolio formulation, solvability issues are addressed to deal with complex issues resulting from adding more goals to the portfolio (Bienstock, 1996; Chang et al., 2000; Jobst et al., 2001; Lin and Wang, 2002; Canakgoz and Beasley, 2008; Ruiz-Torrubiano and Suarez, 2009). For example, Chance Constrained Programming (CCP) offered in (Charnes and Cooper, 1959; Abdelaziz et al., 2007), heuristic algorithms designed in (Beasley et al., 2003; Canakgoz and Beasley, 2008), genetic algorithms are employed in (Lin and Wang, 2002; Ruiz-Torrubiano and Suarez, 2009), Crama and Schyns (2003) used simulated annealing approach. This study formulates the Mixed-Integer Programming (MIP) to model a dynamic portfolio that involves a set of objectives and constraints considering different real-world financial factors. The number of assets involved in all portfolio designs as an essential practical value for the portfolio managers ensured in this study (Stoyan, 2009; He and Qu, 2014). The Stochastic-Goal Mixed-Integer Programming (SGMIP) model developed in this paper posed additional factors besides problem size which are transaction cost, liquidity, diversity, risk and return.

This study stands to answer the question of "How can real-world dynamic (multi-period) portfolio selection be efficiently solved using SGMIP with samples from ASE?" An algorithm composed of a decomposition strategy was designed to solve the multi objectives stock-bond portfolio problem. While prior studies use SGP with one goal and a target wealth constraint to maximize a target value, the portfolio structure as a SGMIP and the algorithm solution to the portfolio problem are the main contributions in this paper. The selection model of the study portfolio involves SGMIP and integrates bond investments as multi types of assets, discrete choice (integer) constraints, and multi-objective while considering stochasticity problem. The algorithm designed in this study was successful at outperforming current state-of-the-art MIP solvers and reports optimal results for many test cases. In addition, the study provides the financial results of the portfolio over a period involving unstable markets.

The outline of this paper is as follows: Section 2 defines the portfolio model. Section 3 describes the algorithm solution method of the SGMIP portfolio. Finally, Section 4 displays the computational and financial results, and discussion and conclusion is explicated in Section 5.

II.LITERATURE REVIEW

This section outlined the decision variables related to the two assets' types that is included in the portfolio, which are stocks and bonds. Thereafter, the SGMIP will be formulated specifically.

The portfolio problem formulation

The portfolio characteristics are well defined, implying that x_i , is the fraction of the portfolio invested in security i that is purchased in the first-stage (t=0). y_{il}^{t} is the fraction of the portfolio invested in security i that is purchased in the second-stage (t>0).$Ø_{il}^{t}$ is the unit price of security i at time t = 0,1,…,m under scenario l = 1,2,… L , i = 1,2,…n .Where $x_i \in R$, and $y_{il}^{t} \in R$, note that the security price is known at t = 0 and there is only one scenario in the first stage. z_{jl}^{t} is the fraction of the portfolio invested in bond j to purchase at time t under scenario l, hence $z_{jl}^{t} \in R$. φ_{jl}^{t} is the price of bond j at time t under scenario l. U_{jl}^{t} is the bond return at maturity. $\hat{B}$ is the initial wealth of the portfolio.

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Obtaining the portfolio problem requires defining the portfolio elements that consist of transaction cost, liquidity, diversity by minimizing the unsystematic risk, risk and return and their vectors referred to stocks and bonds. Including these elements to the portfolio of this study is because they suit the long-term strategy and the stock-bond investment. To define the portfolio elements let's start to maximize the return of the portfolio as in the following equation:

$$\sum_{l=1}^{L}\sum_{i=1}^{n} Ø_{il}^{1} x_i + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{n} plØ_{il}^{t+1}\, \mathrm{y}_{il}^{t} + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{j=1}^{h} plU_{jl}^{t}\, z_{jl}^{t-h_j^*} \tag{1}$$

Where pl denotes the probability of a scenario realization, where $\sum_{l=1}^{L} pl = 1$ and pl >0. To keep the portfolio managing fees at minimum level, minimizing transaction costs is required which entails minimizing the number of transactions between time periods. Thus, defining transaction cost ϖtil to be the following:

$$\overline{\omega}_{il}^{t} = \left| \mathrm{y}_{il}^{t} - \mathrm{y}_{il}^{t-1} \right| \quad \text{i= 1,2,…,n t=2,…,T, } l = 1,…,\text{L} \tag{2}$$

for t=1

$$\overline{\omega}_{il}^{1} = |\, \mathrm{y}_{il}^{t} - x_i \,| \text{ i= 1,2,…,n, } l\text{=1,…,L;} \tag{3}$$

