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Multifractal Analysis of Time-Varying Market Efficiency: Implications for Adaptive Market Hypothesis

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Abstract

This paper investigates the implications of the Adaptive Market Hypothesis (AMH) by studying the nature of cross-correlation between price and volume and assesses whether the price-volume relationship has a long memory. The MultifractalDetrended Fluctuation Analysis (MF-DFA) is used to capture the behavior of price and volume series whereas the MultifractalDetrended Cross-correlation Analysis (MF-DCCA) is used to capture the behavior of price-volume cross correlation.

The findings of this paper assert that the Indian Stock Market is not efficient and provide opportunities for arbitrage from time-to-time. The small fluctuations in price are most likely to be followed by another small increment or vice versa, whereas large (small) fluctuations are followed by small (large) fluctuations. Similarly, a large (small) increment in volume is most likely followed by a small (large) increment in volume. A large (small) increment in volume is most likely to be followed by a large (small) increment in price. These findings have practical implications for traders and investors.

Keywords; *Market efficiency; price-volume; efficient market hypothesis; adaptive market hypothesis; multifractality; long memory*

I. INTRODUCTION

According to the Efficient Market Hypothesis (EMH), asset prices reflect the fundamental value of an asset by incorporating relevant information (See Fama (1970, 1991)). Thus, in an informationally-efficient markets it is hard to achieve above average profits as every investor has an access to all the relevant information simultaneously. It implies that the prices must fluctuate randomly and therefore should not exhibit long memory. In real world, however, such an efficient market is not observed, and it is seen that many investors are able to beat the market and achieve above average profits. Proof of this phenomenon is demonstrated by showing that markets have long memory. Long-term memory indicates an auto-correlated series in which the autocorrelation function decays asymptotically hyperbolically. Such a series may exhibit the properties that are dependent on its past. Such a series allows for arbitrage and is against the tenets of an efficient market (Mandelbrot and Van Ness, 1968). Numerous studies (Cajueiro and Tabak, 2004, 2005; Di Matteo, Aste and Dacorogna, 2003, 2005; Eom, Oh and Kim, 2008) have discussed the connection between long-range dependence and possible efficiency, and have found in many cases that the under-developed markets exhibit long-term memory. Morales, Di Matteo, Gramatica and Aste (2012) and Kristoufek (2012) have described the association between the phases of markets and efficiency of the markets through time-varying Hurst exponent. The importance of the Hurst exponent in the development of the fractal theory to

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understand chaotic theory is acknowledged by its originator Mandelbrot in his paper (Refer Mandelbrot, 1982).

The evidence of long memory has also been provided by Mandelbrot (1971); Urquhart and McGroarty (2016); Bariviera (2017); Caporale, Gil-Alana and Plastun (2018). Lo (2004) explained this phenomenon through the behavioral aspect of the investors in that investors have bounded rationality and they adapt to the changing environment over time. Therefore, achieving the perfect market efficiency is not practically feasible. In the context of non-achievability of efficient markets, Lo (2004) introduced the concept of adaptive market hypothesis (AMH).

One of the implications of the AMH – time-varying efficiency – can be observed in a time series by observing the results from time-varying versions of the serial correlation tests, long memory tests, etc. However, these standard tests are unable to describe the multifractality in a series (Bacry, Delour and Muzy, 2001). These standard tests only allow one to test whether markets are efficient or inefficient in certain period. These tests therefore do not allow one to compare financial markets across different periods and/or markets.

Although Mandelbrot, Fisher and Calvet (1997) claimed that multifractal nature of the assets explains the most of the stylized facts of a time series, the research made way into this field after the work of Kantelhardt et al. (2002). Zunino, Zanin, Tabak, Pérez and Rosso (2009) and Zunino, Tabak, Figliola, Pérez, Garavaglia and Rosso (2008) have argued that there is an increasing empirical evidence for using multifractal analysis to capture the level of inefficiency of markets and highlighted the inverse relationship between level of development of stock markets and level of multifractality. Wang, Liu and Gu (2009) showed that as emerging markets become more efficient over time, the singularity width of the spectrum is narrowed. Thus, multifractality properties may be used to assess the level of market inefficiency. Since then some of the techniques that measure the multifractality of the series have been increasingly used in this domain.

MultifractalDetrended Fluctuation Analysis (MF-DFA) is a preferred method to test the long-memory in a univariate series (Zunino, Zanin, Tabak, Pérez and Rosso, 2009; Horta, Lagoa and Martins, 2014; Rizvi, Dewandaru, Bacha and Masih, 2014; Stošic, Stošic, Stošic and Stanley, 2015; Arshad and Rizvi, 2015; Ferreira, Dionísio and Movahed, 2017). After the seminal work by Podobnik, Horvatic, Petersen and Stanley (2009), a large number of investigations have been made into price-volume relationship. A multivariate version of the test, known as MF-DCCA (MultifractalDetrended Cross Correlation Analysis), measures the long-term cross-correlation between price and volume (Bolgorian and Gharli, 2011; Rak, Drozdz, Kwapien and Oswiecimka, 2015; Fan and Li, 2015; Yuan, Zhuang and Liu, 2012; El Alaoui, 2017). India has seen many developments in both the regulatory and economic environments of stock markets including changeover from open outcry system to screen based trading (SBT), introduction of DEMAT (dematerialization) trading, changes in settlement period of T + 2, and abolition of badla trading. Many of the regulatory changes and entry of FIIs in big way have impacted the way the trading took place in India. These microstructural changes have impacted the efficiency of markets in India (Shah, 1999). Given these phenomenal changes in the Indian Stock market in past three decades, it is reasonable to assume that the price-volume relationship in Indian stock market too would have changed over time reflecting the change in the market efficiency.

There are few studies that assess the market efficiency in India (Stošic, Stošic, Stošic and Stanley, 2015; Razdan, 2002; Manimaran, Panigrahi and Parikh, 2005; Ghosh, Manimaran and Panigrahi, 2011; Dutta, 2010; Hiremath and Kumari, 2014; Hiremath and Narayan, 2016) using different multifractal analysis of a single financial time series. However, barring a study by Hasan and Salim (2017) in price-volume relationship and a study by

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Dutta, Ghosh and Chatterjee (2016) in cross correlation between foreign exchange and stock price, there is a limited research in understanding the market efficiency through changing price-volume relationship. There is no paper conducting multifractal analysis of price-volume relationship done that links the results to the implications of the AMH. This paper is an attempt to establish the link between multifractality in price-volume relationship to the implications of the AMH. The findings of this paper will help in establishing the fact that markets are dynamic in nature and oscillate around the perfect efficiency. Secondly, the more a series is away from market efficiency, the more are the arbitrage opportunities. Such a series also carries more information that is in contradiction with perfect efficient markets. This is the motivation of this paper to investigate the dependence structure between price and volume over the period.

This paper uses the multifractal properties of the time series to test the dependence structure of the price-volume relationship with the following objectives in mind.

