

Bandwidth Efficient Digital Image Watermarking Scheme Using a Concatenation of Three Transforms

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Abstract

Digital watermarking is a preliminary task of ingraining data into digital forms such as visual , audio-visual as well as audible data which helps to identify possession of those data. This technique is useful for copyright protection, tamper-resistant, theft-intimidation etc. it should be fixed in such a way that naked eye cannot bring into the mind & hard to eliminate, as growth of technologies in the digital side is becoming wider and wider chances of clone copies is added. The trending research field in current times is ensuring the penetration of data so that minor party cannot hold it. This paper covers an algorithm for black and white (binary) digital image watermarking mechanism based upon discrete wavelet transform, singular value decomposition and DCST. In this proposed watermarking algorithm, the proprietary image is compressed in the beginning and then watermarking process is carried out. The required bandwidth and storage capacity for image data will be reduced during transmission due to the applied compression before watermarking. The sturdiness of both host image as well as watermark image is tested along with DCST using respective performance measures. Finally, the performance of DCST based watermarking implementation has been compared with existing algorithm based on DOST. Comparative analysis has been carried out in presence of white Gaussian noise attack. The results show that, the performance and achievements of the presented algorithm is superior in comparison to the extant DOST based algorithm.

Keywords; Digital Watermarking , DOST , DCST , DWT , SVD.

I. INTRODUCTION

The need of proficient systems is on a splurge with the advancements in technology for transmission of data. The images transmitted henceforth are at a high risk of being forged nowadays and so it becomes a necessity to add a watermark to them to mark their true ownership. Moreover, it is quite obvious to say that the present copyright laws don't suffice to ensure to deal with security of digital data [1]. For these reasons watermarking has been a subject of attention in recent researches.

Digital watermarking is a task of enclosing the data on a carrier signal in a way that prevents the end user to observe it unless it is extracted digitally. So, this enables the user to prove his authenticity

and his image's uniqueness in case of its ownership being claimed by other party. A concept as simple as this can go a long way in corporate businesses where the competition is all about releasing new patents and henceforth, remaining the top leaders in their respective industries.

The salient features of a watermark are Robustness, Imperceptibility and Security. Robustness guarantees its endurance against numerous signal

Processing and plenty of operations like cropping, transformation, compression, etc. Imperceptibility denotes that the digitally marked image should be virtually tantamount to the source image from the user's perspective. Security implies that an unauthorised pupil may not be able to retrieve and

alter the embedded watermark. Making a developed compromise between the capacity, robustness and imperceptibility parameters is the core motivation for watermarking schemes.

The primary domain wherein the numerous watermarking methodologies are being classified are spatial domains and transform domains[2]. The spatial domain method refers to a procedure in which the pixels are modified straightaway in order to hide the digital mark, whereas in case of domain-based methods, the pixels are modified directly, but rather the transform coefficients are altered in order to hide the watermark bits. Examples of those kind includes Discrete Cosine-function Transform, Discrete Wavelet Transform (DWT) and Singular Value Decomposition (SVD).

In the hybrid watermarking, Discrete Wavelet Transform and Singular Value Decomposition are usually put into use wherever the watermark is embedded into the singular values of cover images in the wavelet sub bands. A entirely new category of watermarking procedures has been studied which performs the function of inserting a visible pattern into structural domain and an invisible logo into altered domain and henceforth it satisfies both imperceptibility and robustness requirements [3].

The primary focus of this paper is on the unnoticed watermarking techniques based on the SVD method in mathematics based upon linear algebra and applied into the divisibility criterion of a real or a complex matrix, with some noteworthy implications in the field of image processing. The usage of SVD is implied twice in here and numerous attacks are carried out to show the robustness of the embedding methods used.