Where $\overline{\omega}_{il}^{0}$ = 0, $\overline{\omega}_{il}^{t}$ equals the fraction of a security that traded between two periods. The previous equation will be minimized in objective function in order to maintain the portfolio cost to a minimum. Maximizing the sector diversity and minimize the portfolio risk is the second objective of this study. Diversifying the portfolio throughout the market sectors offers a reduction in unsystematic risk. To include the sector exposure element, the variable Q(i, s) determine the security i to which sector is belonged; Hence, Q(i, s) = 1, if security i belonged to sector s ,otherwise = 0, where S represents the total of sectors, Q(i, s) ∈ B. To provide the portfolio with the appropriate sector diversification, considering that f_s^t is the fraction of the portfolio, it will be

$$\sum_{s=1}^{S} f_s^t = 1 \qquad t = 0, \dots, T \tag{4}$$

f st ∈ [0, 1], it is now already known that f s0 is a first stage parameter, and when t> 0 , f st is a second stage parameter. The form of sector exposure element will be as follows:

$$\sum_{i=1}^{n} \mathrm{Q}(i, \mathrm{s})\, Ø_{il}^{t} \mathrm{y}_{il}^{t} = f_s^t \sum_{i=1}^{n} Ø_{\mathrm{il}}^{\mathrm{t}}\, \mathrm{y}_{\mathrm{il}}^{\mathrm{t}} + \xi_{\mathrm{sl}}^{\mathrm{t}} \tag{5}$$

Where $\xi_{\mathrm{sl}}^{\mathrm{t}}$ is the sector relaxation variable which is compatible to the fraction of the portfolio f_s^t that invested in sector, it permits the model to find suitable solution for f_s^t if the feasible solution cannot be found with the current variable used. The above constraint can be used for variable x_i by replacing the variable y_{il}^{t} . Finally, to bound the different number of securities and bonds used in the portfolio. Consequently, gtil defined as follows:

$$\text{gtil} = \begin{cases} \text{1, if security i is used in the portfolio at time t} \\ \text{under scenario l (i.e. if xi, yti > 0);} \\ \text{0, otherwise} \end{cases} \tag{6}$$

where gtil ∈ B , there is one scenario in the first stage l= 1 for g_{il}^{0}, considering Gt as the upper bound of stocks to hold in the portfolio and in order to achieve the goal of limiting number (quantity) of security to hold the cardinality constraint will be:

$$\sum_{i=1}^{n} \mathrm{g}_{il}^{t} \leq \mathrm{G}^{\mathrm{t}} \text{t= 0,…, T} \quad \text{, l= 1,…,L.} \tag{7}$$

Assuming $\hat{B}$ is the initial wealth of the portfolio; the constraints of balancing the portfolio are as follows:

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$$\widehat{B} = \sum_{i=1}^{n} \text{Ø}_i^0 x_i + \sum_{j=1}^{h} \varphi_{jl}^0 z_j^0 \quad (8)$$

$$B_l^1 = \sum_{i=1}^{n} \text{Ø}_{il}^1 x_i - \sum_{i=1}^{n} \text{Ø}_{il}^1 y_{il}^1 + \sum_{j=1}^{h} U_{jl}^1 Z_j^{1-h_j} - \sum_{j=1}^{h} \varphi_{jl}^1 Z_j^1 - \tau_l^1 \overline{w}_{il}^1 \quad (9)$$

where, τ is the transaction cost of a security. Thus, equations (8) and (9) guarantees that the total portfolio wealth including dividends is being invested at each time period. Note that, adding upper bounds to the decision variables of security and bond offers more force to be divers, where di and $\tilde{d}_j$ are the maximum fractions of the portfolio to be invested in security i or bond j; respectively. In order to capture passive investment in PS model, these components in addition to performance measures and various goals of the portfolio will be taken in consideration by adding GP to the problem.

III. THE SGMIP DESIGN

This section presents the remaining elements as objectives goals which are return, risk and liquidity.These objectives are solved in separate portfolio problems to obtain the optimal value then constraint them as goal constraints. Any deviation from the optimal value will penalize in the objective function. Displaying the performance measure as the first portfolio goal is to ensure that the portfolio cannot outperform the obtained optimal value. To do so, assume Rtl as a maximum benchmark, the investment is not allowed to outperform at time t and under scenario l. The value of Rtl is obtained after calculating the index return. The performance constraint is as follows:

$$\text{For first stage} \sum_{i=1}^{n} \text{Ø}_{il}^{t+1} x_{il}^t \leq R_l^t + \mathrm{X}_l^t \; t = 1 \;, l = 1 \quad (10)$$

$$\text{For first stage} \sum_{i=1}^{n} \text{Ø}_{il}^{t+1} y_{il}^t \leq R_l^t + \mathrm{X}_l^t \; t = 1, \ldots \mathrm{T} \;, \qquad l = 1, \;, \mathrm{L}$$