1. To test if there is a time-varying long-memory in the return series

2. To test if there is a time-varying long-term cross-correlation between the return series and volume series

The following Section 2 briefly covers the literature on the topic followed by Section 3 describing the MF-DFA and MF-DCCA methodology employed in this study. Section 4 explains the data and conducts an analysis. Section 5 elaborates the results and its implications are discussed in Section 6, and Section 7 concludes the paper.

II. LITERATURE REVIEW

Stock returns have been studied extensively in academic research and models such as CAPM have been developed to understand the dynamics of the returns. Illiquidity, a concept related to trading volume, has been added to this model only recently by Acharya and Pedersen (2005) to study the price equilibrium process. Blume, Easley and O'hara (1994) and Suominen (2001) have investigated the role of volume as carrier of information in financial markets. They suggest that prices do not contain all the information and volume is an important parameter that investors use to derive information. Suominen (2001) shows that investors extract information from the past volume for their investment strategies.

2.1. Price-volume relationship

According to Karpoff (1986, 1987), price-volume relationship is instrumental in understanding how the information flows to the market. Various studies corroborate the positive relationship between price and volume (Assogbavi, Khoury and Yourougou, 1995; Chen, Firth and Rui, 2001; Crouch, 1970; Epps and Epps, 1976; Karpoff, 1986, 1987). Different models describe the price-volume relationship. Copeland (1976), Jennings and Barry (1983), and Morse (1980) elaborate how information arrives sequentially. Clark (1973), Epps and Epps (1976), Lamoureux and Lastrapes (1990), Tauchen and Pitts (1983), and Harris (1987) have argued how price-volume connection can be expressed in terms of mixture of distributions. Admati and Pfleiderer (1988) and Kyle (1985) discuss how the information flows asymmetrically whereas Varian (1985), Harris and Raviv (1993) impact of information based on the difference of opinion on the interpretation of the information. On the other hand, Assogbavi et al. (1995) and Karpoff (1986, 1987) have discussed how asymmetrically volume responds to return due to varied expectations and costs of short selling. Henry and McKenzie (2006) have found asymmetrical bi-directional price-volume relationship.

2.2. Time-Varying Market Efficiency: An implication of the AMH

According to AMH, markets evolve through time and prices only reflect information such as business situations and level of profitable prospects. These business situations indicate the churn out

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competitors in the industry. Although AMH is qualitative in nature, it is verified on its practical implications for the market. The two implications of the AMH: (1) there will be arbitrage opportunities from time to time and (2) there will be time-varying relation between the risk and return can be empirically tested to prove the AMH. Such financial variables that provide arbitrage opportunity imply that they have long memory. The financial time series quantities having long memory are found to exhibit power-law correlations and multifractality (Kwapien et al., 2015; Wang et al., 2013; Yuan et al., 2012). Therefore, multifractal analysis of these financial quantities may reveal the existence of the relationship between these variables.

2.2.1 Multifractality in Prices

Cont (2001) discusses intermittence, a property of a time series having the oscillatory and heterogenic fluctuation, as one of the stylized facts. Such fluctuation over different time scales is found to be fractal in nature (Mandelbrot et al., 1997). To say that a univariate series has a long-term memory implies that the series has long term auto-correlations and thus the series can be predicted. If the series can be predicted, then it implies that markets are not efficient. Similarly, a multivariate series can have long-term cross correlations and it can be predicted if the series has long-term memory. As such highest multifractal properties in a series indicate lowest market efficiency (Han, Wang and Xu, 2019).

The time-varying Hurst effect observed in the fluctuations of a time series captures the long-term memory. On the other hand, Generalized Hurst Exponent approach was employed by Di Matteo et al. (2003) to study scaling properties of markets. Zunino et al. (2008) use multifractal approach (MF-DFA) to illustrate the phase of market development and Rizvi et al. (2014) to study stock market efficiency. Kristoufek and Vosvrda (2013) presented a new quantity of market efficiency using fractal dimension. If the series shows a time-varying dependence structure, then the series adheres to the implications of the AMH. This dependence structure has implications for the Adaptive Market Hypothesis.

The AMH has been tested in Indian stock market by Hiremath and Kumari (2014) using auto-correlation tests, and the test results show oscillating efficiency. Tiwari et al. (2019) employed MF-DFA which uses the Hurst exponent to describe the level of efficiency and find multifractal markets having long-term persistency. Hiremath and Narayan (2016) employed fixed and rolling window technique to find long-range dependence that changes over time but moving toward efficiency in case of Indian Markets. Anagnostidis et al. (2016) also used rolling window technique to find Eurozone stock prices are moving towards their means. Sensoy and Tabak (2015) established that stock markets have long-term memory which varies with time, while Horta et al. (2014) used MFDMA i.e. multifractaldetrended moving average to find existence of long memory during the crisis period. Wang, Liu and Gu (2009) and Cajueiro and Tabak (2004) found that markets are moving closer to efficiency. Tuyon and Ahmad (2016), Noda (2016), Urquhart and Hudson (2013) and Al-Khazali and Mirzaei (2017) have argued that efficiency varies with time and AMH explains the markets better.

2.2.2. Multifractality in Price-Volume relationship

Implications of the AMH are studied by Ferreira (2019), Hasan and Salim (2017), El Alaoui (2017), and Ruan et al. (2016) finding in general that price-volume relationship is multifractal in nature. Other studies finding the price-volume connection are those of Sukpitak and Hengpunya (2016), Wang et al. (2013), and He and Chen (2011). Podobnik et al. (2009), on the other hand, finds absence of any connection between price and volume. However, none of these studies link their results to adaptive market hypothesis.

There are no studies that use MF-DCCA and directly link the results of the studies in stock markets using price-volume relationship to the AMH. This paper bridges this gap by conducting a study using MF-DFA and MF-DCCA to test

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whether there is a long memory between price and volume and attempts to link the results to the implications of the AMH. The findings of this study will be important from the perspective of investors, traders, and regulators to know how the price-volume relationship changes over period in Indian stock market as it will determine their trading strategy in the market. If the market efficiency changes over period as measured by price volume relationship, then it is apparent that EMH does not hold over a period.

The following section describes the MF-DFA and MF-DCCA methods.

III. METHODOLOGY

A series is said to have scale invariant structure if it replicates itself over different time-segments of the series. Fractal analyses are used to determine a specific structure of series that doesn't change with scales chosen. This is achieved through the power law exponent, denoted by *H*. Mathematically, the series $X(t)$ exhibit this type of repeating structure when $X(ct) = c^H X(t)$. There are two kinds of these structures: Monofractal structures, for which power law exponent is constant and multifractal structures, for which power law exponent takes range of values.

Detrended Fluctuation Analysis (DFA) is used to assess the properties that define how the structure of the series changes over time and to capture the existence of long memory in the series. Kantelhardt et al. (2002) developed a multifractal version of the DFA, called multifractal-DFA (MF-DFA), using generalization of DFA that can be applied to a non-stationary time series that may have trend or cannot be normalized.

