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Retrial G - Queue with Phase Types of Service and Immediate Feedback

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Abstract:
In this work, a single service retrial queuing system with negative customers under the Bernoulli vacation schedule and its immediate feedback is addressed. It is proposed the development of steady state Probability Generating Function (PGF) with various size of the system of orbit and mathematical model is obtained by using the Supplementary Variable Technique (SVT).

Keywords:*retrial queue; G-queue; immediate feedback; Bernoulli vacation.*

I. INTRODUCTION

Queueing system with negative customers represents an additional behaviour such as breakdowns, load balancing, signal and call losses. The concept of negative arrivals (called G-Queues) in queues was first introduced by Gelenbe [2] to model neural networks. For the service pattern of the system, positive customers arrive to the queue exponentially and receive service in the regular manner, whereas the negative customers arrive to the system by killing positive customers that are in service and cause to breakdown the server [9]. For more details readers may refer to Kumar and Arivudainambi [6], Choudhury and Ke [1], Rajadurai [7], Sherif and Rajadurai [8]. Another important additional feature that has been discussed widely in queues is the feedback customers. Ke and Chang [5] have discussed the feedback of the customers who appended to the queue to receive feedback service. Kalidas and Kasturi [3] provided a different approach to this aspect called immediate feedback.

II. DESCRIPTION OF THE MODEL

A Markov arrival M/G/1 retrial *G*-queue with two level phases of servicing system and immediate feedback under Bernoulli vacation schedule is proposed to address in this paper [9]. From the base work of Kalidas and Kasturi [3], we assumed the general descriptions. Here, we considered the concepts of arrival, retrial, service, immediate feedback, vacations, breakdown and repairs are considered as different states [4].

Assume the boundary conditions $R(0)=0$, $R(\infty)=1$, $S_i(0)=0$, $S_i(\infty)=1$ (for $i = 1, 2$), $V(0)=0$, $V(\infty)=1$, $H(0)=0$, $H(\infty)=1$ are continuous at initial level $x = 0$.

Hazard rates for different states are (for $i = 1, 2$).

$$a(x)dx = \frac{dR(x)}{1-R(x)},\ \mu_i(x)dx = \frac{dS_i(x)}{1-S_i(x)},\ \gamma(x)dx = \frac{dV(x)}{1-V(x)},\ \xi(x)dx = \frac{dH(x)}{1-H(x)}.$$

The embedded Markov chain $\{Z_n;\ n \in N\}$ is Ergodic if and only if $\rho < 1$ for our system to be stable, where $\rho = r\left(1-R^*(\lambda)\right)-\tau$

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III. STEADY STATE PROBABILITIES

"By the method of SVT, obtain the following equations, $(0 \le j \le m-1)$.

$$\lambda b P_0 = \int_0^\infty \Omega_0(x)\gamma(x)dx + \int_0^\infty R_0(x)\xi(x)dx + q\sum_{l=0}^{m-2}(1-\alpha_{l+1})\int_0^\infty P_{l,0}(x)\mu_2(x)dx + q\int_0^\infty P_{m-1,0}(x)\mu_2(x)dx \quad (3.1)$$

$$\frac{d\psi_n(x)}{dx} + \left(\lambda + a(x)\right)\psi_n(x) = 0, \; n \ge 1 \quad (3.2)$$

$$\frac{dQ_{j,n}(x)}{dx} + \left(\lambda + \delta + \mu_1(x)\right)Q_{j,n}(x) = \lambda(1-b)Q_{j,n}(x) + \lambda b Q_{j,n-1}(x), \; n \ge 0 \quad (3.3)$$

$$\frac{dP_{j,n}(x)}{dx} + \left(\lambda + \delta + \mu_2(x)\right)P_{j,n}(x) = \lambda(1-b)P_{j,n}(x) + \lambda b P_{j,n-1}(x), \; n \ge 0 \quad (3.4)$$

$$\frac{d\Omega_n(x)}{dx} + \left(\lambda + \gamma(x)\right)\Omega_n(x) = \lambda(1-b)\Omega_{j,n}(x) + \lambda b \Omega_{n-1}(x), \; n \ge 0 \quad (3.5)$$

$$\frac{dR_n(x)}{dx} + \left(\lambda + \xi(x)\right)R_n(x) = \lambda(1-b)R_n(x) + \lambda b R_{n-1}(x), \; n \ge 0 \quad (3.6)$$