Where $\mathrm{X}_l^t \geq 0$ is a relaxation element that satisfies the GP model, $\mathrm{X}_l^t \in \mathrm{R}$, and l= 1 for$R_l^0$. As noticed from (10) the performance of the securities is only constrained which permits the portfolio to invest in bond when the investment in securities is not favorable. The second portfolio objective is to minimize the portfolio risk measured with beta. The value of optimal beta β* is calculated by using first stage variables as the accompanying sub-problem:

$$min \mu \sum_{s=1}^{n} \beta_s \, g_s^0 + \; (1-\mu) \left| \sum_{s=1}^{n} \text{Ø}_s^0 x_s - B_\beta \right| \quad (11)$$

Subject to

$$\sum_{s=1}^{n} \mathrm{g}_s^0 \leq \mathrm{G}^0 \quad (12)$$

$$x_s \leq C g_s^0 \qquad \forall \, s \in \Upsilon \quad (13)$$

$$x_s \geq 0 \,, \quad x_s \in \| \qquad \forall \, s \in \Upsilon \quad (14)$$

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$$g_s^0 \in \mathbb{B} \qquad \forall s \in \Upsilon \qquad (15)$$

Where 0< μ <1 and B_β the initial portion of portfolio invested in security. Consequently, $\beta^* = \sum_{s=1}^{n} \beta_s \, (g_s^0)^*$, where $(g_s^0)^*$ is the optimal value resulted from solving the model (11) – (15). The equation (12) presented in the sub-problem model to bind the number of security names in the portfolio as cardinality constraint. Because the calculation of β^* are based on historical price movement it gives the best recognized risk in the first time period t=0. The optimal risk value β_{sl}^* of the security s at time t>0 is computed using SP to facilitate future uncertainty of the market. Then, the following constraint is added to the model:

$$\sum_{i=1}^{n} \beta_i \, g_i^0 \leq \beta^* + \delta^0 \qquad (16)$$

For time t > 0 uncertainty must add to the optimal risk value by including scenarios. Thus, for time t > 0, the optimal security risk becomes β_l^* and when associated with single stock it becomes β_{il}^* as shown:

$$\sum_{i=1}^{n} \beta_{il}^t g_{il}^t \leq \beta_l^* + \delta_l^t \, t = 0, \dots, T \quad , l = 1, \dots L \qquad (17)$$

Penalty variables δ^0 , δ_l^t are accompanied by penalty parameter and minimized in the objective function. The last element considered in the study problem is the liquidity. Liquidity already exists in all financial investment, where usually stocks are the most liquid. Liquidity cost is calculated by the difference between the buying price paid by exigent or a rush purchaser and price received by an exigent seller. Since brokerage firm commissions do not fluctuate with the length of time taken to finish a transaction. Rather, ranges in bid-ask spread specify the liquidity cost (Parra et al., 2001).

$$\text{Percent spread} = \frac{Ask - Bid}{Ask} * 100\% \qquad (18)$$

As the investor aims to invest in instruments with high liquidity, the liquidity will be solved for the optimal value Λ_l^* under each scenario, similar to equations (11) - (15). Thus, the following constrain will be include

$$\sum_{i=1}^{h} \Lambda_{(i,t,l)} \, g_{il}^t \geq \Lambda_l^* - \Lambda^t \qquad , t = 0, \dots, T \quad , l = 1, \dots, L \qquad (19)$$

as a constraint in the main model of the problem, where ¸ $\Lambda^t \geq 0$ is a penalty variable that is minimized in the objective function and accompanied by a penalty parameter.

The SGMIP model will be

$$Min \quad -\mu_1 \left(\sum_{l=1}^{L} \sum_{i=1}^{n} \text{Ø}_{il}^1 \, x_i + \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{i=1}^{n} p_l \, \text{Ø}_{il}^{t+1} y_{il}^t + \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{j=1}^{h} p_l \, U_{jl}^t z_{jl}^{t-h_j^*} \right)$$

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$$+\mu_2\left(\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{n} p_l\,\overline{w}_{il}^{t}\right)+\mu_3\left(\sum_{s=1}^{S}\left|\xi_s^0\right|+\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{s=1}^{S} p_l\left|\xi_{sl}^{t}\right|\right)+\mu_4\left(\delta^0+\sum_{t=1}^{T}\sum_{l=1}^{L} p_l\,\delta_l^t\right)$$
$$+\mu_5\left(\lambda^0+\sum_{t=1}^{T}\sum_{l=1}^{L} p_l\,\lambda_l^t\right)+\mu_6\left(\chi^0+\sum_{t=1}^{T}\sum_{l=1}^{L}\chi_l^t\right)(20)$$

