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Centered Fan Field Region under Wedge Indentation by Numerical Solution Method

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Abstract: A numerical solution method of stress distribution in the deformation region under wedge indentationare presented. The granular materials are assumed to obey Mohr-Coulomb yield condition in a plane strain condition. The numerical methods used to determine the stress field at each position (x,y) in the deforming region under the punch. The formation of the centered fan region was constructed by the network of the stress characteristic lines. The construction of the region is presented by using MATLAB. The solution obtained in this research only refer to the initial movement of the granular materials after the punch. The stress field obtained were then compared to the analytical solution. This solution method provides a simple algorithms that is easy to be applied in deformation and this will subsequently provide a contribution in designing tools and the construction industries involving granular materials.

Keywords:*Granular materials, plasticity, wedge indentation.*

I. INTRODUCTION

The numerical solution method of stress field in a centeredfan regionformed under the wedge indentation is considered. The approximation to the solution for the double-slip and double-spin model was presented in [1]. The authorgeneralized the numerical algorithm that commonly used in metal plasticity into thedeformationandflow ofgranularmaterials by a flat-ended rigid punch. By following closely the method described in [1], the extension was made by [2] and [3]. Reference [2] has applied the method to the axisymmetric problem, while the latter provides the solution for the region under the wedge punch. The work of this paper is the extension of the numerical solution methods by [3].The method was then applied to the second elementary boundary value problem, namely centered fan region. The solution to the wedge indentation problem was once first presented by [4]. The author made the assumption that the granular materials are modelled well by the Mohr-Coulomb failure conditions and the associated flow rule. The slip line approach with the aid of [5] for metal plasticity problemwas applied to soil plasticity problem. Reference [6] proposedan analytical solution method to the wedge indentation problem for dilatant material model developed through [7] and [8]. In this study, the stress distribution in centered fan region beneath a smooth rigid wedge indentation by anumerical solution technique is described. In

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section II, we discussedthe fundamental equations for deformation in regions of high stress concentration. In sectionIII, we mentioned the numerical solution approach for the stress distribution. The construction of stress distributionby using MATLAB are introduced in section IV and the outcomes are in section V.

II. THE EQUATIONSOF THE STRESS

The equilibrium equations for the stress are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \qquad (1)$$
$$\frac{\partial \sigma xy}{\partial x} + \frac{\partial \sigma y}{\partial y} = 0$$

and Mohr-Coulomb yield condition is given by

$$\tau = c - \sigma_n \tan\omega \qquad (2)$$

where c represent the material cohesion and ω represent the material friction angle. In a plastic state, the stress components were given by

$$\sigma_x = -p + q\cos 2\lambda$$
$$\sigma_{xy} = q\sin 2\lambda \qquad (3)$$
$$\sigma_x = -p - q\cos 2\lambda$$

where λ is the inclination angle between the failure line and the x-axis, meanwhile p denote thepressure and q is the shear stress represented by

$$p = -\frac{1}{2}\left(\sigma_x + \sigma_y\right), q = \frac{1}{2}\left(\sigma_x - \sigma_y\right) \qquad (4)$$

From (1), the stress characteristic directions were given by

$$\frac{dy}{dx} = \tan\left[\lambda \mp \left(\frac{\pi + 2\omega}{4}\right)\right] \qquad (5)$$

for α - and β-characteristic line respectively.

By substituting (3) into (1), the governing equation for p and λ were defined by

$$\frac{\partial p}{\partial D_\alpha}\cos\omega + 2q\frac{\partial \lambda}{\partial D_\alpha} = 0 \qquad (6)$$
$$-\frac{\partial p}{\partial D_\beta}\cos\omega + 2q\frac{\partial \lambda}{\partial D_\beta} = 0$$

where $\frac{\partial}{\partial D_\alpha}$, $\frac{\partial}{\partial D_\beta}$ are the directional derivatives along α - and β-characteristic line respectively.

III. NUMERICAL PROCEDURE METHOD

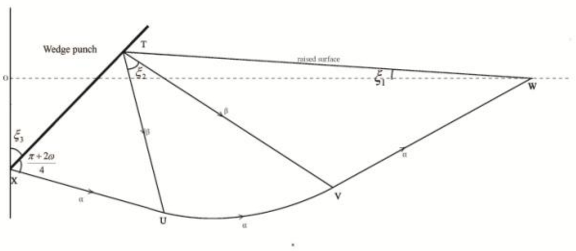

Fig. 1.Deformationregion under wedge punch

Figure 1 shows the deformation region under the wedge indentation of granular materials. When a smooth rigid wedge with semi-angle ξ_3 move downwards into the granular materials surface, the material will deform.Since the configuration is symmetrical, we only consider the right-hand half of the field. There are three distinct regions formed under the indentation which are two triangular regions XUT and TVW and a centered fan region TUV of angle ξ_2. In this study, we consider only the centered fan region, TUV. This deformation region is generated by the stress distribution network and is then constructed using Matlab. For this region, there were two types of elementary boundary value problem involved.