The steady state boundary conditions at $x = 0$ are

$$\psi_n(0) = \int_0^\infty \Omega_n(x)\gamma(x)dx + \int_0^\infty R_n(x)\xi(x)dx + q\int_0^\infty P_{m-1,n}(x)\mu_2(x)dx + q\left\{\sum_{l=0}^{m-2}(1-\alpha_{l+1})\int_0^\infty P_{l,n}(x)\mu_2(x)dx\right\}, \; n \ge 1 \quad (3.7)$$

$$Q_{0,n}(0) = \int_0^\infty \psi_{n+1}(x)a(x)dx + \lambda r\int_0^\infty \psi_n(x)dx + \lambda(1-r)\int_0^\infty \psi_{n+1}(x)dx, \; n \ge 1 \quad (3.8)$$

$$Q_{j,n}(0) = \alpha_j \int_0^\infty P_{j-1,n}(x)\mu_2(x)dx, \; n \ge 0, \; j = 1,2,\ldots m-1. \quad (3.9)$$

$$P_{j,n}(0) = \int_0^\infty Q_{j,n}(x)\mu_1(x)dx, \; n \ge 0, \; j = 0,1,2,\ldots m-1. \quad (3.10)$$

$$\Omega_n(0) = p\left(\sum_{l=0}^{m-2}(1-\alpha_{l+1})\int_0^\infty P_{l,n}(x)\mu_2(x)dx + \int_0^\infty P_{m-1,n}(x)\mu_2(x)dx\right), n \ge 0 \quad (3.11)$$

$$R_n(0) = \delta\sum_{j=0}^{m-1}\left(\int_0^\infty Q_{j,n}(x)dx + \int_0^\infty P_{j,n}(x)dx\right), \; n \ge 0 \quad (3.12)$$

The normalizing condition is

$$P_0 + \sum_{n=1}^\infty \int_0^\infty \psi_n(x)dx + \sum_{n=0}^\infty\sum_{j=0}^{m-1}\left(\int_0^\infty Q_{j,n}(x)dx + \int_0^\infty P_{j,n}(x)dx\right) + \sum_{n=0}^\infty\left(\int_0^\infty \Omega_n(x)dx + \int_0^\infty R_n(x)dx\right) = 1 \quad (3.13)$$

3.2 The steady state solution

The PGFs for all the states,

$$\psi(x,z) = \sum_{n=1}^\infty \psi_n(x)z^n \; ; \; Q_j(x,z) = \sum_{n=0}^\infty Q_{j,n}(x)z^n ; \; P_j(x,z) = \sum_{n=0}^\infty P_{j,n}(x)z^n ;$$

$$\Omega(x,z) = \sum_{n=0}^\infty \Omega_n(x)z^n \; ; \; R(x,z) = \sum_{n=1}^\infty R_n(x)z^n \; ;$$

Making calculations from (3.2) - (3.12), then we get the limiting PGFs results $\psi(x,z), Q_j(x,z), P_j(x,z), \Omega(x,z), R(x,z)$.

Results: The PGF for orbit size for different states,

$$\psi(z) = \int_0^\infty \psi(x,z)dx = \frac{Nr(z)}{Dr(z)} \quad (3.14)$$

$$Nr(z) = bzP_0\left(1 - R^*(\lambda)\right)\left[\begin{array}{l}\left(q + pV^*\left(b(z)\right)\right)\Phi(z)A_b(z) \\ +\delta\left(1 - A(z)\right)\Phi_1(z)H^*\left(b(z)\right) - A_b(z)\end{array}\right]$$

$$Dr(z) = zA_b(z) - \left(R^*(\lambda) + \left(1 - r + rz\right)(1 - R^*(\lambda))\right) \times \left(\begin{array}{l}\left(q + pV^*\left(b(z)\right)\right)\Phi(z)A_b(z) \\ +\delta\left(1 - A(z)\right)\Phi_1(z)H^*\left(b(z)\right)\end{array}\right)$$

$$Q_j(z) = \int_0^\infty Q_j(x,z)dx = \left(\prod_{j=0}^{m-1}\alpha_j A^j(z)\right)\frac{\lambda bP_0\left(1 - S_1^*\left(A_b(z)\right)\right)}{Dr(z)} \times \left\{z - \left(R^*(\lambda) + \left(1 - r + rz\right)(1 - R^*(\lambda))\right)\right\} \quad (3.15)$$