II. PRILIMNARIES

Discrete Wavelet Transform (DWT)

Its an analytical tool used for stratified breaking down of an image. It appears to be the most beneficial in conversion of the mobility signals like images, videos and audio signals[4]. The DWT is an

imminent advancement over the DCT in a way that this method investigates the signal at numerous resolutions, which means it puts forth the images at multiple resolutions during their presentation. The image is basically bifurcated into two portions – one with a high frequency and another with a lower one[4] . In two dimensional detachable bilateral DWT, each level of disintegration generates basically four fillets of data, one equivalent to a low pass fillet(LL), and three other resembling to the horizontal(HL) , vertical(LH) , and diagonal (HH) high pass bands. Attributing to its phenomenal contiguous dimensional-frequency localisation characteristics, the DWT is very well suited in determining the domains in the host image, whereby a watermark can be injected effectively [5]. Henceforth, the two dimensional progressive DWT of an image $g(x,y)$ of size $A \times B$ can be generalised as in statements (1) and (2) [6] .

$$W_{\phi}(j_0, m, n) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(x, y) \phi_{j_0, m, n}(x, y) \quad (1)$$

$$W_{\psi}^i(j_0, m, n) = \frac{1}{\sqrt{mn}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \psi_{j_0, m, n}^i(x, y), i = (H, V, D) \quad (2)$$

Where j, m, n, M, N are integers, $I = (H, V, D)$, j_0 annotates a sensible beginning scale $j_0 = 0$, n indices sequenced from $0 \dots N-1$, m indices go from $0 \dots M-1$, $W_{\phi}(j_0, m, n)$ depicts comparative coefficients of $f(x,y)$ at scale j_0 , $W_{\psi}^i(j, m, n)$ represents horizontal , upright and oblique details coefficients at scales

$$j \geq j_0 \quad A = B = 2^j, \text{ for } j = 0, 1, 2, \dots, j-1$$

$a, b = 0, 1, 2, \dots, 2^j - 1$. The portrait $g(x, y)$ can be regained from a 2-D inverse DWT of W_{ϕ} and W_{ψ} as in Eqn. (3) [6] .

$$f(x, y) = \frac{1}{\sqrt{mn}} \sum_m \sum_n \omega_{\phi}(j_0, m, n) \phi_{j_0, m, n}(x, y) + \frac{1}{\sqrt{mn}} \sum_{i=H, V, D} \sum_m \sum_n \omega_{\psi}^i(j, m, n) \psi_{j, m, n}^i(x, y) \quad (3)$$

Singular Value Decomposition

This basically is a method to that breaks down a matrix into its fractional items which enables faster calculations on that matrix for further uses in the future.

It is a matrix conversion in a linear fashion which works on compressing images and divides images. Using this decomposition, the matrix consisting of the original image data is bifurcated and represented as a product of three different matrices, namely S, V and D.

The SVD works by the breakage of a single matrix into an orthonormal one and another one consisting of all the singular values of the original matrix known as the diagonal matrix [7]. It is an essential equipment in the field of computer vision as a decomposition matrix valued for picture alterations. [8]. The linear decomposition of a picture G with dimensions a x b can be specified as in Eqn. -

$$G=USV^T \quad (4)$$

Where U and V are orthonormal matrices, also signifying the left and right peculiar vectors of the image. S = diag(λ_i) is a orthogonal matrix of the unprecedented values λ_i , $i=1, \dots, m$ which are formulated in shrinking heirarchy. They're depicted as [7]:

$$S = \begin{bmatrix} S1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & Sn \end{bmatrix} \quad (5)$$

DOST (Discrete Orthonormal Stockwell Transform)

The Stockwell transform (ST) imported in [9], is basically a time frequency decomposition that accommodates precisely assigned frequency and phase data of a function (the phase is accredited with respect to time $T=0+$. Provided a function $h(t)$ backed over the real and a linear line R, its ST is specified as follows,

$$\mathcal{H}(\Gamma, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) \exp\left[i\pi\left(\frac{1}{2}(t - \Gamma)^2 f^2\right) - i2\pi ft\right] dt \quad (6)$$

where f is the frequency and t and Γ are both time (or spatial) variables. When put-up against the conventional demarcation of the Fourier Transform (FT) of h,

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-i2\pi ft) dt \quad (7)$$

Its observed that ST is a sort of a windowed Fourier transform, wherein the width of the (normalized) window, centred at Γ , is inversely proportional to the frequency f. The ST is a deeply superfluous depiction of a function since the FT of the function may be retrieved by integration over the Γ -components, i.e.

$$\int_{-\infty}^{\infty} \mathcal{H}(\Gamma, f) d\Gamma = H(f) \quad (8)$$

Hence, Stockwell may be thought of as a definite transform that paves the way for Fourier transforms and wavelets. It is implemented in a multitude of applications, but chiefly so, in signal processing. Also, considerate iterations of ST have been created but they typically suffer from the high proximity of the unceasing transforms mentioned in above. For Images and extended signals, this is much more than what the PCs can accumulate in the RAM.

