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Application of Fractional Calculus Operators in Projectile Motion of a Particle along with Resisting Medium in two Dimensional Spaces

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Abstract
In the present paper, we propose to describe the nature of two dimensional projectile motions of a particle along with resisting medium by the fractional derivative equation. A subsidiary constraint λ is initiated in the differential operator, in order to keep the dimensionality of the physical quantities in the structure. The reciprocal of seconds $(\sec)^{-1}$ is a dimension of this constraint and characterizes the continuation of fractional time components in a certain structure. In the fractional approach, the trajectories of the projectile motion at distinct values of α and distinct fixed values of velocity v_0 and elevation ψ, are until the end of time smaller than the conventional one, nothing like the outcomes achieved in new studies. The entire outcomes achieved in the standard condition may be achieved from the fractional condition when $\alpha = 1$.

Keywords: Fractional derivative equations, Caputo derivative, generalized Mittag-Leffler function, Projectile motion.

1. INTRODUCTION:

Fractional calculus, one of the domain of mathematics, studies fractional derivatives and fractional integrals i.e. operators having non-integer [1, 3]. Fractional calculus is necessary in order to explain them because several physical phenomena have "intrinsic" fractional order representations [4, 5]. When compared to classical calculus, fractional calculus has an advantage that it provides very good tool for the representation of remembrance and inherited performance of different resources and processes [6]. Tools of fractional calculus have been used to restudy several physical phenomena with expansion over the conventional study as well as better correctness on calculating through experimental facts, for example, in agriculture, chemistry, biomedical, and physics together with Lagrangian and Hamiltonian formulation [7, 8]. During the most recent decades, applications have been found by fractional calculus and fractional differential equations in various engineering disciplines [9, 10]. From the physical and engineering point of view, the outcomes of two dimensional fractional projectile motion in a resisting medium are not totally match with the outcomes throughout obtained in the usual conditions because the differential equations contain the physical constraints should not have the dimensionality calculated in the laboratory [11, 12]. Recently [13, 14] proposed efficient technique to construct fractional differential equation is like as a unit of Physical constraints engaged in the equation continue invariant. Here in the manuscript we will examine that when the resistance is proportional to relative velocity [15], therefore, what will be the performance in resisting medium of projectile motion in a fractional two dimensional space. This will be explained that in a fractional approach, the trajectories of the projectile motion at distinct values of α and distinct fixed values of the initial velocity V_0 and elevation ψ are always less than the classical one, unlike the result obtained in ordinary case [11].

2. METHODOLOGY:

It is necessary to have the definition of fractional derivative to examine the dynamic behavior of the fractional structure. In many applied problems, the use of definitions of fractional derivatives is necessary which allows the utilization of physical interpretable initial condition that contains X (0), X ′(0), . . . The Caputo representation fulfils desire conditions for fractional order derivative. In the condition of Caputo, the differential of a constant be vanish (but it is not zero in the sense of Riemann -Liouville) so, the beginning condition for the Fractional Derivative Equations

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can be defined properly, which can be handled by using an analogy with the classical integer case. As a result, the Caputo fractional derivative given by [2] is used by us in this manuscript.

$$ {}_0^cD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^n(x)\;dx}{(t-x)^{\alpha-(n-1)}}, \quad (1)$$

Where $n-1<\alpha\le n$ and Γ (.) be the Euler Gamma function for natural number n∈ N. The fractional derivative order is $0<\alpha\le 1.$ now applying the condition n =1

The Laplace transform for the function f (t) for all real numbers $t\ge 0$,is defined by the following function F(s)

$$F(p) = L\{f(t)\} = \int_0^\infty e^{-pt} f(t)\,dt \quad (2)$$

Now the following equation is defined as the Laplace transform for Caputo fractional differential

$$L\left\{{}_0^cD_t^\alpha f(t)\right\} = p^\alpha F(p) - \sum_{k=0}^{n-1} p^{\alpha-k-1} f(0)^{(k)} \quad (3)$$

The Swedish Mathematician Mittag-Leffler (1903) introduced a function is defined as

$$E_\alpha(x) = \sum_{r=0}^{\infty}\frac{x^r}{\Gamma(\alpha r+1)}, \qquad (\alpha>0) \quad (4)$$

The generalized form for the above Mittag-Leffler function in the following series expansion

$$E_{\alpha,\beta}(x) = \sum_{r=0}^{\infty}\frac{x^r}{\Gamma(\alpha r+\beta)}, \quad (\alpha,\beta>0) \quad (5)$$

has been studied by several authors notably by Mittag-Leffler (1903,1905) and Wiman(1905).

Its Laplace transform is given by the formula

$$\int_0^\infty e^{-pt} t^{\alpha m+\beta-1} E_{\alpha,\beta}^{(m)}(\pm a t^\alpha)\,dt = \frac{m!\,p^{\alpha-\beta}}{(p^\alpha \mp a)^{m+1}}. \quad (6)$$

Therefore, inverse of the Laplace transform be

$$L^{-1}\left\{\frac{m!\,p^{\alpha-\beta}}{(p^\alpha \mp a)^{m+1}}\right\} = t^{\alpha m+\beta-1}E_{\alpha,\beta}^{(m)}(\pm a t^\alpha). \quad (7)$$