A. Riemman Problem

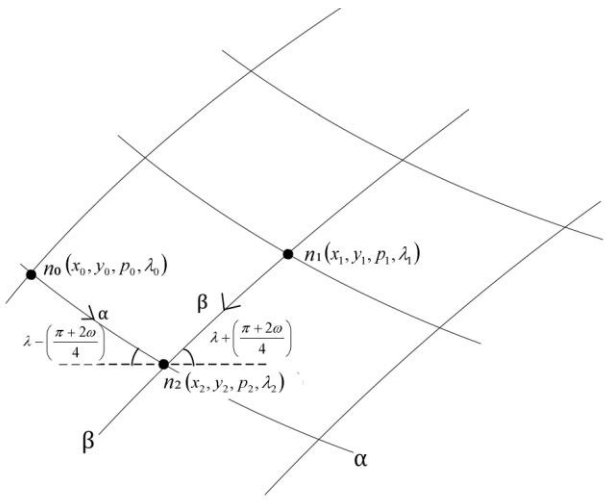

Fig 2: Riemman problem

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Thedeformationfield is determined uniquely whengiventwointersectingcharacteristiclines. Let an α-characteristic line passed through n_0n_2 and β-characteristic line passed through n_1n_2. The position of (x, y) and the stress elements (p, λ) at points n_0, n_1, and n_2 are denoted by $(x_0, y_0, p_0, \lambda_0)$, $(x_1, y_1, p_1, \lambda_1)$, and $(x_2, y_2, p_2, \lambda_2)$ respectively. From the discretization of (5), the approximated solution at point n_2 $(x_2, y_2, p_2, \lambda_2)$ were given by

$$x_2 = \frac{x_1 \tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right] - x_0 \tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right] + y_0 - y_1}{\tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right] - \tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right]} \quad (7)$$

$$y_2 = \frac{(x_1 - x_0)\tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right]\tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right]}{\tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right] - \tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right]} + \frac{y_0 \tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right] - y_1 \tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right]}{\tan\left[\lambda_\beta + \left(\frac{\pi+2\omega}{4}\right)\right] - \tan\left[\lambda_\alpha - \left(\frac{\pi+2\omega}{4}\right)\right]} \quad (8)$$

where

$$\lambda_\alpha = \frac{1}{2}(\lambda_0 + \lambda_2), \lambda_\beta = \frac{1}{2}(\lambda_1 + \lambda_2) \quad (9)$$

at the α-characteristic line n_0n_2 and β-characteristic line n_1n_2 respectively.

The following approximated equations has been made in order to determine the approximation solution for (p_2, λ_2),

$$\partial D_\alpha = D_0^\alpha - D_2^\alpha, \ \partial D_\beta = D_1^\beta - D_2^\beta \quad (10)$$

where ∂D_α is the distance between n_0 and n_2 while ∂D_β is the distance between n_1 and n_2. We therefore have the following approximations from (6),

$$\frac{\partial p}{\partial D_\alpha} = \frac{p_0 - p_2}{D_0^\alpha - D_2^\alpha}, \frac{\partial p}{\partial D_\beta} = \frac{p_1 - p_2}{D_1^\beta - D_2^\beta},$$
$$\frac{\partial \lambda}{\partial D_\alpha} = \frac{\lambda_0 - \lambda_2}{D_0^\alpha - D_2^\alpha}, \frac{\partial \lambda}{\partial D_\beta} = \frac{\lambda_1 - \lambda_2}{D_1^\beta - D_2^\beta} \quad (11)$$

Theequations for stress variables θ_2 and p_2 can therefore be obtainedas,

$$\lambda_2 = \frac{(\lambda_0 + \lambda_2)\cos\omega + 2q_\alpha\lambda_0 + 2q_\beta\lambda_1}{2q_\alpha + 2q_\beta}$$ along α-characteristic line and (12)

$$p_2 = \frac{p_0\cos\omega + 2q_\alpha(\lambda_0 - \lambda_2)}{\cos\omega}$$ along β - characteristic line, (13)

where

$$q_\alpha = \frac{1}{2}(p_0 + p_2)\sin\omega + c\cos\omega$$
$$q_\beta = \frac{1}{2}(p_1 + p_2)\sin\omega + c\cos\omega \quad (14)$$

The initial approximated value for p_2 and λ_2 represented by p_2^0 and λ_2^0 respectively were given as

$$p_2^0 = \frac{1}{2}(p_0 + p_1), \ \lambda_2^0 = \frac{1}{2}(\lambda_0 + \lambda_1) \quad (15)$$

Special Case (Riemman Problem)

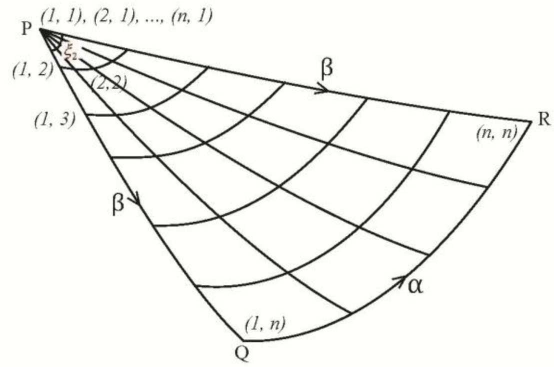

Fig 3: Fan shaped region

This elementary boundary value problem is a special case of the Riemann Problem when one of the characteristic line degenerates to a single point, namely singular point and forms a fan shaped region as shown in Fig. 3, namely centered fan field region.In this case, we considered an α