The DOST and its modifications can all be considered as some minor and otherwise major variations of the Fourier transformations. Every Iteration of this brings with it some modifications pertaining to the similarities in the conjugate coefficients or more so, to the basis functions.

To establish how transforms are combined to form the DOST, we essentially follow the same route as Wang and Orchard [9] in their inception of the fast DOST algorithm. Examine the following basis vector as defined by Stockwell [4], being constructed as a sum of Fourier basis vectors which are relocated in time and then phase-reformed:

$$S_{[V, \beta, \tau]}[kT] = \frac{ie^{-i\pi\tau} \left\{ e^{-i2\pi(k/N - \tau/\beta)(v - \beta/2 - 1/2)} - e^{-i2\pi(k/N - \tau/\beta)(v + \beta/2 - 1/2)} \right\}}{\sqrt{\beta} \cdot 2\sin[\pi(k/n - \tau/\beta)]} \quad (9)$$

Because the DOST is an orthogonal transform, each coefficient is procured by taking an inner product of the signal vector with a basis vector.

The DOST is comparable to the general Fourier-family transform functions(GFT) presented in [4] , leaving out an angular special squared window instead of a truncated Gaussian. This is also the fast algorithm derived in [9] , and as soberved in [7] , it can simply be altered to generate the associated symmetric Discreet Orthonormal Transform applied in here[10].

DCST(Discreet Cosine Stockwell Transform)

Taking into account the DOST as introduced above, it can also be improved by following a slightly different approach to it by applying varied transforms other than the Fast Fourier ones. The Discreet Cosine Transform is touted to have a wide variety of applications throughout the domain of image processing. It's a contiguously differentiable image, which makes it surprisingly good at the tasks of image compression and purity checking. Its affinity to the Fourier Transforms compels to an effortless implication of the DOST algorithm. A DCT on the basis of DOST algorithm(DCST) may be designated by directly substituting the DFT with a DCT, as follows:

$$DCST = (\bigoplus_{k=1}^{i-1} DCT_{ni}^{-1})DCT(10)$$

When the DCT is used, all frequencies are deemed as greater than zero. As a result, high band frequencies are required, so the bifurcation in the frequency field has to be accommodated. The imminent choice left out is to keep up with the dyadic partitioning as before. Provided a signal of strength 2N, the widths of the frequency partitions can be defined as follows:

$$n_1 = 1$$

$$n_i = 2^{i-2}, 2 \leq i \leq N - 1$$

Other implications of the DCST include the magnitude of the basis functions, how a phase

correction might be implemented, and separability for the higher – dimensional transform.

Alpha Blending

In digital image watermarking, alpha fusion procedure can be implied by encasing the watermark into a host image and then extorting it out from the modified / altered image by the use of below equations. The embedding formula is given by [11]

$$S_{a(i,j) \text{ embedded}} = S_a I_a + S_b e I_b \alpha(11)$$

Where $S_{a(i,j) \text{ embedded}}$ depicts the embedded image, $S_a I_a$ depicts the diagonal element of the cover image and depicts the singular values , $S_b e I_b$ depicts the oblique element of the watermarked image and signifies the singular values of the image and α depicts the value of escalating factor. The recovered watermark equation is given by,

$$S_{br_encrypted} = \frac{S_w I_w(i,j) - S_a I_a(i,j)}{\alpha} (12)$$

Noise Attacks

An image noise, simply stated , is an arbitrary shift in the light intensity or the visual spectrum data in the images apprehended. It is the distortion created in the image signals caused by the sources well beyond the image processing's proximity[12]. Images wherein the small noise altercations do eventually grow out exponentially have the aspect that well lit areas generate more noise amplitudes. But in more frequent cases, it is generally additive. A noisy image can be fairly easily modelled as

$$Z(p,q) = I(p,q) + R(p,q) (13)$$

Where, $Z(p,q)$ is noisy image, $I(p,q)$ = function of image noise, $R(p,q)$ = function of the source image. The primary contributors of an image noise are –

- 1) The transfer from one location to another via electronic means.
- 2) Incurrence of sensor heat while capturing the image.