Subject To

Initial investment amount constraints:

For first period:

$$\sum_{i=1}^{n}\text{Ø}_i^0 x_i+\sum_{j=1}^{h}\varphi_{jl}^0 z_j^0 = \hat{B} \qquad (21)$$

For second period:

$$\sum_{i=1}^{n}\text{Ø}_{il}^1 x_i-\sum_{i=1}^{n}\text{Ø}_{il}^1 y_{il}^1+\sum_{j=1}^{h}U_{jl}^1 Z_j^{1-h_j}-\sum_{j=1}^{h}\varphi_{jl}^1 Z_j^1-\tau_l^1\overline{w}_{il}^1 = B_l^1\,,\quad l\in\Omega \qquad (22)$$

For other period

$$\sum_{i=1}^{n}\text{Ø}_{il}^t\, y_{il}^{t-1}-\sum_{i=1}^{n}\text{Ø}_{il}^t y_{il}^t+\sum_{j=1}^{h}U_{jl}^t Z_j^{t-h^*j}-\sum_{j=1}^{h}\varphi_{jl}^t Z_{jl}^t-\sum_{i=1}^{n}\tau_l^t\,\overline{w}_{il}^t = B_l^t t\in \mathrm{T} \qquad (23)$$

The performance of portfolio will not exceed market portfolio (Passive St.) Constraints, when t=0, t > 1:

$$\sum_{i=1}^{n}\text{Ø}_{il}^1 x_i \le R^0+\chi^0 \qquad (24)$$

$$\sum_{i=1}^{n}\text{Ø}_{il}^t y_{il}^t \le R_l^t+\chi_l^t\,,\quad \forall\, l\in\Omega,\quad t\in\overline{T} \qquad (25)$$

$$\sum_{i=1}^{n}\mathrm{Q}_{(i,s)}\,\text{Ø}_i^0 x_i = f_s^0\sum_{i=1}^{n}\text{Ø}_i^0 x_i+\xi_s^0\,,\quad s\in S \qquad (26)$$

$$\sum_{i=1}^{n}\mathrm{Q}_{(i,s)}\text{Ø}_{il}^t\, y_{il}^t = f_s^t\sum_{i=1}^{n}\text{Ø}_{il}^t\, y_{il}^t+\xi_{sl}^t\quad ,l\in\Omega,\mathrm{s}\in\mathrm{S},t\in\mathrm{T},l\in\Omega,\mathrm{t}\in\mathrm{T} \qquad (27)$$

Bound the upper bound # of Assets when t=0, t > 0

$$\sum_{i=1}^{n} g_i^0 \le G^0 \qquad (28)$$

$$\sum_{i=1}^{n} g_{il}^t \le G^t \qquad \forall l\in\Omega,\mathrm{t}\in\overline{\mathrm{T}} \qquad (29)$$

Bound # of bonds when t=0, t > 0

$$\sum_{j=1}^{h}\tilde{g}_i^0 \le \tilde{G}^0 \qquad (30)$$

$$\sum_{j=1}^{h}\tilde{g}_{il}^t \le \tilde{G}^t \qquad \forall l\in\Omega,\mathrm{t}\in\overline{\mathrm{T}} \qquad (31)$$

Minimizing the stock's risk

$$\sum_{i=1}^{n} \beta_i\, g^0 \leq \beta^* + \delta^0 \qquad (32)$$

$$\sum_{i=1}^{n} \beta_{il}^t\, g_{il}^t \leq \beta_l^* + \delta_l^t \;, \qquad l \in \Omega, t \in \overline{\mathrm{T}} \qquad (33)$$

Liquidity constraints

$$\sum_{i=1}^{n} \Lambda_{(i,0)}\, g_i \geq \Lambda^* - \lambda^0 \qquad (34)$$

$$\sum_{i=1}^{n} \Lambda_{(i,t,l)}\, g_{il}^t \geq \Lambda_l^* - \lambda_l^t \qquad \forall\, l \in \Omega, t \in \overline{\mathrm{T}} (35)$$

Transaction cost constraints

$$\overline{w}_{il}^1 = |y_{il}^t - x_i| \,, \qquad \forall i \in \Upsilon \,, l \in \Omega \qquad (36)$$

$$\overline{w}_{il}^t = \left|y_{il}^t - y_{il}^{t-1}\right| \forall i \in \Upsilon \,, l \in \Omega, \mathrm{t} \in \widetilde{\mathrm{T}} \qquad (37)$$