Study of long-range correlation tries to assess the statistical dependence between the points of a series as the time interval between them increases. Usually, there is a slow decay of statistical dependence with increasing time interval. This slow decay (slower than the exponential decay) follows a power law behavior, and this power law behavior can be captured in a statistical scaling exponent referred to as the Hurst exponent (Hurst, 1951).

Multifractal processes exhibit different scaling behavior of series over different time periods for small and large fluctuations. This scaling behavior is assessed through the hierarchy of scaling exponents. A multifractal series shows different scales at different time intervals and analysis of such a series requires the calculation of different scaling exponents (Kantelhardt et al., 2002).

The following section illustrates the procedure for calculating scaling exponents using MFDFA and MF-DCCA methods.

3.1. MF-DFA

Multifractal analysis helps to uncover nonlinear properties of a series. Computationally, conducting DFA or DMA of the time series gets the Hurst exponents. There are two ways to reveal a multifractal structure in a time series (1) generalize the concept of Hurst exponent and (2) construct a multifractal spectrum. The classical way to calculate multifractal spectrum is detailed out in Kantelhardt et al. (2002) and Peng et al. (1995) and is elaborated below. The classical way to calculate multifractal spectrum is detailed out in Kantelhardt et al. (2002), Peng et al. (1995) and Jiang et al. (2019). The methodology below is based on Jiang et al. (2019) and is elaborated below.

Step 1: As the DFA requires a random walk like time series, the time series is ensured to have the random walk like structure before employing DFA. Suppose that such a series is represented by $X(i)$, where i can take values from 1 through N.

Step 2: Calculate the detrended residuals. The Equation (1) calculates detrended residuals

$$\epsilon(i) = X(i) - \widehat{(X)}, \quad (1)$$

where $\hat{X}$ presents the local polynomial linear trend function.

Step 3: These residuals $\epsilon(\mathrm{i})$ are divided into $N_s = int[\frac{N}{s}]$ of equal size s, such that the segments

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do not overlap. The v^{th} segment is given as $S_v = \epsilon((v-1)s+j)$, where j can take values from *1* through *s*. Then using Equation 2, the local detrended fluctuation function, $F_v(s)$, is then calculated as the root mean square of the detrended residuals:

$$[F_v(s)]^2 = \frac{1}{s}\sum_{j=1}^{s}[\epsilon((v-1)s+j)]^2 \qquad (2)$$

• Step 4: In the next step, the fluctuations calculated as above are averaged over all the segments to get the q^{th} order overall detrended fluctuation as shown in Equation 3

$$[F_v(s)]^2 = \frac{1}{s}\sum_{j=1}^{s}[\epsilon((v-1)s+j)]^2 \qquad (3)$$

where q is limited to assume any real value except *0*. When $q = 0$, we can calculate the fluctuation as $ln[F_0(s)] = \frac{1}{N_s}\sum_{v=1}^{N_s} ln[F_v(s)]$ using l'Hôpital rule.

Equation 4 describes the relationship between s and fluctuation function. One can see that the relationship is governed by the power law,

$$F_q(s) \sim s^{h_{(q)}}. \qquad (4)$$

These multifractals can be understood in terms of (1) the function $\tau(q)$, the mass exponent function and (2) the function $f(\alpha)$, the singularity spectrum, where, q denotes the order of moments and α, the singularity strength. These two terms are related as shown in Equation 5 (the Legendre transform)

$$\alpha = d\tau(q)/dq \qquad (5)$$

and the spectrum $f(\alpha)$, which signifies the fractal dimension, is calculated as shown in Equation 6.

$$f(\alpha) = q\alpha - \tau(q). \qquad (6)$$

The singularity spectrum $f\ (\alpha)$ (also known as multifractal spectrum) and q is the slope of the spectrum.

The $\tau(q)$ function can be expressed in two comparable functions D_q and $H_{(q)}$, where D_q denotes the generalized dimensions and is calculated as shown in Equation 7.

$$D_q = \lim_{q' \to q}\left[\frac{\tau(q')}{q'-1}\right], \qquad (7)$$

The term generalized Hurst exponent, $H(q)$ is defined by Equation 8 shown below.

$$H_{(q)} = \lim_{q' \to q}\left[\frac{(\tau(q')+1)}{q'}\right] \qquad (8)$$

3.2. MF-DCCA

Zhou et al. (2008) proposed multifractaldetrended cross-correlation analysis (MF-DCCA) as detailed below. The method was based on the seminal work of Podobnik and Stanley (2008). The methodology below is based on Jiang et al. (2019) and is elaborated below.

In this method, two series having equal number of observations are considered: {X(i)} and {Y(i)}, where i = 1, 2, ..., N. As with the MF-DFA, in this case also each time series is divided into segments of equal size, *s*, such that they do not overlap. Any two such segments of series can be coupled for each v^{th} box $[l_{v+1}, l_{v+s}]$ and are denoted by $X_v(k)$ and $Y_v(k)$ with $k = 1, \ldots, s$, where $l_v = (v-1)s$. Then $\{X_v(k)\}$ and $\{Y_v(k)\}$ represent the local trend functions of for each segment. The cross-correlation in each segment is then determined as shown in Equation 9

$$F_v^2(s) = \frac{1}{s}\sum_{k=1}^{s}[X_v(k) - \widehat{X_v(k)}][Y_v(k) - \widehat{Y_v(k)}]. \quad (9)$$

Equation 10 represents cross-correlation between two series for the q^{th} order.

$$F_{xy}^2(q,s) = \left[\frac{1}{m}\sum_{v=1}^{m} F_v\,(s)^q\right]^{\frac{1}{q}} \qquad (10)$$

Equation 11 expresses the scaling relation

$$F_{xy}(q,s) \sim sH_{xy}(q) \qquad (11).$$

IV. DATA AND ANALYSIS

4.1. Data

For this study, daily closing prices and volume (number of trades) of Sensex, a market index of Indian stock market, from 13th July1995 – from the day the volume data is available – to 6thAugust2019 – to the date the volume data is updated – is collected. The price and volume data are first transformed into return and volume change using natural logarithm. This series is used for further analysis. Refer Table 4.1 for the summary statistics of the data analyzed in this study.

Table 4.1: Descriptive Statistics of Data

	Price	Volume	Return	volChange

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Number of Observations	5959	5959	5959	5959
Minimum	2600.12	423	-0.11	-5.68
Maximum	40267.62	1166709	0.17	5.29
Median	13137.49	148391	0	0
Mean	14149.49	167627.58	0	0
SE of mean	136.56	1513.2	0	0
CI of mean 0.95	267.7	2966.42	0	0.01
Standard Deviation	10541.44	116810.96	0.01	0.29

Note: SE and CI stand for the Standard Error and Confidence Interval of the mean respectively. Confidence Interval is at 95 percent probability level.

