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1:1 Distributed Pulse Coupling Synchronization Algorithm

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Abstract

Synchronization is a universal phenomenon in the nature and the synchronization topic is old and new. The activity of heart muscle cells and brain cells in the body follows a synchronous pattern. By researching the mass synchronization phenomenon in the nature, it is discovered that a lot of synchronization phenomena follow the natural law of distributed pulse coupling synchronization. This paper researches the principle and mechanism of 1:1 distributed pulse coupling synchronization, constructs its mathematic model and theoretical proof, finds out the optimal coupling coefficient and finally verifies through data simulation. The results show that the experimental analysis is consistent with theoretical analysis and prove the correctness of the theoretical analysis.

Keywords: *Time synchronization; Distributed synchronization; Pulse coupling*

1. INTRODUCTION

The simultaneous happening of the same thing is called as synchronization [1], which is a universal phenomenon in the nature. The movement of all objects even including the atomic nucleus and the universe follows the synchronization law. The activity of heart muscle cells and brain cells in the body also follows a synchronous pattern. The crickets chorus in summer night, the fish swim freely in a pond and thousands of fireflies can flicker simultaneously in Southeast Asia and Africa. These examples are the natural demonstrations of synchronization phenomenon. One of the most common laws in the universe is that moving objects tend to be synchronized, from atoms to animals, from humans to planets [1]. When a woman spends a long time with other women, they often find that their physiological cycle gradually tends to be the same day. When a wonderful drama performance ends, the theater will be quiet in the first seconds, then sound a clap and finally be full of claps. These claps will be messy in different rhythms at the beginning and tend to be simultaneous after seconds. At last, all of the audience will cheer together with a common rhythm.

The synchronization has been researched for long. In the 16th century, Galileo, a famous physicist discovered the isochronous principle of single pendulum vibration by researching the swing of a church's chandelier. In the 17th century, Huygens, a physicist of Holland invented the timing tool - pendulum clock according to the above finding. The research on synchronization has never been interrupted in centuries. In 1990, Steven Strogatz, an American scientist researched the pulse coupling synchronization mechanism of biological vibrator. In 2000, *Nature* published an article construing the mechanism of audience clap synchronization from the perspective of nonlinear dynamics. In recent years, with the rise of IOT, cloud computing, big data, AI and new information technology, it has been found that the importance of keeping network time synchronization in distributed wireless networks such as wireless sensor network, social network and complex network, leading to the emergence of a large number of research outcomes on distributed synchronization. These outcomes are concentrated in the research of synchronization protocol[3-6] and the parameter estimation of synchronization signal[7-9] in the early years. However, there are more research outcomes in preset time synchronization[10-12] and much fewer in synchronization mechanism.

Balanov A et al. defined the synchronization process as: the time scale of different systems of repeatedly adjusting their oscillators[13]. They researched a large number of synchronization phenomena and classified the synchronization into three ways: 1:1 forced synchronization, 1:1 coupling synchronization and n:m synchronization. The synchronization process is actually a process of continuous coupling of time information between entities [2]. 1:1 forced synchronization is that the synchronizing entities will continuously receive the time information of reference entities and regulate their time parameters until the time information of the synchronizing entities are completely consistent with the reference entities. This way is universal in the nature, for instance, a flock of sheep in the grassland and a flock of geese in the sky always have a leader. A person following a cripple may adjust his/her pace and try to walk as same as the cripple in front. n: m synchronization refers to that when n nodes are in synchronization state, other m nodes will reach the same time state with these n nodes according to the relevant mechanism. The cricket chorus in summer, the simultaneous flicker of fireflies and the open dances of women all belong to such synchronization. 1: 1 coupling means that each synchronizing party constantly sends its time information to the counterparty and regulate its time information as per the time information received until their time information reach a same value. This phenomenon is also universal in the nature. For instance, the two persons who spend a long time with each other may be similar in disposition and the two women who spend a long time with each other may find their physiological cycle gradually tend to be same. 1:1 coupling synchronization is a kind of distributed synchronization, which can be widely used in the clock synchronization of distributed wireless networks because it needn't refer to the root node of the clock.

