TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 2685 - 2687

An M/G/1 Retrial Queue with Working Vacation under N Policy

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Article Info
Volume 82
Page Number: 2685 - 2687
Publication Issue:
January-February 2020

Article History
Article Received: *14 March 2019*
Revised: *27 May 2019*
Accepted: *16 October 2019*
Publication: *18 January 2020*

Abstract:
A single server retrial queue with working vacation and N policy is addressed in this work. The steady state Probability Generating Function (PGF) for the system size is obtained by using the method of supplementary variable technique (SVT). The results are illustrated numerically for various parameters.

Keywords:*Retrial queues, working vacations, vacation interruption, N policy.*

I. INTRODUCTION

Theoretical research into the properties of queues first of all started by A.K.Erlang in 1903. The theory was further developed by Mollins in 1927 and then by Thomson D. Fry. A systematic approach to the problem was made by D.G.Kendall in 1951 by using model terminology and since then significant work has been done in this direction. The concept of retrial queues has a great efforts and interest by many researchers (Artalejo and Corral [3]).

Artalejo [2] analyzed an M/G/1 queue with constant repeated attempts and server vacation, using embedded Markov chain. The server takes a vacation only when there are no customers in the system. The server enters the N-policy vacation until N customers accumulated. N-policy queue with an additional service is discussed by Choudhury and Paul [4]. Arivudainambi et al. [1], Gao et al. [5] considered a retrial queue with working vacations.

II. DESCRIPTION OF THE MODEL

Here, retrial queue with WVs under *N*-policy is considered. In this work, we extend the work of Rajadurai et al. [6] by incorporating the concept of *N* policy WV.

Considered the general WV policy in this mode, if there are at least *N* customers in the orbit at a service completion instant, the server will stop the vacation and come back to the normal busy period, which means the vacation interruption happens. If the number of customers in the orbit is less than *N*, the server will continue the vacation. When a vacation ends, if there are at least *N* customers in the orbit, the server switches to the normal working level. Otherwise, the server begins another vacation. Otherwise, the server begins another vacation.

Assume that, $R(0)=0$, $R(\infty)=1$, $S_b(0)=0$, $S_b(\infty)=1$, $S_v(0)=0$, $S_v(\infty)=1$ are continuous at $x = 0$. Where hazard rates for repeated attempts, normal service, lower rate service, and repair are

$$a(x)dx = \frac{dR(x)}{1-R(x)}, \quad \mu_b(x)dx = \frac{dS_b(x)}{1-S_b(x)} \quad and \quad \mu_v(x)dx = \frac{dS_v(x)}{1-S_v(x)}$$

The embedded Markov chain $\{Z_n;\, n \in N\}$ is ergodic if and only if $\rho < R^*(\lambda)$, where $\rho = \lambda\beta^{(1)}$.

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III. STEADY STATE PROBABILITIES

The steady state equations and solutions are developed in this section.

3.1.The steady state equations

"By the SVT, following system of equations are governed,

$$(\lambda+\theta)Q_n = \theta Q_n,\ 0\le n\le N-1$$

$$(\lambda+\theta)Q_0 = \int_0^\infty \Omega_{b,0}(x)\mu_b(x)dx+\int_0^\infty \Omega_{v,0}(x)\mu_v(x)dx. \quad (3.1)$$

$$\frac{dP_n(x)}{dx}+[\lambda+a(x)]P_n(x)=0,\ n\ge 1. \quad (3.2)$$

$$\frac{d\Omega_{b,0}(x)}{dx}+[\lambda+\mu_b(x)]\Omega_{b,0}(x)=0,\ \ n=0. \quad (3..3)$$

$$\frac{d\Omega_{b,n}(x)}{dx}+[\lambda+\mu_b(x)]\Omega_{b,n}(x)=\lambda\Omega_{b,n-1}(x),\ \ n\ge 1. \quad (3.4)$$

$$\frac{d\Omega_{v,0}(x)}{dx}+[\lambda+\theta+\mu_v(x)]\Omega_{v,0}(x)=0,\ \ n=0. \quad (3.5)$$

$$\frac{d\Omega_{v,n}(x)}{dx}+[\lambda+\theta+\mu_v(x)]\Omega_{v,n}(x)=\lambda\Omega_{v,n-1}(x),\ \ n\ge 1. \quad (3.6)$$

The steady state boundary conditions are at $x = 0$ are

$$P_n(0)=\int_0^\infty \Omega_{b,n}(x)\mu_b(x)dx+\int_0^\infty \Omega_{v,n}(x)\mu_v(x)dx,\ n\ge 1 \quad (3.7)$$

$$\Omega_{b,0}(0)=\int_0^\infty P_1(x)a(x)dx+\theta\int_0^\infty \Omega_{v,0}(x)dx,\ n=0. \quad (3.8)$$

$$\Omega_{b,n}(0)=\int_0^\infty P_{n+1}(x)a(x)dx+\lambda\int_0^\infty P_n(x)dx+\theta\int_0^\infty \Omega_{v,n}(x)dx,\ n\ge 1. \quad (3.9)$$

$$\Omega_{v,n}(0)=\begin{cases}\lambda Q_n, & 0\le n\le N-1\\ 0, & n\ge N\end{cases} \quad (3.10)$$