The Figure 4.1a shows the plot of Sensex price series. The plot clearly indicates a trend in the series. The Figure 4.1b shows the plot of Sensex volume series. The series displays volatility and exhibits instances of clustering. The Figure 4.2a shows the plot of Sensex Return series and the Figure 4.2b shows the plot of Sensex Volume Change. Both the returns and volume show clear sign of volatility clustering, but the return series is more volatile.

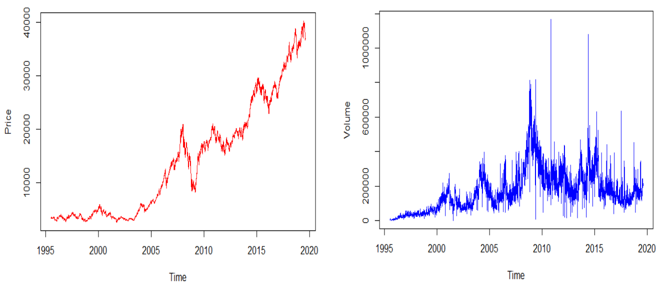

Figure 4.1 Plot of Sensex Price and Volume from 1995 to 2019

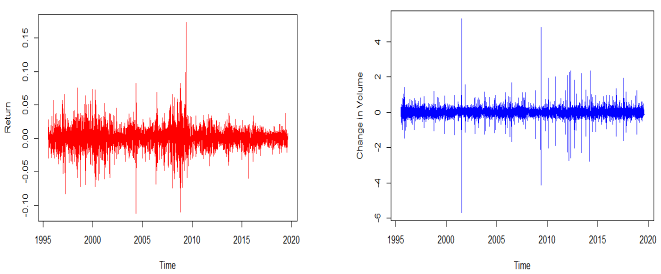

Figure 4.2IV Plot of Sensex Return and Volume Change from 1995 to 2019

4.2. Analysis

The DFA requires that the series under study is a random walk (Peng et al., 1995). Therefore, the time series is transformed into a random walk before the test is applied. Since both return and volume change under study are noise like structures, they are converted into random walks by deducting the mean and then integrating the series. Next, the values of scale, the order of moments (q) and m are set as required in MFDFA. The minimum size of the segment is taken to be 10 as a 'rule of thumb', as in smaller segments a trend will be over-fitted and there is a chance of error in computation of local fluctuation using root mean square (RMS). A choice between -10 to 10 is made for q, and m is set at 1. The output of the analysis is explained with Hurst exponent and multifractal spectrum, the interpretation of which is explained below.

4.2.1 Interpretation of Hurst exponent

For a series having multifractality, $H(q)$ declines as q increases. Alternatively, a nonlinear relationship between $\tau(q)$ and q indicates that the time series has a multifractal nature. For a series having long-memory, the fluctuation function ($F_q(n)$) will increase with n.

Table 4.2 explains the interpretation of Hurst exponent values.

4.2.2 The interpretation of multifractal spectrum

The presence of a multifractal structure in a series indicates that the series has long-range correlations exhibited on different intrinsic time scales. The multifractal spectrum reveals this multiscaling structure. Computationally, the multifractal spectrum denotes the aberrations in fractal structure over period. Kantelhardt et al. (2002) corroborate that $h(q)$ represent small fluctuations when q is positive and represent large fluctuations when q is negative. Thus, right and left side of the spectrum denote small and large fluctuations respectively.

If the spectrum is left shifted from $\alpha = 0.5$, the series is anti-persistent. For example, if left side of the spectrum (which describe large fluctuations) are left shifted from $\alpha = 0.5$, then the large fluctuations are anti-persistent.

Table 4.2: Interpretation of Hurst Exponent

Hurst Exponent Value	Interpretation(s)
$0 \leq H < 0.5$	The data are fractal and therefore, the EMH cannot be corroborated. The series are anti-persistent (or mean-reverting). It also means that returns are negatively correlated. Such a market is riskier for individual participants. It denotes anti-persistence or negative long-memory, which indicates that the changes in this kind of series is significantly different from that can be observed in a random series. It also signifies a change of a trend.
$H = 0.5$	The data are random which corroborates the EMH. In other words, prices move randomly and have no memory. In such a situation, traders cannot beat the market.
$0.5 < H \leq 1$	The series is fractal and persistent. The EMH cannot be supported. Returns have long memory.

Note: Collated by authors from various papers quoted in this study.

V. RESULTS

The entire analysis was done using tidyverse package (Wickham et al., 2019), MFDFA package (Laib et al., 2019) in software R (R Core Team, 2019), and R studio (RStudio Team, 2019). The Figure 5.1 shows the multifractal properties of the price series and the legend in Table 5.1 can be used to read the plots.

The plot of Fluctuation function f_q in Figure 5.1 displays that $\log_2 F(q)$ increases for large

Table 5.1: Legend to read Spectrum

Variable	Description
$F_q(n)$	Fluctuation function denotes change in the fluctuation of a series for different orders across different time segments.
$H(q)$	Scaling exponent. Scaling exponent is the slope of the regression fit of log version of $F_q(n)$ against n.
$\tau(q)$	Multifractal exponents = $qh(q) - 1$. These exponents are nonlinear if the series has multifractal structure.
$f(\alpha)$	Singularity spectrum. It represents fractality dimension of subperiod of the series.

Note: n is non-overlapping segment length, q is the order of fluctuation function and α is singularity strength.

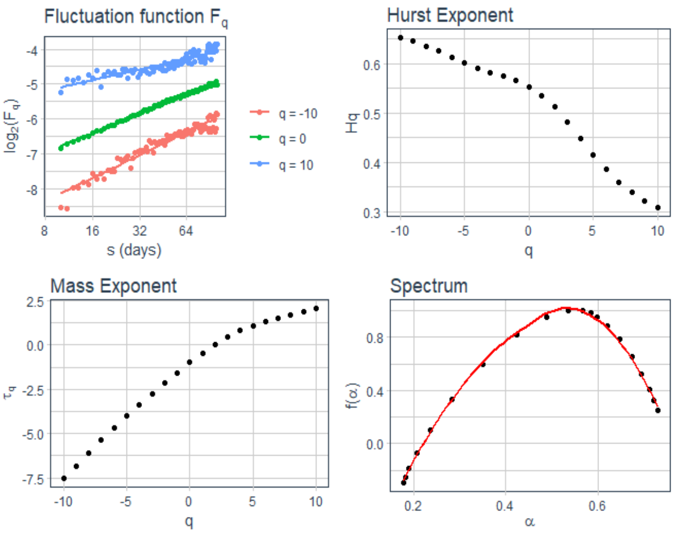

Figure 5.1: Plot of Price Spectrum

value of s, demonstrating an existence of long-range power law correlation in the series (Refer Equation 4). The plot of H_q versus q shows a decreasing trend indicating multifractality of the series. The multifractality of the series is also confirmed by the non-linear relationship between mass exponent τ_q and q.