- 3) With variability of the ISO factor with the capability of the lens to absorb light.

White Gaussian Noise

Gaussian Noise is a demographic noise function having a probability density function akin to the normal distribution, also known as the Gaussian distribution. Random Gaussian function is attached to the image function to generate this noise. The parameter inherently governing its magnitude is the Standard Deviation. Noise Magnitude is directly proportional to the Sigma (Standard Deviation) value. It is also known as electronic noise since it originates from amplifiers or signal detectors.

Additive Gaussian noising is a technique of supplementing a typical gaussian noise purposefully to exploit(corrupt) an image, henceforth hindering with its appearance and quality.[13] And so , the White Gaussian Noise is observed to be additive in nature . It has the tag ‘white’ associated with it simply because of the technique that it possesses the continuity to stay operational across the frequency band of the image onto which its typically applied. Moreover, the additive property implicates that it has the tendency to mimic the arbitrary ongoing processes of the nature. It very well serves the function of operating on the double values that are predominantly dealt with in the improved algorithm.

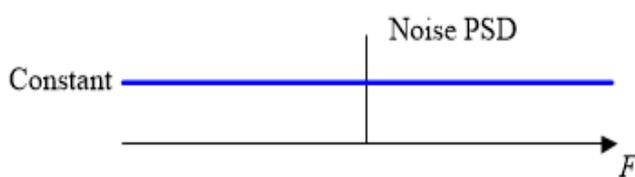


Figure1 : White gaussian noise function

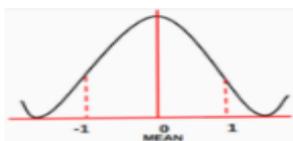


Figure2:Chart for PSD

III. PROPOSED METHODS

The primary aim is to first insert into watermark a different image by altering the Lower Intricate coefficients of a multi-res. image [19, 20]. Lower intricate presents refer to the spaces not inculcating the edges and the borders of images, where the frequency is typically lower and embedding of the watermark at these places is very much beneficial against the attacks dealt in here. The same logic suffices for the other attacks too wherein the lower frequency modifications are carried out in the lower frequency regions to minimise the impact of noise attacks on it. The primary image is decomposed into two levels - the sub-bands LH2, HL2, and HH2 are disregarded and all the operations are solely carried out on the LL2 fillet. It is, however to be observed that all the components of the watermark are injected in completeness and no data is left yet to be inserted . The adopted method for the embedding procedure is given below.

EMBEDDING :

1. Host image (Fig. 3 , Fig. 16) is converted into a double type data and DCST compression is then applied to it.
2. The Discreet Wavelet Transformation(DWT) (2 Dimensional) is carried out twice(upto the second level) onto the compressed host image rendering LL2 , HL2, LH2 and HH2 sub bands.
3. As mentioned earlier, the LL2 sub band is chosen for the proceeding operations that are to be performed on the image obtained in Step2.
4. The image obtained in step 3 is resized to fit in for the application of SVD onto it. The SVD is then applied onto it.
5. The watermark image is read and resize (the same procedure that was done on the host image as mentioned in the previous step).
6. Mat2gray function is applied onto the watermark image to predominantly set all the values of the

image matrix in the range of 0 to 1(a grey scale image is hence procured).

7. Watermark image converted to double type, SVD is performed onto it for the final embedding process.
8. Watermark is embedded onto the host image, and sub bands are rebuilt (using SVD).

Additive white Gaussian noise is adjusted onto the processed image so as to test for its robustness and imperceptibility.

EXTRACTION:

1. Possibly corrupt watermarked image is read for analysis and extraction.(Fig. 3 , Fig. 16)
2. DCST decomposition is applied to it.
3. DCST coefficients are subdivided upto the second level(same as in embedding procedure)
4. Singular values obtained from the image using SVD.
5. Watermark is extracted using the alpha blending technique based upon equation 12.
6. Finally , inverse DCST and DWT are applied to procure the watermark from the pre-processed image.

Obtained watermarks and noisy images obtained from both the methods are then compared.