Stock

$$x_i \leq C\mathrm{g}_\mathrm{i}^0 \forall i \in \Upsilon \qquad (38)$$

$$y_{il}^t \leq C\mathrm{g}_{il}^\mathrm{t} \forall i \in \Upsilon, l \in \Omega, t \in \overline{\mathrm{T}} \qquad (39)$$

Bond

$$Z_j^0 \leq C g_j^o j \in \Xi (40)$$

$$Z_{jl}^t \leq C \tilde{g}_{jl}^t \forall j \in \Xi, l \in \Omega, t \in \overline{\mathrm{T}} (41)$$

Stock

$$x_i \leq d_i \;, \qquad \forall i \in \Upsilon \qquad (42)$$

ytil ≤ di ∀ i ∈ ϒ, l ∈ Ω, t ∈ T (43)

Bond

$$Z_j^0 \leq \tilde{d}_j \forall j \in \Xi \qquad (44)$$

$$Z_{jl}^t \leq \tilde{d}_j \forall j \in \Xi, l \in \Omega, t \in \qquad (45)$$

Non-negative Constraints

$$x_i \geq 0, \qquad x_i \in \mathrm{R} \qquad \forall i \in \Upsilon (46)$$

$$y_{il}^t \geq 0, \quad y_{il}^t \in \mathrm{R} \quad \forall i \in \Upsilon, \quad l \in \Omega, \quad t \in \overline{\mathrm{T}} \qquad (47)$$

$$Z_j^0 \geq 0 \,, \quad Z_j^0 \in \mathrm{R} \quad \forall j \in \Xi \qquad (48)$$

$$Z_{jl}^t \geq 0 \,, \quad Z_{jl}^t \in \mathrm{R} \quad \forall j \in \Xi, \quad l \in \Omega, t \in \overline{\mathrm{T}} \qquad (49)$$

$$\overline{w}_{il}^t \geq 0, \quad \overline{w}_{il}^t \in \mathrm{R} \quad \forall i \in \Upsilon, l \in \Omega, t \in \overline{\mathrm{T}} \qquad (50)$$

$$\delta^0 \geq 0, \quad \delta^0 \in \mathrm{R} \qquad (51)$$

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$$\delta_l^t \geq 0\,, \quad \delta_l^t \in \mathrm{R} \quad \forall l \in \Omega, \quad t \in \overline{\mathrm{T}} \tag{52}$$

$$\lambda^0 \geq 0, \quad \lambda^0 \in \mathrm{R} \tag{53}$$

$$\lambda_l^t \geq 0, \quad \lambda_l^t \in \mathrm{R} \quad \forall l \in \Omega, t \in \overline{\mathrm{T}} \tag{54}$$

$$g_i^0 \in B, \quad \tilde{g}_j^0 \in B \forall i \in \Upsilon, \quad j \in \Xi \tag{55}$$

$$g_{il}^t \in B, \quad \tilde{g}_{jl}^t \in B \forall i \in \Upsilon, j \in \Xi\,, \quad t \in \overline{\mathrm{T}} \tag{56}$$

$$\xi_s^0 \in R, \quad \xi_{sl}^t \in \mathrm{R} \quad s \in \hat{\mathrm{S}}, l \in \Omega, \quad t \in \overline{\mathrm{T}} \tag{57}$$

(Stoyan and Kwon, 2011)

In equations (38) to (41), C is a large constant and expresses the binary decision variables, and Ω is the scenario generation. The solution method is discussed in the next section.

Solution Method

Solving the large SMIP depends on decomposing the problem into two sub-problems security and bond (see Fig. 1), after constructing set of constraints and relaxing others by adding penalty variables. At the same time, the relaxations (penalty variables) are minimized in the objective function. The stock sub-problem in its turn decomposed according to the sector into three divisions (sub-problems). Each sub-problem are solved separately, then, the resulting values combined in the master problem presented in (20) – (57) searching for optimality. The model either reaches the optimality or needs adjusting the initialization.

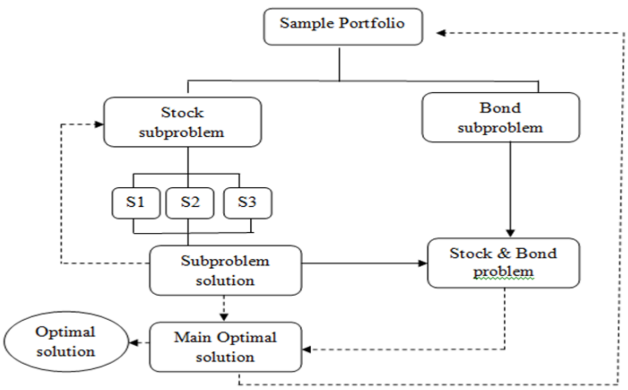

Figure 1 outlined the algorithm design.