The spectrum shows that small fluctuations are persistent or having positive long memory. The right side of the spectrum indicates the small fluctuations, meaning an increase in return will probably be observed after an increase or vice versa. The large fluctuations are anti-persistent (or mean reverting) meaning that series will move toward mean in the long-term.

Figure 5.2 shows the multifractal properties of the volume series.

The plot of Fluctuation function fq in Figure 5.2 shows that $log_2F(q)$ increases with s, implying the series having long-range power law correlation (Refer Equation 4). The plot of H_q versus q shows a decreasing trend confirming the multifractality of the series. The multifractality of the series is also confirmed by the non-linear relationship between

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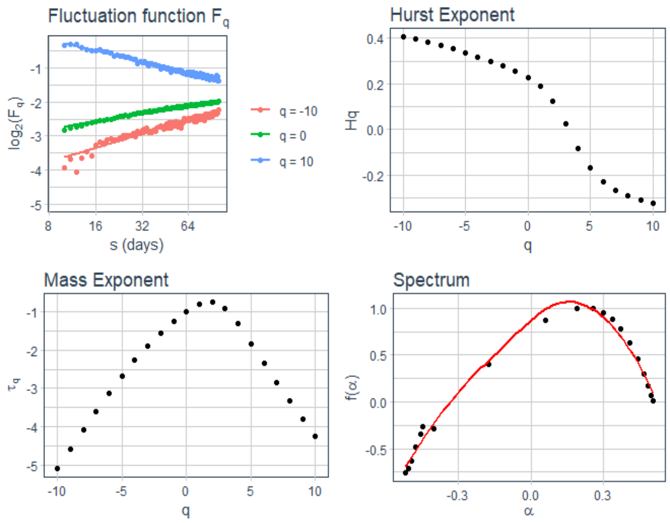

Figure 5.2: Plot of Volume Spectrum

mass exponent τ_q and q.

The spectrum shows that small and large fluctuations are anti-persistent implying large (small) changes are most probability will be followed by small (large) changes.

Figure 5.3 shows multifractal properties of both price and volume series on a single plot for comparison. One can observe that volume series is far left shifted from α = 0.5 and wider than the price series indicating that the volume series is much more anti-persistent and far from efficiency.

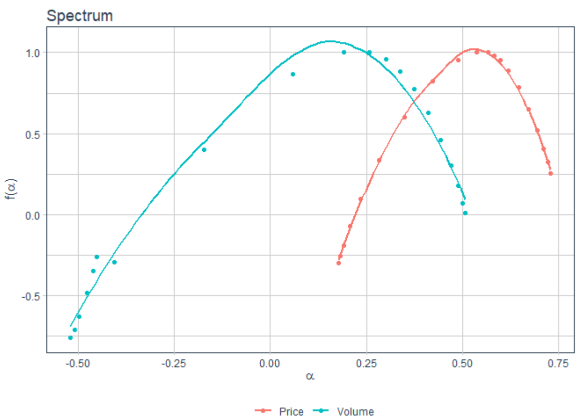

Figure 5.3: Plot of Price and Volume Spectrum

Figure 5.4 shows the price-volume relationship is multifractal in nature. The spectrum of price-volume cross correlations is far left shifted from the α = 0.5 indicating anti-persistency in price series.

Figure 5.5 shows the plot of price, volume and price-volume cross-correlation spectrum on a single plot for comparison.

The plot clearly shows that the cross-correlation between price and volume is wider and much more left shifted from α = 0.5 compared to that of price or volume spectrum. The price-volume cross correlations are clearly showing negative cross-correlations meaning that they have long memory.

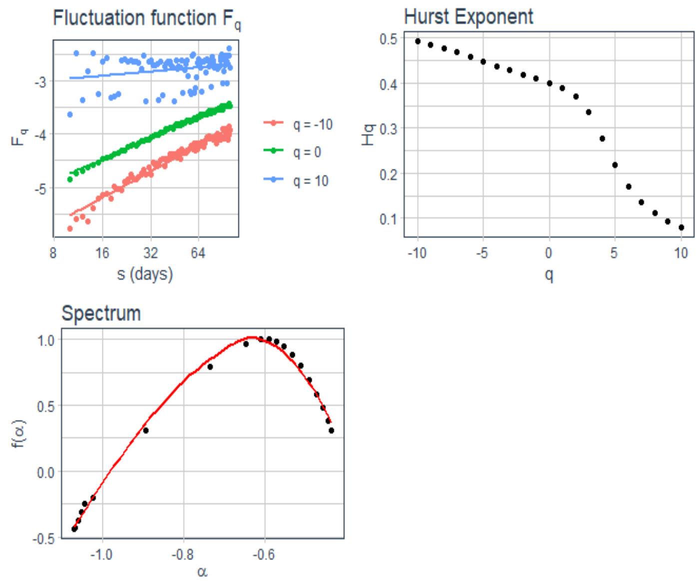

Figure 5.4: Plot of Price-Volume Cross-correlation Spectrum

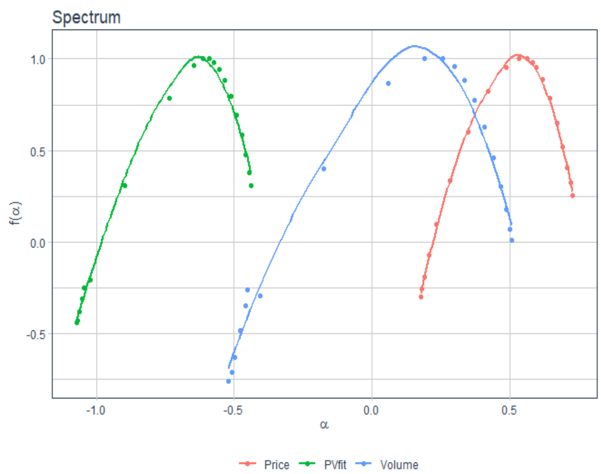

Figure 5.5: Plot of Price, Volume and Price-volume Cross-correlation Spectrum

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VI. DISCUSSION

The findings of this paper in price-volume cross-correlation contradict with that of Podobnik et al. (2009) in showing that Indian stock market indicate the cross-correlation between price and volume. The results agree with the studies of Ferreira (2019); Hasan and Salim (2017); El Alaoui (2017); Wang et al. (2009); He and Chen (2011); Ruan et al. (2016). The results in the return series concur with the findings of Tiwari et al. (2019); Hiremath and Narayan (2016) and contradict with that of Anagnostidis et al. (2016) and partially agree with Hiremath and Kumari (2014); Horta et al. (2014). In general, the results of this paper tend to agree if the study is conducted in the developing regions of the world and disagree otherwise.

The results of this study confirm that the Indian markets have long-memory and chances of arbitrage arise over period as explained by the Hurst exponent. The volume carries information regarding the information and can be used as an informative statistic in predicting the price series as indicated by the spectrum graphs. Lastly, the results corroborate the implications of the AMH in that arbitrage chances arise from time to time. This paper uniquely links the relationship between price-volume to the implications of the AMH using MF-DFA and MF-DCCA methods. The results of this study will help traders to continue using trading strategies as the markets are inefficient. The future scope of research in this domain will be to identify the time periods in which volume carries more information about the price. This may have implications for the policy makers of a country.