IV. EXPERIMENTAL RESULTS:-

In this section, the experimental observations of the proposed scheme are analysed. The proposed technique was applied in form of Matlab code and the performance is evaluated using the correlations coefficients of the obtained images after the white Gaussian attacks. The watermark retrieval was evaluated on the basis of the PSNR values obtained while extracting the watermarking from the watermark embedded image. Cameraman and Lena are adopted as the cover images and Wikipedia image is adopted as the image to be watermarked into the aforementioned images. The test is carried

out using the various SNR values ranging from 0 till 100 and the output images are compared at the intervals of 10 units (SNR). The PSNR values of the extracted watermarks are noted and the results are represented in graphical formats.

CAMERAMAN IMAGE:



Fig. 3 Host image Cameraman and watermark image

PSNR AND CORRELATION CHARTS :

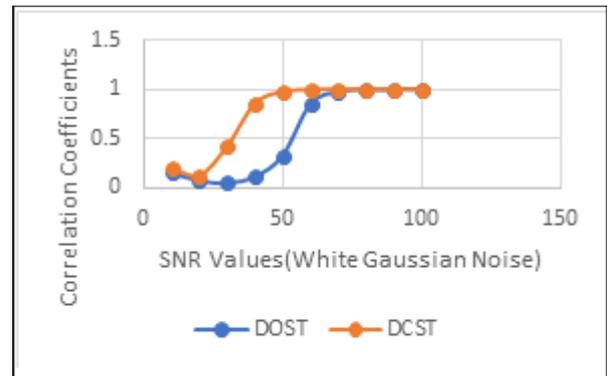


Fig. 4 Correlation DOST_DCST Cameraman

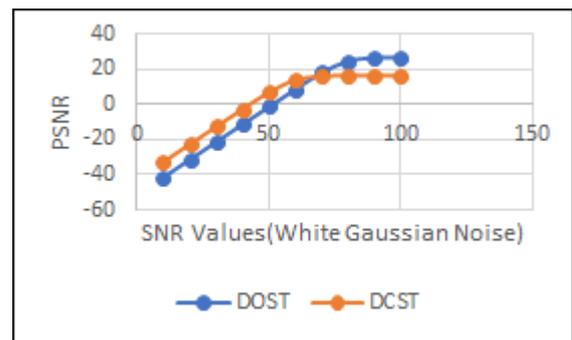


Fig. 5 PSNR DOST_DCST Cameraman

Correlation coefficients and PSNR comparisons for Altered host images and Watermarks (For varied SNR values) :

SNR – 10

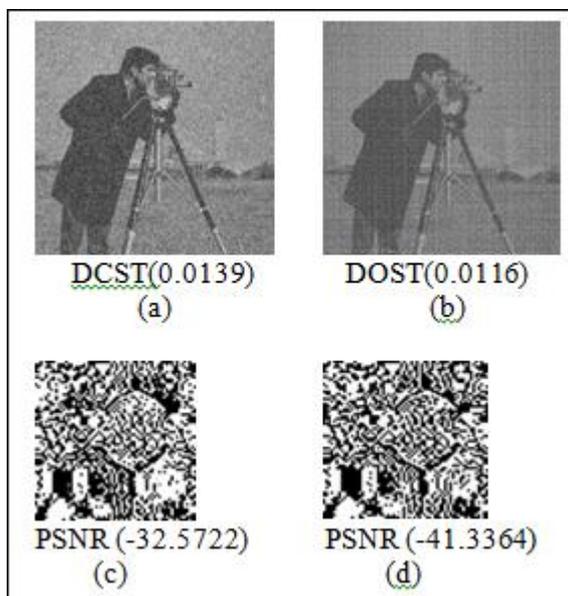


Fig. 6 Cameraman image with SNR 10

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 10 indicates the clarity of DCST extracted image (a) is better than that from DOST image (b) of Fig. 6. The extracted watermarks of both (a) and (b) are unrecognizable given the smaller SNR value of 10. As is visible from Fig.6 neither of the two watermarked images (c) and (d) reveal anything about the true identity of the original watermark. But its to be noted by looking at the images (a) and (b) , the DCST based images may come out to be significantly better than those the DOST extracted ones. This pattern will prevail soon enough and will be visible to many of the varied readings of the SNR values.