The normalizing condition is

$$\sum_{n=0}^{N-1}Q_n+\sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx+\sum_{n=0}^{\infty}\left(\int_0^\infty \Omega_{b,n}(x)dx+\int_0^\infty \Omega_{v,n}(x)dx\right)=1. \quad (3.11)$$

3.2.The steady state solutions

To solve the above equations, by defining GFs as,

$$Q(z)=\sum_{n=0}^{N-1}Q_n z^n;\ P(x,z)=\sum_{n=1}^{\infty}P_n(x)z^n;$$

$$\Omega_b(x,z)=\sum_{n=0}^{\infty}\Omega_{b,n}(x)z^n;\ \ \Omega_v(x,z)=\sum_{n=0}^{\infty}\Omega_{v,n}(x)z^n$$

From (3.1) - (3.10) by z^n and taking summation and make calculations, we obtain the limiting probabilities $P(x,z),\ \Omega_b(x,z)$ and $\Omega_v(x,z)$ and from the above results.

Theorem 3.1 *If the system in stability condition, the PGF of orbit size when server being idle, busy, on vacation are given by*

$$P(z)=\int_0^\infty P(x,z)dx$$

$$=\left\{\frac{z\left(1-R^*(\lambda)\right)Q(z)\left[S_v^*\left(A_v(z)\right)+V(z)S_b^*\left(A_b(z)\right)-1\right]}{Dr(z)}\right\} \quad (3.29)$$

$$Dr(z)=A_b(z)\left(z-\left(R^*(\lambda)+z(1-R^*(\lambda))\right)S_b^*\left(A_b(z)\right)\right)$$

$$\Omega_b(z)=\int_0^\infty \Omega_b(x,z)dx$$

$$=\left\{\begin{matrix}\lambda Q(z)\left(1-S_b^*\left(A_b(z)\right)\right)\\ \left[\left(S_v^*\left(A_v(z)\right)-1\right)\left(R^*(\lambda)+z(1-R^*(\lambda))\right)+zV(z)\right]\end{matrix}\right\}\Big/ Dr(z) \quad (3.30)$$

$$\Omega_v(z)=\int_0^\infty \Omega_v(x,z)dx=\left\{\frac{\lambda Q(z)V(z)}{\theta}\right\} \quad (3.31)$$

where $A_b(z)=\lambda(1-z)$ and $A_v(z)=\ \theta+\lambda(1-z).$

$$V(z)=\frac{\theta}{\theta+\lambda(1-z)}\left[1-S_v^*\left(A_v(z)\right)\right]$$

By the normalizing condition, $Q(1)+P(1)+\Omega_b(1)+\Omega_v(1)=1$

$$Q(1)=\left\{\frac{R^*(\lambda)-\lambda\beta^{(1)}}{R^*(\lambda)+(\lambda/\theta)\left(1-S_v^*(\theta)\right)-S_v^*(\theta)\lambda\beta^{(1)}}\right\} \quad (3.32)$$

Corollary: Under the stability condition,

The PGF of system size, $K_s(z)=\dfrac{Nr_s(z)}{Dr(z)}=Q(z)+P(z)+z\left(\Omega_b(z)+\Omega_v(z)\right).$

The PGF of orbit size, $K_o(z)=\dfrac{Nr_o(z)}{Dr(z)}=Q(z)+P(z)+\Omega_b(z)+\Omega_v(z).$

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ISSN: 0193 - 4120 Page No. 2685 - 2687

IV. Performance Measures

4.1. System state probabilities

(i) Probability that the server is idle during the retrial

$$P(1) = \left\{ \frac{Q(1)\left(1 - R^*(\lambda)\right)\left(1 - S_v^*(\theta)\right)\left((\lambda/\theta) + \lambda\beta^{(1)}\right)}{R^*(\lambda) - \lambda\beta^{(1)}} \right\}$$

(ii) Probability that the server is busy

$$\Omega_b(1) = \left\{ \frac{\lambda Q(1)\beta^{(1)}\left(1 - S_v^*(\theta)\right)\left((\lambda/\theta) + R^*(\lambda)\right)}{R^*(\lambda) - \lambda\beta^{(1)}} \right\}$$

(iii) Probability that N policy working vacation,

$$\Omega_v(1) = \left\{ \lambda Q(1)\left(1 - S_v^*(\theta)\right)/\theta \right\}$$

4.2. Mean system size and orbit size

(i) Mean orbit size $L_q = K_O'(1) = \lim_{z \to 1} \frac{d}{dz} K_O(z).$

(ii)

(ii) Mean system size $L_s = K_S'(1) = \lim_{z \to 1} \frac{d}{dz} K_S(z).$

(iii) The average time a customer spends in the system (W_s) and queue (W_q)" $W_s = L_s/\lambda$ and $W_q = L_q/\lambda$.

V. Special Cases

Case1: $R^*(\lambda) \to 1$. It reduced to single server queue with *N* WV policy.

Case 2:N=0. It reduced to single server retrial queue with WV.

Case3: (N,θ)→(0,0) tends to Single WV in M/G/1 retrial queue.

VI. Conclusion

In this work, *N* policy WV in presence of retrial queues is discussed. The PGF for system size and orbit size are found by using the SVT. System measures like the mean queue and system size are found. Practical application of the work is in flow control policy of IBM's system networks architecture.

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