VII. CONCLUSION

This paper conducts multifractal analysis of price, volume and price-volume relationship for the data between 1995 to 2019. It is found that price, volume and their cross-correlation show multifractal properties and have long memory. The price series shows both persistent and anti-persistent behavior, whereas volume and price-volume cross-correlation shows anti-persistent behaviors. The long-memory in price-volume relationship is more prominent than that of price and volume series alone. The long memory in the series indicate inefficiency of the market and chances for arbitrage exist in this market. The varying Hurst exponents at different time scales indicate that arbitrage opportunities show up over period, and hence supports the implications of the AMH. This paper confirms the nonlinear cross-correlation between the price and volume series.

REFERENCES

[1] Acharya, V.V. and Pedersen, L.H. (2005), “Asset pricing with liquidity risk”, Journal of Financial Economics, Vol. 77, No. 2, pp. 375–410.

[2] Admati, A.R. and Pfleiderer, P. (1988), “A theory of intraday patterns: volume and price variability”, Review of Financial Studies, Vol. 1, No. 1, pp. 3–40.

[3] Al-Khazali, O. and Mirzaei, A. (2017), “Stock market anomalies, market efficiency and the adaptive market hypothesis: evidence from Islamic stock indices”, Journal of International Financial Markets, Institutions and Money, Vol. 51, pp. 190–208, doi:10.1016/j.intfin.2017.10.001.

[4] Anagnostidis, P., Varsakelis, C., and Emmanouilides, C.J. (2016), “Has the 2008 financial crisis affected stock market efficiency? the case of eurozone”, Physica A: Statistical Mechanics and its Applications, Vol. 447, pp. 116–128.

[5] Arshad, S. and Rizvi, S.A.R. (2015), “The troika of business cycle, efficiency and volatility. An east Asian perspective”, Physica A: Statistical Mechanics and its Applications, Vol. 419, pp. 158–170.

[6] Assogbavi, T., Khoury, N., and Yourougou, P. (1995), “Short interest and the asymmetry of the price-volume relationship in the Canadian stock market”, Journal of Banking and Finance, Vol. 19, No. 8, pp. 1341–1358.

May – June 2020
ISSN: 0193-4120 Page No. 16646 – 16660

[7] Bacry, E., Delour, J., and Muzy, J.F. (2001), "Modelling financial time series using multifractal random walks", Physica A: Statistical Mechanics and its Applications, Vol. 299, No. 1–2, pp. 84–92.

[8] Bariviera, A.F. (2017), "The inefficiency of bitcoin revisited: A dynamic approach", Economics Letters, Vol. 161, pp. 1–4.

[9] Blume, L., Easley, D., and O'hara, M. (1994), "Market statistics and technical analysis: The role of volume", The Journal of Finance, Vol. 49, No. 1, pp. 153–181.

[10] Bolgorian, M. and Gharli, Z. (2011), "A multifractaldetrended fluctuation analysis of gold price fluctuations", ActaPhysicaPolonica B, Vol. 42, No. 1, pp. 159–169.

[11] Cajueiro, D.O. and Tabak, B.M. (2004), "The hurst exponent over time: testing the assertion that emerging markets are becoming more efficient", Physica A: Statistical Mechanics and its Applications, Vol. 336, No. 3-4, pp. 521–537.

[12] Cajueiro, D.O. and Tabak, B.M. (2005), "Ranking efficiency for emerging equity markets ii", Chaos, Solitons& Fractals, Vol. 23, No. 2, pp. 671–675.

[13] Caporale, G.M., Gil-Alana, L., and Plastun, A. (2018), "Is market fear persistent? a long memory analysis", Finance Research Letters, Vol. 27, pp. 140–147.

[14] Chen, G.M., Firth, M., and Rui, O. (2001), "The dynamic relation between stock returns, trading volume, and volatility", Financial Review, Vol. 36, No. 3, pp. 153–174.

[15] Clark, P.K. (1973), "A subordinated stochastic process model with finite variance for speculative prices", Econometrica, Vol. 41, No. 1, pp. 135–155.

[16] Cont, R. (2001), "Empirical properties of asset returns: Stylized facts and statistical issues", Quantitative Finance, Vol. 1, No. 2, pp. 223–236, doi:10.1080/713665670.

[17] Copeland, T.E. (1976), "A model of asset trading under the assumption of sequential information arrival", The Journal of Finance, Vol. 31, No. 4, pp. 1149–1168.

[18] Crouch, R.L. (1970), "The volume of transactions and price changes on the New York stock exchange", Financial Analysts Journal, Vol. 26, No. 4, pp. 104–109.

[19] Di Matteo, T., Aste, T., and Dacorogna, M.M. (2003), "Scaling behaviors in differently developed markets", Physica A: Statistical Mechanics and its Applications, Vol. 324, No. 1-2, pp. 183–188.

[20] Di Matteo, T., Aste, T., and Dacorogna, M.M. (2005), "Long-term memories of developed and emerging markets: Using the scaling analysis to characterize their stage of development", Journal of Banking & Finance, Vol. 29, No. 4, pp. 827–851.

[21] Dutta, S. (2010), "Multifractaldetrended fluctuation analysis of Sensex fluctuation in the Indian stock market", Canadian Journal of Physics, Vol. 88, No. 8, pp. 545–551.

[22] Dutta, S., Ghosh, D., and Chatterjee, S. (2016), "Multifractaldetrended cross correlation analysis of foreign exchange and Sensex fluctuation in Indian perspective", Physica A: statistical mechanics and its applications, Vol. 463, pp. 188–201.

[23] El Alaoui, M. (2017), "Price–volume multifractal analysis of the Moroccan stock market", Physica A: Statistical Mechanics and its Applications, Vol. 486, pp. 473–485.

[24] Eom, C., Oh, G., and Kim, S. (2008), "Statistical investigation of connected structures of stock networks in financial time series", Journal of the Korean physical society, Vol. 53, pp. 3837–3841.

[25] Epps, T.W. and Epps, M.L. (1976), "The stochastic dependence of security price changes and

[26] transaction volumes: Implications for the mixture-of-distributions hypothesis", Econometrica, Vol. 44, No. 2, pp. 305–321.

[27] Fama, E.F. (1970), "Efficient capital markets: A review of theory and empirical work", The

May – June 2020
ISSN: 0193-4120 Page No. 16646 – 16660

Journal of Finance, Vol. 25, No. 2, pp. 383–417.URL: http://www.jstor.org/stable/2325486

[28] Fama, E.F. (1991), “Efficient capital markets: Ii”, The Journal of Finance, Vol. 46, No. 5, pp. 1575–1617.