SNR – 40



Fig. 9 Lenna image with SNR 40

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 40 indicates quite a bit of rise in the brightness of (a) and the image (b) appears to be more crisp and clearer than before. The DCST based extracted watermark has already started to gain visibility while (d) is still very lightly visible. As observed from Fig.9 (c) is showing some meagre signs of the original watermarks while (d) is sill very much corrupted and its hardly possible to make out anything from it yet. A pattern , as discussed in the previous findings , can be made out that the watermarked images taken out by the help of DCST transforms are significantly better than those of the DOST transforms.

SNR – 70

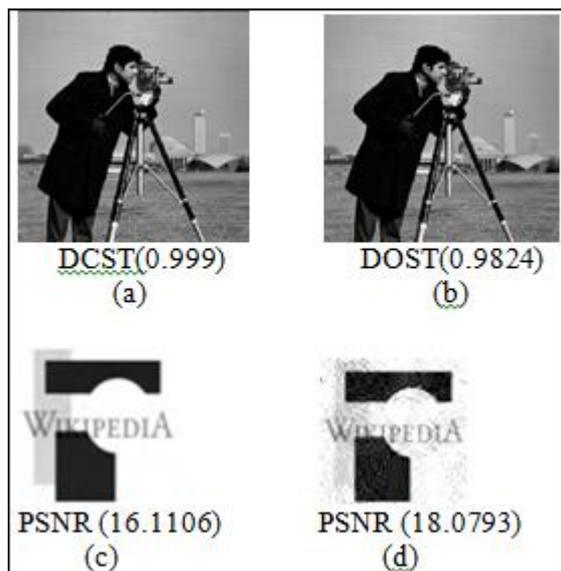


Fig. 12 Cameraman image with SNR 70

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 70 indicates that (a) and (b) have almost equalled in terms of perception by a normal human eye. DCST watermark has become completely visible while (b) is now almost visible, albeit some inherent noise present in it. Looking closely at the other two images of the Fig.12((c) and (d)), There's a bit of a grainy texture that's visible in the image obtained from DOST while those from the DCST is very much close to the original watermark. As is the case with most of previous images, the images treated with the DCST are much inferior and much better than those of the DOST transforms.

SNR – 100

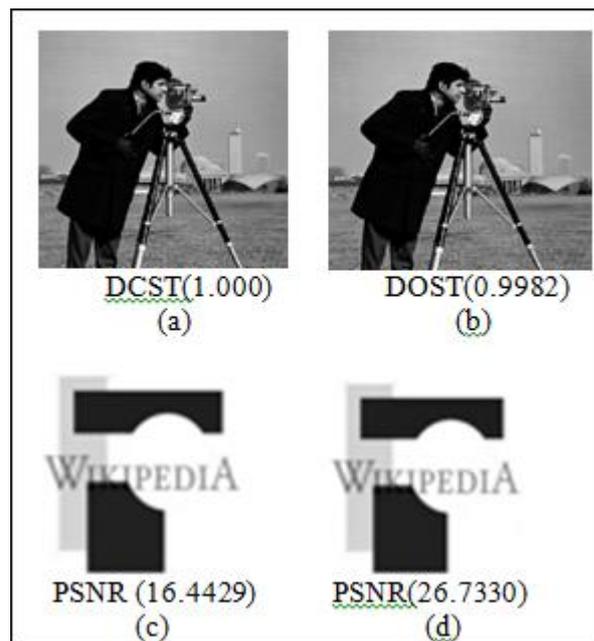


Fig. 15 Cameraman image with SNR 100

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

Based upon the above observations, it can be inferred that it took a considerable amount of time for the values of the DOST based images to shoot up in crispness and quality – at around 60 units – a time quite a bit behind than the same images procured from the DCST based algorithm. In Fig.15 although it is now evident that both the images finally come out to be the same at the SNR values nearby 100, but it won't always be the case that the signals are received and transmitted completely and many a times there may be someone or the interfering with images trying to corrupt it knowingly or unknowingly. So the most important thing the DCST surely held up to was that it was consistent in showing quite a good amount of results even in the lower spectrum of the SNR scale (from 10 to 100)

LENNA IMAGE :



Fig. 16 Host image Lena and watermark image

PSNR and CORRELATION Comparisons :