This paper researches and analyzes the 1:1 distributed pulse coupling synchronization mechanism, establishes a mathematical model, proposes a 1:1 distributed pulse coupling synchronization algorithm, proves its convergence, and analyzes and explores the optimal coupling coefficient of the algorithm

2. 1:1 DISTRIBUTED COUPLING SYNCHRONIZATION MATHEMATICAL MODEL

Stephens Strogatz thinks that the flicker rhythm of fireflies is controlled by an oscillator which can be reset inside[1]. The oscillator refers to the entity that circulates automatically and repeats its behavior in a long or short regular time interval. For instance, the flicker of fireflies, the revolution of planets and the activity of pacemaker cells can be considered to the oscillatory activities of the oscillator. The oscillator defined by Strogatz is equivalent to the crystal oscillator in the sensor. A single oscillator will oscillate freely according to its own natural frequency. When two or more oscillators gather, if certain physical or chemical processes make them interact with each other, the oscillators will be called as coupling oscillators. The oscillators will transmit and receive their time information to/from each other through pulse coupling. Many people think that the interaction of oscillators is facilitated by instantaneous pulse signals, which is known as pulse coupling, that is to say, the pulse coupling between oscillators ignores the transmission delay of time information between them. The synchronization mechanism of fireflies is as follows: in a flock of glittery fireflies, each firefly is constantly transmitting and receiving signals so that it can change the rhythm of other fireflies and will be changed by the rhythm of other fireflies[1].

According to the above analysis and research on synchronization mechanism, the mathematical model of 1:1 distributed pulse coupling synchronization is established. Each entity engaged in coupling synchronization is defined as a node, which is represented by $V=(1,2,3\ldots N)$, where N is the number of nodes. These nodes form a network G through time information coupling. Each node must be able to conduct information coupling with at least one node in the network, so isolated node is not allowed in the network, that is, G is a connected network. $t_i(n)$ is defined as the time information of n time of the i node in G. Obviously, if there is $t_i(n)=t_j(n)$ in $n=0,1,2,\ldots\infty$ any time, the nodes i and j in the network reach time synchronization.

Supposed that the two nodes i and j are pulse coupled, the process will be as follows. Node i will transmit its current time information $t_i(n)$ to node j, and node j will regulate its next time information after receiving the time information of node i. The regulation quantity is the product $\alpha_{ji}(t_j(n)-t_i(n))$ of the difference $t_j(n)-t_i(n)$ between the time information $t_j(n)$ of the current node and the time information $t_i(n)$ of the received node i and the coupling coefficient $\alpha_{ji}(\alpha_{ji}>0)$ between node i and node j, which is defined as the coupling increment, indicated as Δ_{ji} i.e. $\Delta_{ji}=\alpha_{ji}(t_j(n)-t_i(n))$. Similarly, when node j receives the time information of node i, it will regulate its time information and transmit its current time information $t_j(n)$ to node i. After node i receives the time information of node j, it will regulate its next moment time information. The regulation quantity is the product $\alpha_{ij}(t_i(n)-t_j(n))$ of the difference $t_i(n)-t_j(n)$ between the time information $t_i(n)$ of the node i and the time information $t_j(n)$ of the current node and the coupling coefficient $\alpha_{ij}(\alpha_{ij}>0)$ between node i and node j, which is defined as the coupling increment, indicated as $\Delta_{ij}=\alpha_{ij}(t_i(n)-t_j(n))$ .In the distributed system, the processing power between nodes is equal, that is, in the distributed network, when the nodes are 1:1 coupled, the coupling coefficient between nodes i and j is equal, i.e. $\alpha_{ij}=\alpha_{ij}$. Therefore, the coupling coefficient between nodes i and j is defined as a normal number $\alpha(\alpha>0)$. In this way, the coupling increment between nodes i and j satisfies $\Delta_{ji}=-\Delta_{ij}$. The synchronization process of 1:1 mutual coupling is described by Formula (1).

$$\begin{cases} t_i(n+1)=t_i(n)+\alpha(t_j(n)-t_i(n))=t_i(n)+\Delta_{ji} \\ t_j(n+1)==t_j(n)+\alpha(t_i(n)-t_i(n))=t_j(n)-\Delta_{ij} \end{cases} \quad (1)$$

3. 1:1 DISTRIBUTED COUPLING SYNCHRONIZATION ALGORITHM

Supposed that N nodes to be synchronized form a connected graph G and G has K edges with the sequence of $v_1,v_2,\ldots\ldots,v_K$. In these K edges, a switching signal $s(k)$ is set. In the discrete time slot $(n,n+1]$, $n=0,1,\ldots\ldots\infty$, if $s(k)=1$,the two nodes i and j of the k-edge v_K will be coupled with time information; if $s(k)=0$, the time information of the two nodes i and j of the k-edge v_K will remain unchanged, with the specific algorithm as follows:

- Initialization at the beginning of synchronization: $n=0$, $k=1$, make $s(k)=1$, enter the step 3).
- $k=k+1$, $s(k)=1$, enter the step 3).
- The pair nodes i and j of v_k couple the time information according to Formula (1). All other nodes in the network remain unchanged

in the time slot $(n, n+1]$, as shown in Formula (2).

$$\begin{cases} t_i(n+1) = t_i(n) + \alpha(t_j(n) - t_i(n) = t_i(n) + \alpha\Delta_{ji} \\ t_j(n+1) = t_j(n) + \alpha(t_i(n) - t_j(n) = t_j(n) - \alpha\Delta_{ji} \\ t_k(n+1) = t_k(n) \qquad k \neq i, j \end{cases} \quad (2)$$

- Timeslot $n = n+1$, enter the step 2).

4. LEMMA 1 (LASALLE INVARIANT SET PRINCIPLE[14])

Supposed $x(n+1) = f(x(n))$ is a discrete time-varying system, where $f: D \to R^N$ is a continuous function in $D \subset R^N$ and n is discrete time. Supposed Ω is a positive invariant set, $V: D \to R^N$ is a difference function, and $\Delta V = V(n+1) - V(n) \leq 0$, M is the largest invariant set in S, then when $n \to \infty$,every solution in Ω will tend to M.

- **Theorem I.** For a connected network G, if all nodes are coupled with time information according to the algorithm proposed in this paper, the time information of all nodes in the network G will gradually converge to the mean of their initial values.
- **Proof:** Firstly we will prove that the algorithm proposed in this paper converges to a certain value.

Without loss of generality, we supposed that node i and node j exchange clocks in time slot $(n, n+1]$ and update clocks according to Formula (2)

Defining a Lyapunov function: $V(n) = \sum_{m=1}^{N} (x_m(n))^2$, we can obtain from Formula (2) that:

$$\begin{aligned} \Delta V &= V(n+1) - V(n) \\ &= t_j^2(n+1) + t_i^2(n+1) - t_j^2(n) - t_i^2(n) + \sum_{m=1, m\neq i,j}^{N} t_m^2(n+1) - \sum_{m=1, m\neq i,j}^{N} t_m^2(n) \\ &= [t_i(n) + \alpha\Delta_{ji}(n)]^2 + [t_j(n) - \alpha\Delta_{ji}(n)]^2 - t_j^2(n) - t_i^2(n) \\ &= t^2{}_i(n) + 2\alpha\Delta_{ji}(n)t_i(n) + \alpha^2\Delta^2{}_{ji}(n) + t^2{}_j(n) - 2\alpha\Delta_{ji}(n)t_j(n) + \alpha^2 t^2{}_{ji}(n) - t^2{}_i(n) - t^2{}_j(n) \\ &= 2\alpha\Delta_{ji}(n)t_i(n) - 2\alpha\Delta_{ji}(n)t_j(n) + 2\alpha^2\Delta^2{}_{ji}(n) \\ &= 2\alpha\Delta_{ji}(n)\left[t_i(n) - t_j(n)\right] + 2\alpha^2\Delta^2{}_{ji}(n) \\ &= -2\alpha\Delta^2{}_{ji}(n) + 2\alpha^2\Delta^2{}_{ji}(n) \\ &= 2\Delta^2{}_{ji}(n)(\alpha^2 - \alpha) \end{aligned} \quad (3)$$

Only when $(\alpha^2 - \alpha) \leq 0$, if $a > 0$ and $a < 1$, $\Delta V \leq 0$

So from Lemma 1, only when $0 < a < 1$, the above algorithm will converge, i.e.

$$\lim_{n\to\infty} t_1(n) = \lim_{n\to\infty} t_2(n) = \ldots = \lim_{n\to\infty} t_N(n) \quad (4)$$

Formula (4) shows that if all nodes are coupled with time information according to the algorithm proposed in this paper, the time information of all nodes in the network G will converge gradually.

According to convex optimization theory[15]: make $\frac{d\Delta V}{d\alpha} = 2\alpha - 1 = 0$, $\alpha = \frac{1}{2}$ will be the optimal coefficient, i.e. when the coupling coefficient $\alpha = \frac{1}{2}$, the algorithm will realize the fastest convergence speed.