$$P_j^i(z) = \int_0^\infty P_j(x,z)dx \quad (3.16)$$

$$= \left(\prod_{j=0}^{m-1}\alpha_j A^j(z)\right)\frac{\lambda bP_0 S_1^*\left(A_b(z)\right)\left(1 - S_2^*\left(A_b(z)\right)\right)}{Dr(z)} \times \left\{z - \left(R^*(\lambda) + \left(1 - r + rz\right)(1 - R^*(\lambda))\right)\right\}$$

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$$\Omega(z)=\int_0^\infty \Omega(x,z)dx$$

$$=\frac{p\lambda bP_0\Phi(z)A_b(z)\left(1-V^*\left(b(z)\right)\right)}{b(z)\times Dr(z)} \quad (3.17)$$

$$\left\{z-\left(R^*(\lambda)+\left(1-r+rz\right)(1-R^*(\lambda))\right)\right\}$$

$$R(z) \; =\int_0^\infty R(x,z)dx$$

$$=\sum_{j=0}^{m-1}\left(\prod_{j=0}^{m-1}\alpha_j A^j(z)\right)\frac{\delta\lambda bP_0(1-A(z))\left(1-H^*\left(b(z)\right)\right)}{b(z)\times Dr(z)} \quad (3.18)$$

$$\times\left\{z-\left(R^*(\lambda)+\left(1-r+rz\right)(1-R^*(\lambda))\right)\right\}$$

Applying normalizing condition,

$$P_0+\psi(1)+\Omega_v(1)+R(1)+\sum_{j=0}^{m-1}\left(Q_{jb}(1)+P_{jb}(1)\right)=1\,.\;\square$$

$$P_0=\frac{1-r\left(1-R^*(\lambda)\right)-\tau}{(b\tau-r)\left(1-R^*(\lambda)\right)-\tau+\lambda b\left(1-r(1-R^*(\lambda))\right)\omega+1}$$

The PGF of orbit size and system size are,

$$K_o(z)=P_0+\psi(z)+\Omega(z)+R(z)+\sum_{j=0}^{m-1}\left(Q_j(z)+P_j(z)\right)$$

$$K_s(z)=P_0+\psi(z)+\Omega(z)+R(z)+\sum_{j=0}^{m-1}z\left(Q_j(z)+P_j(z)\right)$$

where

$$\tau=\lambda b\left(\frac{1-\Phi(1)}{\delta}+p\gamma^{(1)}+(1-A(1))\Phi_1(1)g^{(1)}\right)$$

$$\omega=p\Phi(1)\gamma^{(1)}+\left(\prod_{j=0}^{m-1}\alpha_j A^j(1)\right)\frac{(1-A(1))}{\delta}\left(1+\delta g^{(1)}\right)$$

$$\mathrm{A}(z)=S_1^*\left(A_b(z)\right)S_2^*\left(A_b(z)\right)$$

IV. PERFORMANCE MEASURES

(i) Mean orbit size $L_q=K_o'(1)=\lim_{z\to1}\frac{d}{dz}K_o(z).$

(ii) Mean system size $L_s \;=\; K_s'(1) \;=\; \lim_{z\to1}\frac{d}{dz}K_s(z).$

(iii) The average time a customer spends in the system (W_s) and queue (W_q)" $W_s=L_s/\lambda$ and $W_q=L_q/\lambda.$

V. SPECIAL CASES

Case (i): Let δ=0, b = r = 1; p = 0; $R^*(\lambda)\to1$; This reduces to two phase levels and instant feedback queue.

Case (ii): Let $\alpha_j=\mu_2=\delta=0:b=r=1$; Results are reduced to Bernoulli vacations in retrial queue.

Case (iii): $\alpha_j=\mu_2=\delta=p=0;b=r=1$; This indicates retrial queues.

VI. CONCLUSION

The analysis of an M/G/1 retrial G- queue with phase types of service under the Bernoulli vacation and immediate feedback is studied in this paper. Using SVT, the system size is derived. Performance measures of the service system under suitable conditions are discussed. The real-life application is in e-mail system.

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