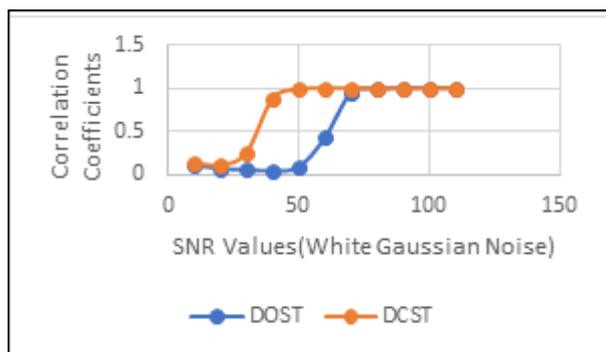


Fig. 17 Correlation DOST_DCST Lenna

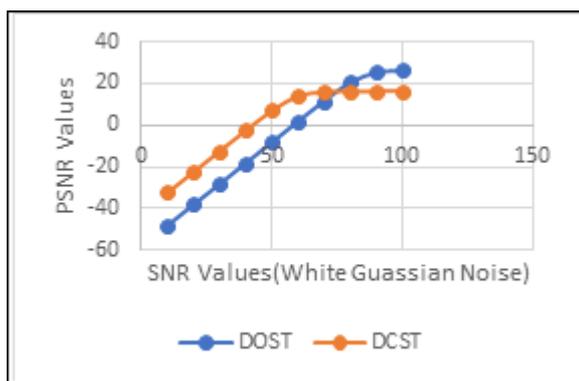


Fig. 18 PSNR DOST_DCST Lenna

Correlation coefficients and PSNR comparisons for altered host images and watermarks (For varied SNR values) :

SNR – 10



Fig. 19 Lenna image with SNR 10

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 10 indicates some severe visibility hindrances in case of (a) while the (b) is quite visible ,albeit with some prominent noise in it. Watermark in both the cases are equal in the sense that the receiver can't make out any conclusions as yet from them. As is visible from Fig.19 neither of the two watermarked images (c) and (d) reveal anything about the true identity of the original watermark. But its to be noted by looking at the images (a) and (b) , the DCST based images may come out to be significantly better than those the DOST extracted ones. This pattern will prevail soon enough and will be visible to many of the varied readings of the SNR values.

SNR - 40

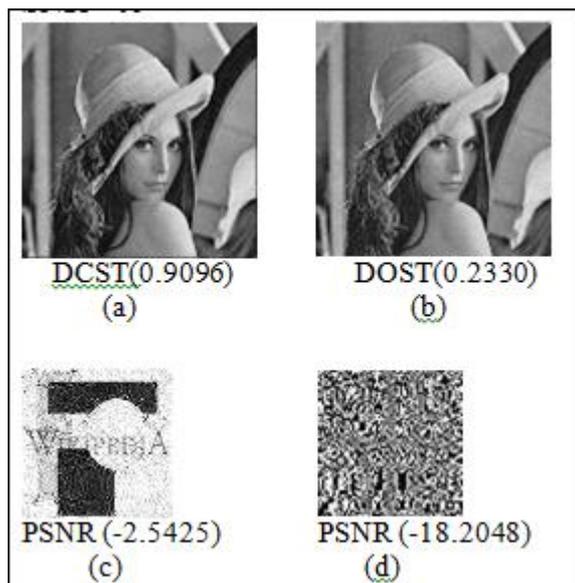


Fig. 22 Lenna image with SNR 40

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 40 indicates a slight increase in image (a) and a step increase in the sharpness of (b)

A significant increase in the quality of the watermark (c). That said, image (d) still doesn't reveal enough for the user to make out any information. As observed from Fig.22 (c) is showing some meagre signs of the original watermarks while (d) is still very much corrupted and it's hardly possible to make out anything from it yet. A pattern, as discussed in the previous finding (SNR 10), can be made out that the watermarked images taken out by the help of DCST transforms are significantly better than those of the DOST transforms.