Fig. 1. Chart Flow of SGMIP Decomposition. This Figure shows the SGMIP Decomposition between stocks and bonds, the stocks on its turn decomposed into three sub-problems relative to the sectors in ASE. The main model consists of the results of the stocks three sub-problems and the bonds.

The strengths of the algorithm refer to the model specific decomposition of stock and bond sub-problems. The other strength of the algorithm comes from the relaxation γ added to the name to hold constraint G0, Gt in (28) and (29). Accordingly, the model parameters such as G0, Gt, B0, Bt and the portfolio benchmark R0, Rt are divided between sectors in the security sub-problem, then collected in the master problem as follows:

$$\mathrm{G}^0 = G_1^0 + G_2^0 + G_3^0 \tag{58}$$

$$\mathrm{G}^t = G_1^t + G_2^t + G_3^t \tag{59}$$

$$\mathrm{B}^0 = B_1^0 + B_2^0 + B_3^0 \tag{60}$$

$$\mathrm{B}^t = B_1^t + B_2^t + B_3^t \tag{61}$$

$$\mathrm{R}^0 = R_1^0 + R_2^0 + R_3^0 \tag{62}$$

$$\mathrm{R}^t = R_1^t + R_2^t + R_3^t \tag{63}$$

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The solutions of the stock sub-problems are added to the bond sub-problem in the master problem. The master problem measures the optimality of the solution by checking if the portfolio goals fall within the benchmark criteria (62) to (63) and hold the same constraints. Then, either accept the optimal solution or need improvements by increasing the parameters related with the variables that have a value lower than the criteria and resolution. Generating the initial solution using a decomposition algorithm facilitates solving the SGMIP quickly.

The optimal solution will be reached if Z is set to be zero or if all penalty variables are equal to zero. Otherwise, limiting the extent of penalty variables can be done by adjusting the penalties trying to locate an optimal value to the sub-problems. According to Taha (2007), if the original inequality is of type $\leq$ and its penalty $+ \gamma -$ is > 0, then the goal of inequality is satisfied. The optimality of the portfolio depends on the extent (size) of the relaxation variables. When the penalty parameters are large enough, few of relaxation variables will arise. This may increase the CPU time when running the CPLEX. Also, it will be more efficient to perform the penalty adjustment regarding the optimal solution of the parameters obtained from solving the sub-problems, since the sub-problems can be solved faster than the master problem. The following Section will portray the results of the proposed SGMIP model in ASE.

The Results of SGMIP

This paper solves the two-stage SGMIP problem portrayed in (20)–(57) using daily returns of the Amman Stock Exchange (ASE) and Zero-coupon rate bonds. The SP model considers every stock that continuously listed and traded in the ASE from January 2010 to December 2014. It results to approximately 100 securities for the time period (Amman Stock Exchange, 2014). Also, it considers all issued bonds over the same time period, which amount to be two bonds. The ASE100 Float Index composites as the portfolio benchmark R0. For forecasting the second stage, three market scenarios are designed and further used decomposition for sector sub-problem as mentioned in Section 3. Considering all these issues, the SGMIP contained over 518 decision variables and 791 constraints. The SGMIP problem is solved using the previously mentioned algorithm with IBM ILOG CPLEX software version 12.7, on Intel ® core, 2.53 GHz i3 CPU. The decomposition of the algorithm improved memory allocation and CPU time.

Results of Sample Portfolio

This section presents the sample portfolio returns in percent over the ASE Index return and compares the SGMIP portfolio to the benchmark portfolio, in which a single proportionate scenario for the portfolio is solved permitting CPLEX to run completely. Fig. 2 presents the results of an equally-weighted sample portfolio monthly return in comparison to the index monthly returns before implementing the algorithm designed in Section 3. The index portfolio has a slight difference in monthly return over the sample portfolio.

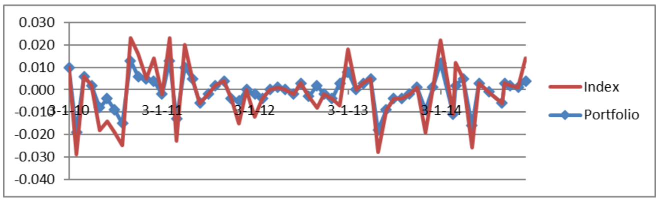

Fig. 2. The monthly return of ASE Index and the sample portfolio at first stage.