[29] Fan, Q. and Li, D. (2015), “Multifractal cross-correlation analysis in electricity spot market”, Physica A: Statistical Mechanics and its Applications, Vol. 429, pp. 17–27.

[30] Ferreira, P. (2019), “Assessing the relationship between dependence and volume in stock markets: A dynamic analysis”, Physica A: Statistical Mechanics and its Applications, Vol. 516, pp. 90–97.

[31] Ferreira, P., Dionísio, A., and Movahed, S. (2017), “Assessment of 48 stock markets using adaptive multifractal approach”, Physica A: Statistical Mechanics and its Applications, Vol. 486, pp. 730–750.

[32] Ghosh, S., Manimaran, P., and Panigrahi, P.K. (2011), “Characterizing multi-scale self-similarbehavior and non-statistical properties of fluctuations in financial time series”, Physica A: Statistical Mechanics and its Applications, Vol. 390, No. 23-24, pp. 4304–4316.

[33] Han, C., Wang, Y., and Xu, Y. (2019), “Efficiency and multifractality analysis of the Chinese stock market: Evidence from stock indices before and after the 2015 stock market crash”, Sustainability, Vol. 11, No. 1699, pp. 1–15.

[34] Harris, L. (1987), “Transaction data tests of the mixture of distributions hypothesis”, Journal of Financial and Quantitative Analysis, Vol. 22, No. 2, pp. 127–141.

[35] Harris, M. and Raviv, A. (1993), “Differences of opinion make a horse race”, Review of Financial studies, Vol. 6, No. 3, pp. 473–506.

[36] Hasan, R. and Salim, M.M. (2017), “Power law cross-correlations between price change and volume change of Indian stocks”, Physica A: Statistical Mechanics and its Applications, Vol. 473, pp. 620–631.

[37] He, L.Y. and Chen, S.P. (2011), “Nonlinear bivariate dependency of price–volume relationships in agricultural commodity futures markets: a perspective from multifractaldetrended cross correlation analysis”, Physica A: Statistical Mechanics and its Applications, Vol. 390, No. 2, pp. 297–308.

[38] Henry, T. and McKenzie, M. (2006), “The impact of short selling on the price-volume relationship: Evidence from Hong Kong”, Journal of Business, Vol. 79, No. 2, pp. 671–691.

[39] Hiremath, G. and Narayan, S. (2016), “Testing the adaptive market hypothesis and its determinants for the Indian stock markets”, Finance Research Letters, Vol. 19, pp. 173–180,doi:10.1016/j.frl.2016.07.009.

[40] Hiremath, G.S. and Kumari, J. (2014), “Stock returns predictability and the adaptive market hypothesis in emerging markets: evidence from India”, Springer Plus, Vol. 3, No. 428, pp. 1–14.

[41] Horta, P., Lagoa, S., and Martins, L. (2014), “The impact of the 2008 and 2010 financial crises on the hurst exponents of international stock markets: Implications for efficiency and contagion”, International Review of Financial Analysis, Vol. 35, pp. 140–153.

[42] Hurst, H.E. (1951), “Long-term storage capacity of reservoirs”, Trans AmerSoc Civil Eng, Vol. 116, pp. 770–799.

[43] Jennings, R.H. and Barry, C.B. (1983), “Information dissemination and portfolio choice”, Journal of Financial and Quantitative Analysis, Vol. 18, No. 1, pp. 1–19.

[44] Jiang, Z.Q., Xie, W.J., Zhou, W.X. and Sornette, D. (2019), “Multifractal analysis of financial markets: a review”, Reports on Progress in Physics, Vol. 82, No. 12, p.125901.

[45] Kantelhardt, J.W., Zschiegner, S.A., Koscielny-Bunde, E., Havlin, S., Bunde, A., and Stanley, H.E. (2002),

May – June 2020
ISSN: 0193-4120 Page No. 16646 – 16660

"Multifractaldetrended fluctuation analysis of nonstationarytime series", Physica A: Statistical Mechanics and its Applications, Vol. 316, No. 1-4, pp. 87–114.
[46] Karpoff, J.M. (1986), "A theory of trading volume", The Journal of Finance, Vol. 41, No. 5, pp. 1069–1087.
[47] Karpoff, J.M. (1987), "The relation between price changes and trading volume: A survey", Journal of Financial and Quantitative Analysis, Vol. 22, No. 1, pp. 109–126.
[48] Kristoufek, L. (2012), "Fractal markets hypothesis and the global financial crisis: Scaling, investment horizons and liquidity", Advances in Complex Systems, Vol. 15, No. 06, p. 1250065.
[49] Kristoufek, L. and Vosvrda, M. (2013), "Measuring capital market efficiency: Global and local correlations structure", Physica A: Statistical Mechanics and its Applications, Vol. 392, No. 1, pp. 184–193.
[50] Kwapien, J., Oswi ̨ecimka, P., and Drozdz, S. (2015), "Detrended fluctuation analysis made flexible to detect range of cross-correlated fluctuations", Physical Review E, Vol. 92, No. 5, p. 052815.
[51] Kyle, A.S. (1985), "Continuous auctions and insider trading", Econometrica: Journal of the Econometric Society, Vol. 53, No. 6, pp. 1315–1335.
[52] Laib, M., Telesca, L., and Kanevski, M. (2019), MFDFA: MultiFractalDetrended Fluctuation Analysis, r package version 1.1.URL: https://CRAN.R-project.org/package=MFDFA
[53] Lamoureux, C.G. and Lastrapes, W.D. (1990), "Heteroskedasticity in stock return data: volume versus garch effects", The journal of finance, Vol. 45, No. 1, pp. 221–229.
[54] Lo, A.W. (2004), "The adaptive markets hypothesis", The Journal of Portfolio Management, Vol. 30, No. 5, pp. 15–29.
[55] Mandelbrot, B. (1971), "Analysis of long-run dependence in economics: the r s technique", Econometrica, Vol. 39, No. 4.
[56] Mandelbrot, B.B. (1982), "The fractal geometry of nature. 1982", San Francisco, CA.
[57] Mandelbrot, B.B., Fisher, A.J., and Calvet, L.E. (1997), "A multifractal model of asset returns",Cowles Foundation Discussion Paper 1164, Sauder School of Business Working Paper.URL: https://ssrn.com/abstract=78588
[58] Mandelbrot, B.B. and Van Ness, J.W. (1968), "Fractional Brownian motions, fractional noises and applications", SIAM review, Vol. 10, No. 4, pp. 422–437.
[59] Manimaran, P., Panigrahi, P.K., and Parikh, J.C. (2005), "Wavelet analysis and scaling properties of time series", Physical Review E, Vol. 72, No. 4, p. 046120.
[60] Morales, R., Di Matteo, T., Gramatica, R., and Aste, T. (2012), "Dynamical generalized hurst exponent as a tool to monitor unstable periods in financial time series", Physica A: statistical mechanics and its applications, Vol. 391, No. 11, pp. 3180–3189.
[61] Morse, D. (1980), "Asymmetrical information in securities markets and trading volume", Journal of Financial and Quantitative Analysis, Vol. 15, No. 05, pp. 1129–1148.
[62] Noda, A. (2016), "A test of the adaptive market hypothesis using a time-varying AR model in Japan", Finance Research Letters, Vol. 17, pp. 66–71, doi:10.1016/j.frl.2016.01.004.
[63] Peng, C.K., Havlin, S., Stanley, H.E., and Goldberger, A.L. (1995), "Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series", Chaos: An Interdisciplinary Journal of Nonlinear Science, Vol. 5, No. 1, pp. 82–87.
[64] Podobnik, B., Horvatic, D., Petersen, A.M., and Stanley, H.E. (2009), "Cross-correlations between volume change and price change", Proceedings of the National Academy of Sciences, Vol. 106, No. 52, pp. 22079–22084.