Then we prove that this stable value is the mean of their initial values

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$$\begin{aligned}
&\frac{1}{N}\sum_{m=1}^{N} t_m(n+1) \\
&=\frac{1}{N}\sum_{m=1,m\neq i,j}^{N}[t_m(n)+t_i(n+1)+t_j(n+1)] \\
&=\frac{1}{N}\sum_{m=1,m\neq i,j}^{N}[t_m(n)+t_i(n)+\alpha\Delta_{ji}(n)+t_j(n)-\alpha\Delta_{ji}(n)] \\
&=\frac{1}{N}\sum_{m=1,m\neq i,j}^{N}[t_m(n)+t_i(n)+t_j(n)] \\
&=\frac{1}{N}\sum_{m=1}^{N} t_m(n)
\end{aligned} \tag{5}$$

Thus,

$$\lim_{n\to\infty}\frac{1}{N}\sum_{m=1}^{N}x_m(n+1)=\lim_{n\to\infty}\frac{1}{N}\sum_{m=1}^{N}x_m(n)=\ldots=\lim_{n\to\infty}\frac{1}{N}\sum_{m=1}^{N}x_m(0)=\frac{1}{N}\sum_{m=1}^{N}x_m(0) \tag{6}$$

Formula (4) and Formula (6) show that each node of the network uses the distributed coupling synchronization algorithm, and the final convergence value is the mean of its initial time information.

In conclusion, Theorem 1 is proved.

5. SIMULATION AND DISCUSSION

The 1:1 distributed pulse coupling synchronization algorithm proposed in this paper is verified and analyzed by computer simulation. It is assumed that there are five nodes engaged in the coupling to form a connected graph as shown in Figure 1 below. In a coupling time slot, there are a pair of nodes to couple the time information. This process is repeated between different node pairs, and the normalized phase value is used to represent the time information of nodes. The synchronization time is measured in seconds.

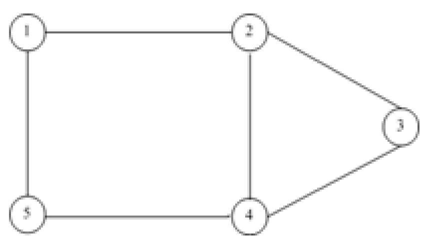

Fig. 1 Coupling Connection Graph

Firstly, let the initial time values of these five nodes be uniform distribution and Gaussian distribution. With the process of coupling synchronization, the time information values of these five nodes are kept at the mean of their initial values, as shown in Fig. 2 and Fig. 3

Secondly, the coupling coefficient $\alpha=0.2$, $\alpha=0.5$ and $\alpha=0.7$ is simulated, as shown in Figure 4. The simulation results show that when coupling coefficient $\alpha=0.5$, the coupling synchronization convergence speed is the fastest, and the data simulation and theoretical deduction are consistent.

When the coupling coefficient $\alpha=0.5$, the distributed coupling algorithm can be abbreviated as:

$$\begin{cases} t_i(n+1)=\dfrac{1}{2}(t_i(n)+t_j(n)) \\ t_j(n+1)=\dfrac{1}{2}(t_i(n)+t_j(n)) \\ t_k(n+1)=t_k(n) \qquad k\neq i,j \end{cases} \tag{7}$$

Formula (7) shows that when a pair of nodes are in coupling synchronization, the simplest information processing mode is to take the mean of the received information and their current information value as the update time information value after receiving the time information of the counterparty. The convergence speed of this processing mode is the fastest.

Finally, the error states of different coupling coefficients are simulated and analyzed. As reference [16], this paper uses $E(n)=\sum_{i=1}^{N}\left[(x_i(n)-\frac{1}{N}\sum_{m=1}^{N}x_m(0))^2\right]$ to express synchronization error.

The average error is expressed in a $\overline{E}(n)=\frac{1}{N}E(n)=\frac{1}{N}\sum_{i=1}^{N}\left[(x_i(n)-\frac{1}{N}\sum_{m=1}^{N}x_m(0))^2\right]$.

Figure 5 shows the synchronization error of coupling coefficient $\alpha=0.2$, $\alpha=0.5$ and $\alpha=0.7$ respectively. The simulation results show that the synchronization error is the smallest when the coupling coefficient $\alpha=0.5$ after the same coupling time; to achieve the same error accuracy, the synchronization time is the least when the coupling coefficient $\alpha=0.5$.