2.1. FRACTIONAL OF TWO DIMENSIONAL MOTIONS:

In recent times, an efficient approach to formulate fractional differential equations, maintaining the unity of the physical constraint invariant, has been expected in [23]. We describe these techniques to examine the motion in resisting medium of a moving particle the two-dimensional space in this manuscript. In two- dimensional spaces motion of a projectile moving particle through a resistive force proportional to its velocity contain the following form of equations [15]

$$M\frac{dV_x}{dt} = -\lambda M V_x \quad (8)$$

$$M\frac{dV_y}{dt} = -Mg - \lambda M V_y \quad (9)$$

Wherever λ be the positive real invariable it denotes the power of the retarding force and mass for projectile is M which is measured in Kg. Sec^{-1} is the dimensionality in inverse of seconds. Let us assume the beginning conditions are $V_X(0) = V_{0X} = V_0 \cos\psi$ and $V_Y(0) = V_{0Y} = V_0 \sin\psi$, Where ψ be the direction of distance from the ground and v_0 be initial velocity for projectile motion. Equations (8) and Equations (9) have the solution are given by [15]

$$X(t) = \frac{V_{0X}}{\lambda}(1-e^{-\lambda t}), \quad (10)$$

$$Y(t) = -\frac{gt}{\lambda} + \frac{1}{\lambda}(V_{0Y} + \frac{g}{\lambda})(1-e^{-\lambda t}). \quad (11)$$

Replacing $\frac{d}{dt}$ by $\lambda^{\beta-\alpha}\frac{d^\alpha}{dt^\alpha}$. (12)

During the above equation (8) and equation (9), the constraint λ must have a dimension of the inverse of second's s^{-1} to continue a consistent set of units. Therefore, the fractional derivative equations as given below with order $0<\alpha\le 1$.

$$\frac{d^\alpha V_X}{dt^\alpha} = -\lambda^{\alpha-\beta+1}V_X, \quad (13)$$

$$\frac{d^\alpha V_Y}{dt^\alpha} + \lambda^{\alpha-\beta+1}V_Y = -g\,\lambda^{\alpha-\beta}. \quad (14)$$

On the applying Laplace transform in equations (13) and (14) and also taking inverse Laplace, we get

$$V_X(t) = V_{0X}E_{\alpha,\beta}(-(\lambda t)^\alpha), \quad (15)$$

And

$$X(t) = \frac{V_{0X}}{\lambda}[1-E_{\alpha,\beta}(-(\lambda t)^\alpha)]. \quad (16)$$

Using the same technique, we obtain the following solutions in terms of Mittag-Leffler function

$$V_Y(t) = -\frac{g}{\lambda} + (g/\lambda + V_{0Y}) E_{\alpha,\beta}(-(\lambda t)^{\alpha}), \quad (17)$$

And

$$Y(t) = -\frac{g}{\lambda^2 \Gamma(\alpha+\beta)} (\lambda t)^{\alpha} + \frac{1}{\lambda} (g/\lambda + V_{0Y})(1 - E_{\alpha,\beta}(-(\lambda t)^{\alpha})) \quad (18)$$

If we put $\alpha = \beta = 1$ in equation (16) and (18) then, we get the formulas in equation (10) and (11) for the ordinary case.

The Mittag-Leffler functions in the sequence extension form are given below:

$$E_{\alpha,\beta}(-(\lambda t)^{\alpha})). = \frac{1}{\Gamma(\beta)} - \frac{(\lambda t)^{\alpha}}{\Gamma(\alpha+\beta)} + \frac{(\lambda t)^{2\alpha}}{\Gamma(2\alpha+\beta)} - \frac{(\lambda t)^{3\alpha}}{\Gamma(3\alpha+\beta)} + ..., \quad (19)$$

In equations (16) and (18), the trajectory of the projectile can be drawn for differential equations used with distinct values in the order $0 < \alpha \leq 1$. On determining the time T imposed on whole trajectory, the range R can be obtained, subsequently, putting this value in the equation (16). On putting t = T, time T is obtained, when Y = 0. Then we get from equation (18)

$$(\lambda T)^{\alpha} = \frac{\lambda \Gamma(\alpha+\beta)}{g} (g/\lambda + V_{0Y})(1 - E_{\alpha,\beta}(-(\lambda T)^{\alpha})). \quad (20)$$

An analytic expression cannot be obtained for T, because equation (20) is a transcendental equation and therefore, we get time T required for the whole trajectory as given below

$$(\lambda T)^{\alpha} = \Gamma(2\alpha+\beta) \left[\frac{1}{\Gamma(\alpha+\beta)} - \frac{g}{\Gamma(\alpha+\beta)\{g + \lambda V_{0Y}\}} + \frac{(\lambda T)^{2\alpha}}{\Gamma(3\alpha+\beta)} - \frac{(\lambda T)^{3\alpha}}{\Gamma(4\alpha+\beta)} + ... \right] \quad (21)$$

Therefore, the range R will be obtained on replacing $t \to T$ in equation (16) as below

$$R = X(T) = \frac{V_{0X}}{\lambda} \left[1 - E_{\alpha,\beta}(-(\lambda T)^{\alpha}) \right] \quad (22)$$

3. RESULT AND DISCUSSION:

In this manuscript we have studied using fractional differential equations in projectile motion of a particle in a resisting medium for two dimensional space. A subsidiary constraint λ is initiated in the Differential operator for maintaining the dimensionality of the physical quantities in the structure. The constraint k characterizes the continuance for fractional time components in the particular structure and has a dimension of second's reciprocal $(\mathrm{sec})^{-1}$. In this outcomes inequalities $V_0 \sin \psi > 0$ and $\psi < \pi/2$ are satisfied, the projectile's trajectories for distinct values of α and distinct fixed values of velocity V_0 evaluated. Therefore, applying the fractional calculus technique is anytime smaller than the conventional technique. Total outcomes may be calculated in the ordinary conditions may be also calculated with the fractional calculus conditions when $\alpha = \beta = 1$. The expansion of this technique may be done for three-dimensional motion.

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