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-characteristic line to degenerates to a single point. In region PQR, the β -characteristic lines are straight and all meet at a point, P, and the angle of QPR is denoted by ξ_2 . At the singular point, P, stress variables λ and p are discontinuous and at each point along β -characteristic line, PQ these values are known. Suppose that the degenerated α -characteristic is divided into n points which are denoted by $(1,1),(2,1),(3,1),\ldots,(n,1)$. The approximated value of λ and p at point P were given by

$$\lambda_{(i,1)} = \lambda_{(i-1,1)} + \frac{\xi_2}{n-1} \tag{18}$$

$$p_{(i,1)} = \frac{p_{(i-1,1)}(\cos\omega - \sin\omega)(\lambda_{(i,1)} - \lambda_{(i-1,1)}) + 2c\cos\omega}{(\cos\omega + \sin\omega)(\lambda_{(i,1)} - \lambda_{(i-1,1)})} + \frac{2c\cos\omega(\lambda_{(i,1)} - \lambda_{(i-1,1)})}{(\cos\omega + \sin\omega)(\lambda_{(i,1)} - \lambda_{(i-1,1)})} \tag{19}$$

where $i = 1,2,3,\ldots,n$. Then, from the known values along QR and at a singular point,P, the stress distributionthroughout the region PQR may be defined.

IV. THE CONSTRUCTION OF THE STRESS FIELD

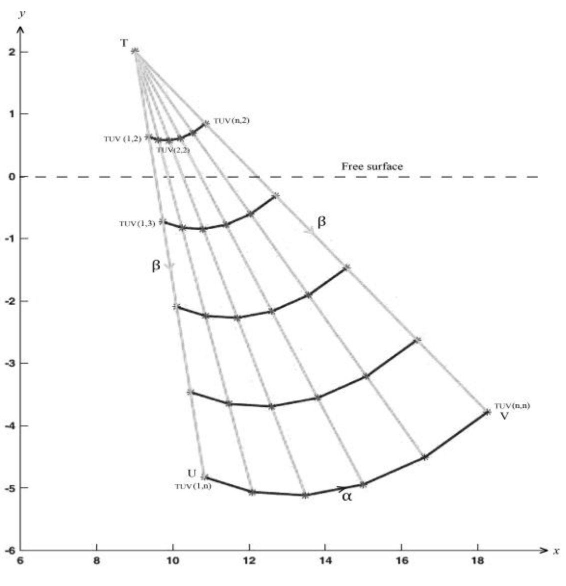

Fig 4: Centered fan field in deformation region

Let $TUV(i,1)$ where $i = 1,2,3,\ldots,n$ denoted the initial n points at T as illustrated in Fig. 4. The coordinates $(x,y)$of all initial points are identical to the initial point $TUV(1,1)$ with distinct values of pressure, p and angle λ . The initial value of p and angle λ at T are known from(18) and (19). Now, the point $TUV(2,1)$ and adjacent point $TUV(1,2)$ is considered. Let the α -characteristic and β -characteristic passed through $TUV(1,2)$ and $TUV(2,1)$ respectively intersect at $TUV(2,2)$. From the known values at point $TUV(1,2)$ and $TUV(2,1)$, the coordinates (x,y) and the stress variables (p,λ) at point $TUV(2,2)$ were obtained. Then, the construction continues to the next point $TUV(3,2)$. Repeating the steps mentioned in section III, the stress distributionare defined throughout the region TUV and the construction of the stress field is shown in Fig. 4.

V. RESULTS AND CONCLUSION

In this research, the wedge semi-angle, is taken as $\xi_3 = 45^\circ$, granular material frictional angle, $\lambda = 30^\circ$, and the cohesion, $c = 2$. For a region TUV in Fig. 4, the calculated values of p and λ were then compared to the solution given by [6] that was solve analytically. The comparison is shown in Fig. 5.

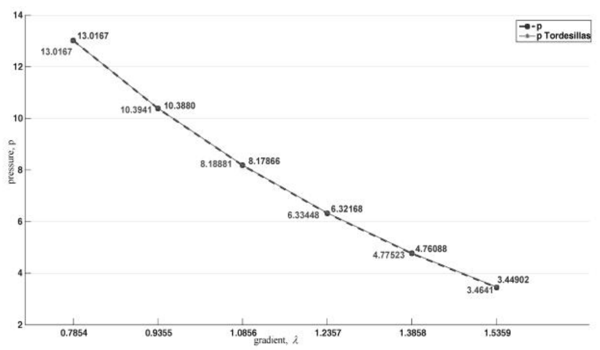

Fig 5: Comparison of the value p versus λ with T or desillas's analytical method

As shown in Fig. 5, the results show a very small differences between the analytical and the numerical methods. To conclude, the numerical method for the construction of the stress distribution in the granular materials deformation region under the wedge indentation found in this

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study is convenient to beused for the related field.

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