SNR – 70

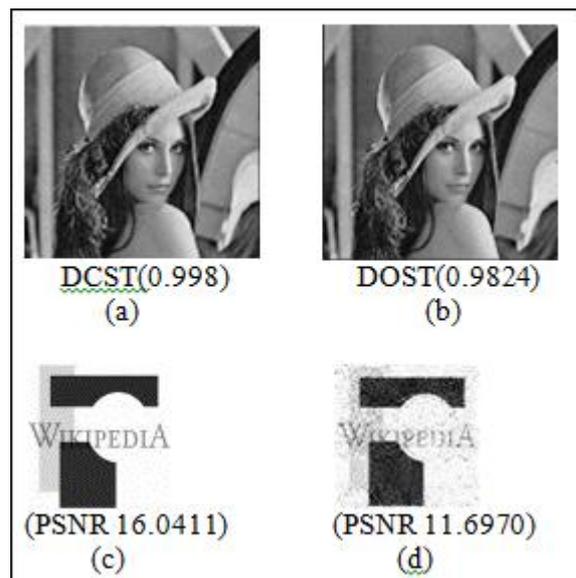


Fig. 25 Lenna image with SNR 70

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

SNR 70 indicates the same observations found in the previous SNR(60) reading.

Increased visibility of watermarks (c) and (d) now exposes the inner contents of the watermark properly (Similar to the SNR 40 watermark based on DCST). Looking closely at the other two images of the Fig.25((c) and (d)), there's a bit of a grainy texture that's visible in the image obtained from DOST while those from the DCST is very much close to the original watermark. As is the case with most of previous images, the images treated with the DCST are much inferior and much better than those of the DOST transforms.

SNR – 100

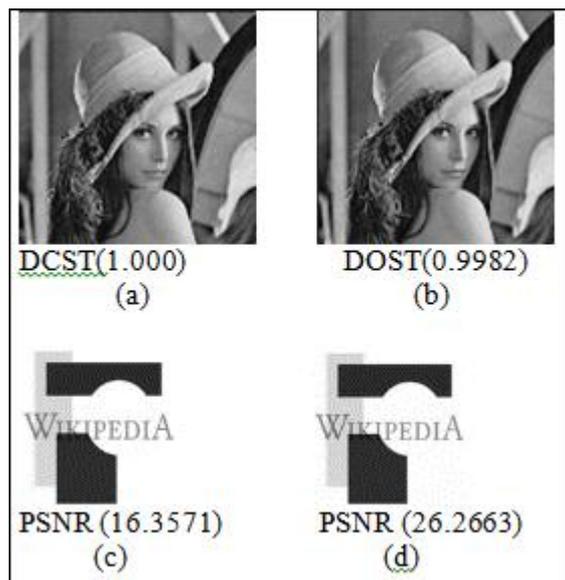


Fig. 28 Lenna image with SNR 100

(a) and (b) - Noisy watermarked images

(c) and (d) - Extracted watermark images

Based upon the above observations, it can be inferred that it took a considerable amount of time for the values of the DOST based images to shoot up in crispness and quality – at around 70 units – a time quite a bit behind than the same images procured from the DCST based algorithm. In Fig.28 although it is now evident that both the images finally come out to be the same at the SNR values nearby 100, but it won't always be the case that the signals are received and transmitted completely and many a times there may be someone or the other interfering with images trying to corrupt it knowingly or unknowingly. So the most important thing the DCST surely held up to was that it was consistent in showing quite a good amount of results even in the lower spectrum of the SNR scale (from 10 to 100).

V. CONCLUSION

In this paper, a digital watermarking technique based on DOST, DWT and SVD using alpha blending technique has been formulated. It basically is an invisible watermarking technique in which a binary image is utilized as watermark data and its pixels are embedded invisibly into the host image which is essentially a grey scale image of the same

dimensions. A thorough analysis of two watermarking schemes – DOST and DCST was carried out by taking into account two images – Cameraman and Lenna which act as host images in the analysis. They are then watermarked into by a single watermark image and the parameters like PSNR value of the extracted watermark and the correlation coefficients are used to compare the dynamic efficiency of these schemes. After testing them on a set of varied SNR values, a conclusion was reached upon, indicating to the observations that the proposed method based on Discrete Cosine Wavelet Transform, despite being on par with the existing DCST algorithm at the compression rates, was significantly better in retaining the watermark as well as rendering it more robustness and imperceptibility during the embedding and extraction processes.

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