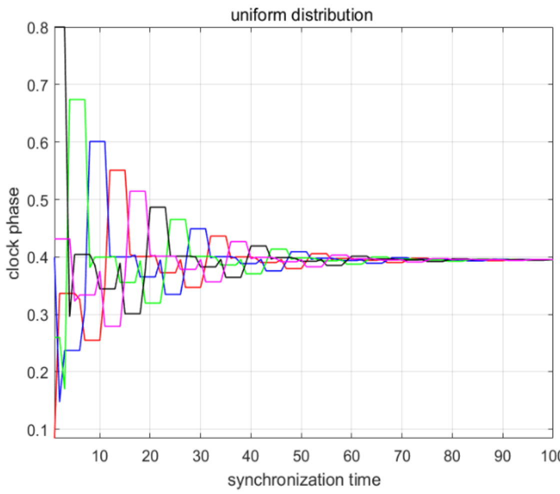

Fig. 2 Convergence of Uniform Distribution

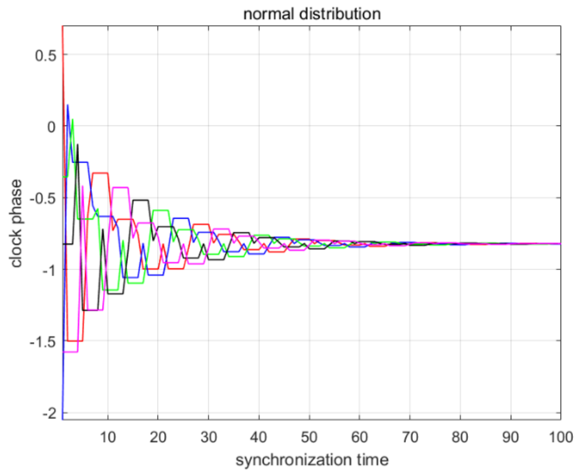

Fig. 3 Convergence of Gaussian distribution

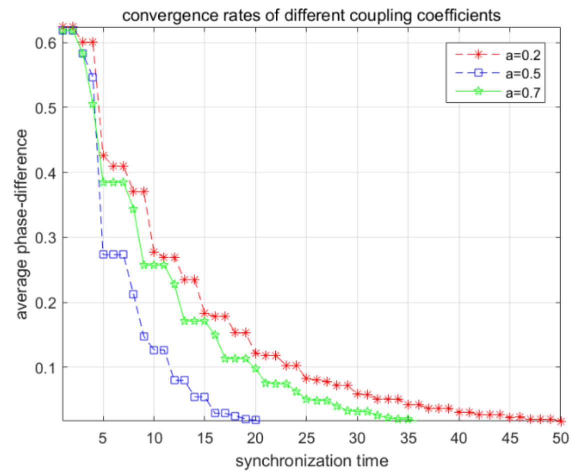

Fig. 4 Convergence Rates of Different Coupling Coefficients

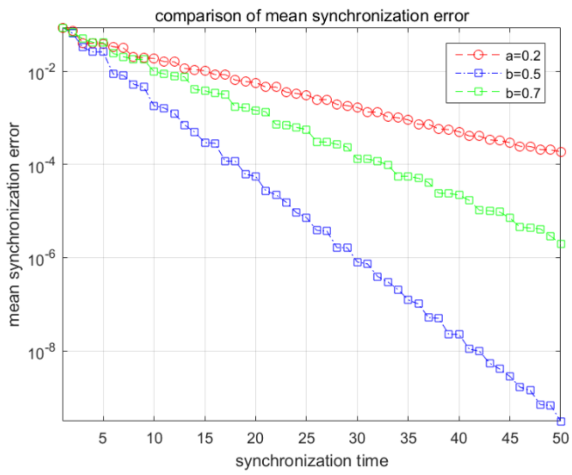

Fig. 5 Comparison of Synchronization Errors of Different Coupling Coefficients

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6. CONCLUSION

Synchronization is a universal phenomenon in nature. The movement of all objects even including the celestial bodies and atomic nuclei follows the law of synchronization. There is synchronization between living and inanimate bodies. Synchronization does not depend on intelligence, life and natural selection but comes from the most profound source of everything: the laws of mathematics and physics. Synchronization is realized through information coupling. This paper explores the principle and mechanism of distributed coupling synchronization, carries out mathematical modeling and theoretical derivation, and finds out the optimal coupling coefficient. The distributed coupling synchronization can be applied to large-scale wireless sensor networks. In practical engineering applications, the message propagation delay and the complex communication mechanism in wireless communication environment need to be considered, which is the future research direction.

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