This figure presents the monthly return of the sample portfolio compared with the ASE100 index during the period from January 2010 to December 2014. (Colored)

After implementing the algorithm, the second stage of the equations is applied to the pure stocks portfolio. A dynamic portfolio (sectors decomposed) is solved by CPLEX which run for 02:00, 03:02 and 04:36 seconds for the financial, services and industrials sectors, respectively. A single scenario dynamic portfolio (sectors decomposed) resulted in decreasing the losses and achieving the optimal return of benchmark portfolio. Fig. 2 revealed the SGMIP dynamic general portfolio for a single

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scenario compared to the benchmark portfolio. Table 1 show that the portfolio achieved a 24% of the return. The SGMIP portfolio is distributed on the market's three sectors. It contains seven different stocks from the financial sector, four from the service sector, and four from the industrial sector. The performance (based on daily return) of each sector i.e., financial, services and industrials, in the portfolio exhibited 6.1%, 22.8% and -4.9 %, respectively.

Table 1: Performance of SGMIP pure stock portfolio/ Stage one – Information

	Financial Sector	**Service Sector**	**Industrial Sector**	**Portfolio**
SGMIP pure Portfolio (Sample)	0.061	0.228	-0.049%	0.24
Index Portfolio	0.019	0.033	0.008	0.17

Figures 3 to 5 show that the returns (monthly) range from -0.03 to 0.03 in the financial sector, -0.04 to 0.06 in the service sector, and -0.07 to 0.04 in the industrial sector, indicating that investing in the service sector improved the SGMIP portfolio return and remained within the index range. The results of this study at this stage supported the findings of Stoyan (2009).

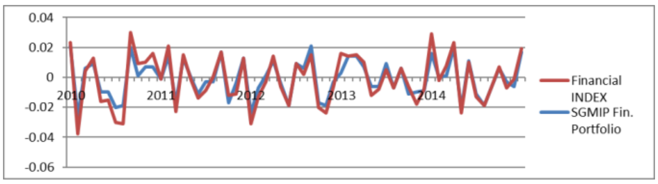

Fig. 3. Financial Sector SGMIP portfolio stage one compared to Financial Sector of Index

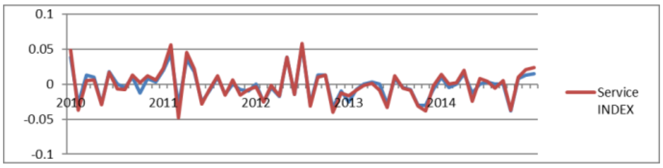

Fig. 4. Service Sector SGMIP portfolio stage one compared to Service Sector of Index.

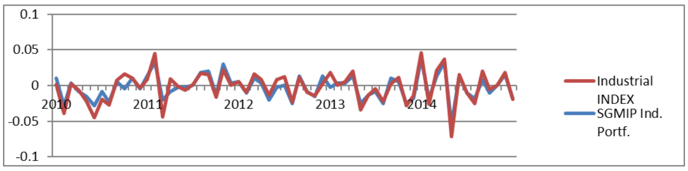

Fig. 5. Industrial Sector SGMIP portfolio stage one compared to Industrial Sectors of Index.

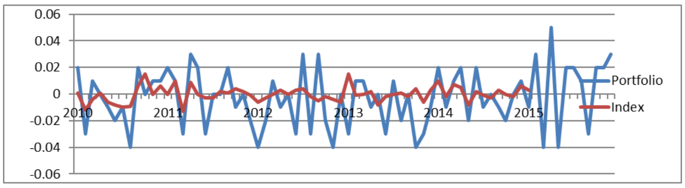

Fig. 6. Worst case SGMIP portfolio compare with Index- Second stage.

The second stage comprises of three scenarios of the above SGMIP portfolio which are lw, li, lb. In all scenarios, the SGMIP portfolio is managed to outperform the index portfolio's total return. The worst scenario of the SGMIP portfolio is presented in Fig. 6, where SGMIP portfolio aggressively fluctuates compared to the index. The SGMIP portfolio achieved 29.2% total return which is greater than the total return of the index portfolio. This result may refer to the beta value of the SGMIP portfolio. The high value of the beta seems to be the reason behind the high return, in line with the direct relationship between risk and return.

Interestingly, the results from the portfolio algorithm designed in the first stage found almost no difference in the performance between the sample portfolio and index portfolio, as shown in Fig. 2. An improvement in the portfolio return is achieved after applying the algorithm. The portfolio gain in the second stage appeared to be connected to the algorithm design (speed up) and uncertainty condition. Figure 6 reveals the results of the second stage between the worst scenario of the SGMIP

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portfolio performance and the index portfolio. In the second stage, the worst scenario takes place in July 2010 where the index portfolio overtakes the SGMIP portfolio by 0.03, as illustrated in Figure 6. These results can be enhanced for some months if the best scenario is taken, which is presented in Fig. 7.