May – June 2020
ISSN: 0193-4120 Page No. 16646 – 16660

[65] Podobnik, B. and Stanley, H.E. (2008), "Detrended cross-correlation analysis: a new method for analyzing two nonstationary time series", Physical review letters, Vol. 100, No. 8, p. 084102.

[66] R Core Team (2019), R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria.URL: https://www.R-project.org/

[67] Rak, R., Drozdz, S., Kwapien, J., and Oswiecimka, P. (2015), "Detrended cross-correlations between returns, volatility, trading activity, and volume traded for the stock market companies",EPL (Europhysics Letters), Vol. 112, No. 4, p. 48001.

[68] Razdan, A. (2002), "Scaling in the Bombay stock exchange index", Pramana, Vol. 58, No. 3, pp. 537–544.

[69] Rizvi, S.A.R., Dewandaru, G., Bacha, O.I., and Masih, M. (2014), "An analysis of stock marketefficiency: Developed vs Islamic stock markets using mf-dfa", Physica A: Statistical Mechanics and its Applications, Vol. 407, pp. 86–99.

[70] RStudio Team (2019), RStudio: Integrated Development Environment for R, RStudio, Inc.,Boston, MA.URL: http://www.rstudio.com/

[71] Ruan, Q., Jiang, W., and Ma, G. (2016), "Cross-correlations between price and volume in Chinese gold markets", Physica A: Statistical Mechanics and its Applications, Vol. 451, pp. 10–22.

[72] Sensoy, A. and Tabak, B.M. (2015), "Time-varying long-term memory in the European unionstock markets", Physica A: Statistical Mechanics and its Applications, Vol. 436, pp. 147–158.

[73] Shah, A. (1999), "Institutional change in India's capital markets", Economic and Political Weekly, Vol. XXXIV, No. 3-4, pp. 183–194.

[74] Stošic, D., Stošic, D., Stošic, T., and Stanley, H.E. (2015), "Multifractal properties of pricechange and volume change of stock market indices", Physica A: Statistical Mechanics and itsApplications, Vol. 428, pp. 46–51.

[75] Sukpitak, J. and Hengpunya, V. (2016), "The influence of trading volume on market efficiency: The dcca approach", Physica A: Statistical Mechanics and its Applications, Vol. 458, pp. 259–265.

[76] Suominen, M. (2001), "Trading volume and information revelation in stock market", Journal of Financial and Quantitative Analysis, Vol. 36, No. 04, pp. 545–565.

[77] Tauchen, G.E. and Pitts, M. (1983), "The price variability-volume relationship on speculative markets", Econometrica: Journal of the Econometric Society, Vol. 51, No. 2, pp. 485–505.

[78] Tiwari, A.K., Aye, G.C., and Gupta, R. (2019), "Stock market efficiency analysis using long spans of data: A multifractaldetrended fluctuation approach", Finance Research Letters, Vol. 28, pp. 398–411.

[79] Tuyon, J. and Ahmad, Z. (2016), "Behavioural finance perspectives on Malaysian stock market efficiency", Borsa Istanbul Review, Vol. 16, No. 1, pp. 43–61.

[80] Urquhart, A. and Hudson, R. (2013), "Efficient or adaptive markets? evidence from major stock markets using very long run historic data", International Review of Financial Analysis, Vol. 28, pp. 130–142, doi:10.1016/j.irfa.2013.03.005.

[81] Urquhart, A. and McGroarty, F. (2016), "Are stock markets really efficient? evidence of the adaptive market hypothesis", International Review of Financial Analysis, Vol. 47, pp. 39–49, doi:10.1016/j.irfa.2016.06.011.

[82] Varian, H.R. (1985), "Price discrimination and social welfare", The American Economic Review, Vol. 75, No. 4, pp. 870–875.

[83] Wang, D.H., Suo, Y.Y., Yu, X.W., and Lei, M. (2013), "Price–volume cross-correlation analysis of csi300 index futures", Physica A:

May – June 2020
ISSN: 0193-4120 Page No. 16646 – 16660

Statistical Mechanics and its Applications, Vol. 392, No. 5, pp. 1172–1179.

[84] Wang, Y., Liu, L., and Gu, R. (2009), "Analysis of efficiency for Shenzhen stock market based on multifractaldetrended fluctuation analysis", International Review of Financial Analysis, Vol. 18, No. 5, pp. 271–276.

[85] Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L.D., François, R., Grolemund, G., Hayes, A., Henry, L., Hester, J., Kuhn, M., Pedersen, T.L., Miller, E., Bache, S.M., Müller, K., Ooms, J., Robinson, D., Seidel, D.P., Spinu, V., Takahashi, K., Vaughan, D., Wilke, C., Woo, K., and Yutani, H. (2019), "Welcome to the tidyverse", Journal of Open Source Software, Vol. 4, No. 43, p. 1686, doi:10.21105/joss.01686.

[86] Yuan, Y., Zhuang, X.t., and Liu, Z.y. (2012), "Price–volume multifractal analysis and its application in Chinese stock markets", Physica A: Statistical Mechanics and its Applications, Vol. 391, No. 12, pp. 3484–3495.

[87] Zhou, W.X. et al. (2008), "Multifractaldetrended cross-correlation analysis for two nonstationary signals", Physical Review E, Vol. 77, No. 6, p. 066211.

[88] Zunino, L., Tabak, B.M., Figliola, A., Pérez, D., Garavaglia, M., and Rosso, O. (2008), "A multifractal approach for stock market inefficiency", Physica A: Statistical Mechanics and its Applications, Vol. 387, No. 26, pp. 6558–6566.

[89] Zunino, L., Zanin, M., Tabak, B.M., Pérez, D.G., and Rosso, O.A. (2009), "Forbidden patterns, permutation entropy and stock market inefficiency", Physica A: Statistical Mechanics and its Applications, Vol. 388, No. 14, pp. 2854–2864.

[90] Jiang, Z.Q., Xie, W.J., Zhou, W.X. and Sornette, D. (2019), "Multifractal analysis of financial markets: a review", Reports on Progress in Physics, Vol. 82, No. 12, p.125901.