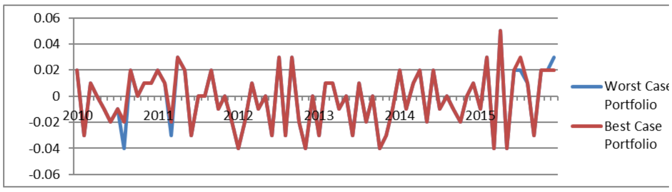

Fig. 7. Best and worst case comparison- Second stage.

Results of Stocks and Bonds Portfolio

This subsection includes the bonds sample to the dynamic pure stocks portfolio and applies the decomposition algorithm. The performance of the resulted portfolio is displayed in the first and second stages as a dynamic portfolio.

In the first stage, the performance of the sample stock and bond portfolio that includes all the objectives and decomposed by sectors equals 30.7%. This portfolio consists of 14 name-to-hold stocks and does not invest in bonds. It consists of six stocks from the financial sector, three stocks from the service sector, and five stocks from the industrial sector. The algorithm portfolio of pure stocks 30.7% outperforms the index portfolio by 1.3% times in the first stage.

In the second stage, the three scenarios are best, stable, and worst. The portfolios of the best and stable scenario invest in stocks and bonds, while the worst invests in bonds only. This fascinating result satisfied the objective of the SGMIP model in investing in mixed stocks and bonds portfolio. The objective is to allow the portfolio to abandon risky assets to safe investments in bond. The best scenario invests in seven name-to-hold stocks and two different bonds. The portfolio results in loss of 9.8% in total return from investing in security and achieves a profit of 5.1% from bonds (see Table 2). The stable scenario invests in eight name-to-hold stocks and two different bonds. The portfolio achieved -12.17% of total return from investing in stocks and 5.1% from investing in bonds. The algorithm portfolio of pure stocks outperforms the SGMIP portfolio of stocks and bonds after decomposition in the best, stable and worst scenarios which achieved 29.2% in total.

Table 2: Performance of the SGMIP portfolios/ stage 2

Type of Portfolio	Best Scenario	Stable Scenario	Worst Scenario
Portfolio- stock & bond	-0.098	-0.122	----
Average return of two bonds	0.051	0.051	0.051

IV. CONCLUSION

This paper presents a solution approach for a complex portfolio selection problem that captures the uncertainty in financial markets. The results of this study documented that examining the pure stock portfolio, the combination of stock and bond portfolio and a large number of managing constraints allows the portfolio to hold a strategy that beat the benchmark portfolio in certain stages. Comparing with the original works of Konno and Kobayashi (1997) and Stoyan (2009), the portfolio of this paper managed additional factors than Konno and Kobayashi (1997), which captures uncertainty in stock price movements and its risk. Moreover, this paper manages two types of a dynamic portfolio than Konno and Kobayashi (1997) and Stoyan (2009), which are pure stocks portfolio and stocks and bonds portfolio. Modeling various portfolio goals and managing different characteristics to attain the preferred portfolio features.

Due to the algorithm construction, the complex portfolio problem solved efficiently. Due to the sub-problem decomposition strategy that reduced the MIP portfolio problem size and the relaxation parameters, the algorithm gains its strength. The SGMIP algorithm model can be applied to similar

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problems. The SGMIP algorithm has a strong effect on solution speed. The SGMIP model managed to reduce the risk of both portfolios and outperform the performance of the benchmark in two-stage pure portfolio and in the first stage of stock and bond portfolio. Also, the model has a fascinating result that when the investment condition getting worst the portfolio turns to invest completely in bonds, avoiding the losses that occur in best and stable scenarios. Because of that, the model can be enhanced with the addition of other portfolio managing characteristics less common to use or adding constraints goals and scenarios.

This study provides theoretical implications on the multi-objective stochastic goal mixed integer programming to gauge optimum portfolio selection. It bridges the gaps in prior literature who mostly focus on single-objective goal integer programming. The study provides clarity to contemporary study and offersa significant recommendation for future research. From the practical perspective, this study solves the two-stage SGMIP problem by using daily returns of the Amman Stock Exchange (ASE) and Zero-coupon rate bonds. It offers significant implications for investors and analysts about the performance of the SGMIP portfolios Portfolio-stock & bond on the best, stable and worst Scenario which will allow them to be able to abandon risky assets to safe investments in stock and bond. The limitation of this study is that the results are only applicable and useful to countries with similar policies and regulations. Hence, further study can examine a comparative study in the context of the stock exchange between developed and developing economies for wider research coverage.

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